Financial Stabilisation and Interest Rate Modelling in the United Kingdom

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DECLARATION

I declare that the thesis submitted is my own work and has not been submitted for another degree, either at University College Cork or elsewhere.

Signed by_______________

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INTRODUCTION

This Ph.D. thesis models and explains the movement of the short-term interest rate in the United Kingdom (UK) using statistical models building on economic theories. It investigates three main areas and makes a number of contributions to the existing monetary policy literature in the UK. Firstly, the thesis investigates alternative methods to construct financial conditions indices (FCIs) and creates an optimal FCI for the UK. The estimated optimal FCI is the one that best predicts economic activity in the UK. Secondly, this study is the first in the literature to test the existence of a short-term inflation target for an inflation-targeting central bank like the BOE. Thirdly, this thesis investigates the most appropriate methodology for modelling the short-term interest rate in the UK using new data and a new estimator.

Review of the Monetary Policy in the UK

The Bank of England's (BOE) monetary policy objective is to deliver price stability – low inflation – and subject to that, to support the Government's economic objectives including those for growth and employment. Currently, price stability is defined as 2% year-on-year increase in the Consumer Price Index (CPI). The BOE seeks to meet its inflation target and to support economic development by adjusting the short-term interest rate. The level of the interest rate is decided by the Monetary Policy Committee (MPC). However, the Bank provides little information on how they set the interest rate. As disclosed in the 'Monetary Policy Trade-offs and Forward Guidance' (MPC, August 2013), the MPC acknowledges that it considers multiple inflation and economic activity indicators while setting the short-term interest rate. This makes the process of setting the interest rate quite complicated and lacking transparency and hence creates much difficulty for investors and researchers in attempting to understand and predict the interest rate.

The BOE's inflation target has changed twice since its introduction in October 1992. Initially, price stability was defined as a retail price index (RPI) inflation range of 1-4% for one year. In mid-1995, a specific target was introduced for the RPI inflation rate of 2.5%. In November 2003, the measure of inflation was changed to the CPI and the target was restated as 2% inflation in the CPI. The BOE stresses that it attempts to meet its target over the medium term. It does not attempt to keep the inflation rate at the published target at all times, because this may cause undesirable volatility in economic activity (see, MPC, March 2013). This raises a question as to whether the BOE has an implicit and unpublished inflation target that is based on the Bank's short-term consideration.

Literature Review and Motivation of Study

In the existing literature, there is a consensus that understanding changes in the interest rate should provide benefit to financial market participants. As demonstrated by the MPC (June 2012), the official interest rate is at the starting point of the monetary transmission mechanism. Changes in the level of the official interest rate will be immediately transmitted to other short-term wholesale money market rates including the mortgage rate and the bank deposit rate. Although these rates may not always move by the exact amount of the official rate change, they are highly correlated with a central bank's official short-term interest rate. The MPC (June 2012) also emphasises that in the first round of the monetary transmission mechanism both the interest rate policy actions and announcements affect the money market rate and the confidence of consumers and investors, as well as affecting the exchange rate and asset prices. In the second round, the resulting changes in financial markets affect spending, savings and the investment behaviour of both individuals and firms, which leads to a change in aggregate demand and output that in turn impacts on the inflation rate. Thus, accurate interest rate forecasting is considered important for predicting changes in financial markets and other macroeconomic indicators. Accurate interest rate modelling should improve interest rate forecasting.

Given the above benefits of interest rate modelling and the complicated and non-transparent decision-making process within central banks, economic studies have an ongoing interest in modelling the interest rate. For example, Taylor (1993) found that the federal funds rate in the United States (US) could be modelled as a function of inflation and output only for the period 1987-1992. This relationship became known as the Taylor rule. Prior to the Taylor (1993) rule, Friedman (1983), McCallum (1988) and Wicksell (1898, 1993) had developed several monetary policy rules, but these did not feature in discussions on monetary policy setting. One explanation is that those rules fail to match either the instruments used by central banks or the stylised facts about the response of policy to economic shocks.

Drawing on extensive readings, Beechey and Osterholm (2012) conclude that when estimating the Taylor rule economic studies may have different objectives. The first strand attempts to obtain the inflation target from inflation data (for instance, Ireland, 2007). The second strand aims to examine whether a central bank works to stabilise its inflation rate around the targeting value as indicated by Taylor (1993). Following Taylor (1993), empirical studies have tried to use the Taylor rule to describe and explain interest rate changes in various countries. However, there is still debate on how to empirically model the short-term interest rate.

There is no consensus in the monetary literature about the appropriate way to model the short-term interest rate. Several empirical studies (for instance, Clarida, Gali and Gertler, 1998, 2000; Chadha, Sarno and Valente, 2004; Ireland, 2007; Castro, 2011) assume that a central bank minimises a symmetric quadratic loss function and therefore employs a linear estimator. Ireland (2007) uses the Maximum Likelihood to estimate the implicit inflation target in the US (the first strand). Castro (2011) uses the Generalised Method of Moments and discovers that monetary authorities promote inflation and output stabilisation in the Eurozone, US and UK (the second strand).

Furthermore, there are studies (such as Kuzin, 2006; Trecroci and Vassalli, 2010, Nakajima, 2011a, 2011b) arguing that the estimators used in the aforementioned literature like Ireland (2007) and Castro (2011) are not suitable. This is because central banks may change their monetary policy implementation over time. Thus, Primiceri (2005), Trecroci and Vassalli (2010) and Nakajima (2011a, 2011b) advocate using time-varying parameter (TVP) estimators. Some complicated time-varying parameter estimators are developed and applied to the US data but have not been used for the UK as yet. Therefore, it is particularly interesting to examine whether the interest rate in the UK can be modelled well with a linear estimator or a time-varying parameter estimator. This study uses a time-varying parameter VAR with stochastic volatility (TVP-VAR-SV) model as a TVP estimator which not only considers changes in policy implementation but also takes stochastic volatility into account.

In addition to the debate on choosing the appropriate estimator, there is no universally-agreed conclusion about the accepted version of the Taylor rule in literature. Some empirical analysis supports the use of the initial Taylor rule that models the interest rate in response to changes in the inflation rate and the output gap (see, the US Fed in Castro, 2011), while others (see, the US Fed in Clarida et al., 2000) argue that central banks should take into account all available information and adjust the interest rate on the basis of the expected inflation rate and the expected output gap instead of their historical levels.

More recently, both theoretical and empirical studies (see, for instance, Clarida et al., 1998; Goodhart and Hofmann, 2002; Corsetti and Pesenti, 2005; Teranishi, 2012) discover that the initial Taylor rule focusing on stabilising inflation and output around their targeting values may not be sufficient to model interest rates in some countries. An augmented Taylor rule that considers financial conditions is required. To examine the reaction of the interest rate to changes in the domestic financial markets, a traditional method in the literature is to do a preliminary exercise by constructing a financial conditions index which summarises all financial information in the market. However, the index constituents and construction methodologies of FCIs (see, for instance, Castro, 2011; Martin and Milas, 2013; Koop and Korobilis, 2014) differ in each study. There is no universally-agreed method in the FCI literature for selecting the combination of financial constituents in an FCI and the weight attached to each indicator. Because of the different estimation methods and financial constituents involved, the obtained FCIs differ from each other in the literature. This leads to opposite conclusions about whether a central bank is changing its monetary policy in response to financial market conditions for the same country examined. In the case of the UK, Castro (2011) estimates a Taylor rule and concludes that the BOE does not react to changes in his estimated FCI. Martin and Milas (2013) investigate the response of the BOE to FCIs between 1992 and 2010. They discover that the response parameter on the FCI is not significant for the pre-2007 period but turns out to be significant after 2007.

Motivated by the above divergence in the monetary literature regarding the choice of an appropriate version of Taylor rule and the estimator, this thesis investigates three main areas and makes three principal contributions (as mentioned at the starting of this section). Firstly, it produces an optimal FCI for the UK using the best weighting method and the best combination of index constituents. Secondly, it is the first in the literature to test the existence of a short-term inflation target for an inflation-targeting central bank such as the BOE. Thirdly, this thesis models the interest rate in the UK using new measures of inflation expectations, the output gap and financial conditions. It also investigates the most appropriate method for modelling the UK interest rate using a new estimator.

Organisation of Study

The first area this thesis addresses is the optimal estimate of an FCI for the UK. This includes Chapter 1 and 2. The estimated FCI is then used in other chapters (Chapter 4 and 5) to test whether the BOE adjusts the interest rate in response to changes in the domestic financial market. The most important assumption in estimating the optimal FCI is that the best FCI predicts economic activity as well as possible. This is derived from the monetary transmission mechanism.

There are three choices in the FCI construction: variable inclusion, variable weighting and index rebalancing. The first choice discusses which financial variables should be included at the time of creating an FCI. The second one selects the optimal method for index weighting. The third focuses on further adjustments (changing the composition of the FCI over time) in order to correctly track a specific financial market. The third choice distinguishes itself from the first by studying (i) whether any new constituents should be included in the index and (ii) whether there are any existing variables that should be removed out of the index at each point in time.

Chapter 1 investigates the optimal variable-weighting method. It selects a small number of indicators of financial conditions and develops different methods of weighting these constituents for an FCI (on the second choice). Following the previous literature (such as Hatzius, Hooper, Mishkin, Schoenholtz and Watson, 2010; Koop and Korobilis, 2014), it selects the best weighting for the FCI based on its ability to forecast macroeconomic activity.

As concluded in Hatzius et al. (2010), all FCI weighting methods fall into two categories: a weighted-sum approach and a principal-component (PC) approach.

Drawing on extensive readings, Chapter 1 develops a new weighted-sum method, known as the ‘two-step’ process, that attempts to overcome the two shortcomings in the existing weighted-sum methods: (i) almost all current studies estimating an aggregate demand equation have ignored the role of stochastic volatility (SV), and (ii) current FCI studies using weighted-sum methods did not consider the impact of economic activity on the financial system.

For comparative purposes, Chapter 1 also creates another FCI using a time-varying parameter factor-augmented VAR (TVP-FAVAR) with SV as a PC method. The TVP-FAVAR with SV model has the primary advantage over the traditional principal component analysis by allowing the relationship between variables to vary over time. It not only seeks to extract the co-movement of multiple variables but also takes the purpose of extracting principal components into account – in this case the ability to forecast economic activity. This is the first time the TVP-FAVAR with SV model has been used for UK financial data.

The results in Chapter 1 indicates that although the proposed ‘two-step’ process is superior to other existing weighted-sum methods, the weighted-sum approach (including the ‘two-step’ process) underperforms relative to the PC approach in creating an FCI. The TVP-FAVAR with SV model is found to be the best weighting method to create an FCI in relation to its purpose of forecasting economic activity.

Chapter 2 studies the optimal combination of constituent financial variables in an FCI at each point in time. It enlarges the number of indicators in the FCI and explores the optimal FCI at each time point (the first and third choices jointly) that best forecasts economic activity.

In order to decide the optimal index constituents at each point in time, Chapter 2 uses dynamic model averaging (DMA) for a larger information set. This is the first time in the literature to use the DMA technique to estimate FCIs. As explained by Koop and Korobilis (2014), the DMA model takes all possible combinations of financial indicators into account. With the conclusion obtained in Chapter 1 that the TVP-FAVAR with SV model is the best weighting-method, Chapter 2 develops an optimal FCI using a joint model of the DMA model and the TVP-FAVAR with SV model (henceforth, DMA-TVP-FAVAR).

Chapter 3 involves the second area this thesis attempts to address. It questions the traditional thinking of a constant inflation target within an inflation-targeting central bank, the BOE. In Chapter 3, this study hypotheses that in addition to setting the announced inflation target based on its medium to long-run considerations, the BOE may have another inflation objective for its short-term considerations. To distinguish this from the BOE's announced inflation target, this study introduces a new term, the implicit short-term inflation target, to the literature. It is the first to test the existence of a short-term inflation target.

Chapter 3 is mainly motivated by two facts. Firstly, there is short-term persistence in the inflation rate of the UK. As concluded in earlier studies such as Ireland (2007), inflation persistence cannot happen without ongoing shifts in a central bank's inflation target. Secondly, the MPC (March 2013) announces that it brings the inflation rate to the medium/long-run target gradually because the attempt to keep inflation at the announced value may cause undesirable volatility in economic activity. This reflects the short-run trade-offs to be made between inflation and output variability while setting the interest rate.

To test the hypothesis regarding the short-term inflation target, Chapter 3 employs the New Keynesian structural model as described in Ireland (2007). A key improvement in Chapter 3 is to introduce more forward-looking elements into the Ireland (2007) structural model. The results obtained in Chapter 3 suggest that the BOE sets the short-term inflation target while gradually bringing the inflation rate to the announced inflation target. This implicit short-term inflation target was time-varying in response to the exogenous shock, the cost-push shock and the technology shock. Among these three types of shocks, the technology shock dominated the changes in the BOE's short-term objective of inflation.

The third area this thesis addresses is the modelling of the interest rate using new measures of inflation, output and financial conditions and a new estimator for the UK. It consists of two chapters, Chapter 4 and 5. They attempt to examine whether a central bank works to stabilise inflation, economic activity and the domestic financial market. Chapter 4 includes linear estimates of the Taylor rule and Chapter 5 uses a time-varying parameter estimator.

Chapter 4 contributes to the literature by improving input data (explanatory variables) for estimation. To appropriately account for changes in inflation expectations, Chapter 4 uses the MPC's mean projection of the inflation rate. It also creates an optimal output gap measure by considering the BOE's monetary policy transmission mechanism. Using the DMA-TVP-FAVAR model, Chapter 4 obtains this new output gap measure to summarise the most valuable information in various economic activity indicators. It uses the FCI developed in Chapter 2 as the new FCI measure.

Although the Generalised Method of Moments (GMM) estimator in Chapter 4 has been previously used in much of the existing literature such as Castro (2011), Chapter 4 uses new measures of inflation expectations, economic activity and financial conditions as compared to the existing monetary studies in the UK. Therefore, the conclusion obtained from Chapter 4 should provide further insight into the BOE's decisions.

Using the GMM method, Chapter 4 confirms that the Bank was working to promote the stabilisation of economic activity, the inflation rate and financial markets. Although the BOE have not explicitly stated that it considers financial stability as a policy objective, the interest rate in the UK can be modelled better by augmenting the Taylor rule with financial markets. In addition, the subsample analysis in Chapter 4 suggests that the BOE has changed its monetary policy implementation. For two sample periods, one up to 2008 and the other including post 2008, it obtains very different parameter estimates. This motivates the further investigation on interest rate modelling by using a time-varying parameter estimator.

Chapter 5 models the short-term interest rate with a TVP-VAR-SV model. This model has several advantages including (i) taking into account the impact of the interest rate, (ii) allowing for the VAR parameters to evolve over time and (iii) considering time-varying volatility of each variable. Furthermore, Chapter 5 explicitly incorporates the effective zero lower bound on the policy rate. To the author's knowledge, this is the first time the TVP-VAR-SV model is employed to describe and explain the interest rate reaction function in the UK.

The empirical results in Chapter 5 justifies the use of time variation to address questions concerning the response of the interest rate to economic and financial shocks. Regarding the impact of the interest rate on the UK economy, Chapter 5 shows that the estimated time lags for the peak effects of the interest rate on the inflation rate and real output are consistent with the estimation of the MPC. In other words, it takes some time for interest rate changes to bring the inflation rate and output back to targeting values.

Policy Contributions

Jointly, this thesis addresses several principal questions in modelling the interest rate in the UK such as ‘whether the short-term inflation target should be taken into account while explaining changes in the interest rate of the UK’, ‘whether the initial Taylor rule should be augmented for stabilising financial markets’, ‘which measures should be used as inputs in the Taylor rule’ and ‘which estimator should be taken to describe changes in the short-term interest rate of the UK’. Both central bankers and financial market participants should benefit from this research.

From the perspective of policy makers, this thesis provides them with an optimal measure of financial conditions, a better measure of economic activity in summarising the information in relation to the further inflation rate and the estimate of the BOE’s unannounced short-term inflation target. This could be used to examine whether the Bank’s short-term inflation objective is in line with its long-run objective of maintaining price stability. Furthermore, this thesis also calculates time lags for the effect of the interest rate on economic activity and the inflation rate, which is important for policy makers to evaluate the effectiveness of their monetary policy.

Market participants will also benefit from this research. Individuals and institutions are always concerned about changes in the policy interest rate, as both policy actions and announcement have impacts on their wealth, consumption behaviour and investments. This thesis provides market participants with more insightful conclusion on how to explain and model the interest rate in the UK. The FCI obtained in this thesis is another important piece of information. The estimated FCI not only summarises the conditions of the UK financial market but also acts as the best predictor for future economic activity.

CHAPTER 1

CONSTRUCTING A FINANCIAL CONDITIONS INDEX FOR THE UNITED KINGDOM: A COMPARATIVE ANALYSIS

1.1 Introduction

The global financial crisis of 2008-9 triggered renewed concern about the relationship between the condition of the financial system and macroeconomic performance. This has led to efforts by authors such as Hatzius, Hooper, Mishkin, Schoenholtz and Watson (2010), Paries, Maurin and Moccero (2014) and Wacker, Lodge and Nicoletti (2014) to develop an indicator, a financial conditions index (FCI), to represent the current state of the financial sector. Their studies indicate that an FCI that summarises information on the current state of the financial system can serve as a good leading indicator of economic activity. In addition, central bankers need to have knowledge about the state of the financial sector because of its effect on the channels through which monetary policy is transmitted to the real economy. FCIs are useful to assess the state of the overall financial sector which are in turn useful to guide monetary policy. They are also useful to judge the effectiveness of their policies. Thus, Paries et al. (2014) argue that although analytically challenging, FCIs are appealing for central banks to help guide their implementation of monetary policy.

The focus of this chapter is on the identification of the optimal weighting method to weight constituent financial variables in an FCI for the United Kingdom (UK). Following Hatzius et al. (2010) and Koop and Korobilis (2014), the optimal method is chosen based on its ability to forecast economic activity. In this study, the sample period used to calculate FCI weights runs from 1993:I to 2013:II, and the prediction evaluation period begins one year later from 1994:I to 2013:II for 1-4 quarters ahead.

There are two methods used in the literature to construct FCIs. The first is the weighted sum approach in which structural models, vector autoregressive (VAR) models, aggregate demand equations are used to estimate the weights and secondly principal component analysis (PCA). However, both suffer from a number of disadvantages. For example, they are computationally difficult to estimate, they assume a constant weight on each index constituent and they fail to take structural changes into account. This study examines alternative weighting methods to construct an FCI for the UK that does not suffer from the previous defects.

The UK is chosen, because the work which has been done in this topic is generally focused on the United States (US) and the Euroarea. In order to analyse the UK's monetary policy, Castro (2011), Guichard, Haugh and Turner (2009) and Goodhart and Hofmann (2001) have provided the Bank of England (BOE) with different estimates of FCIs using VAR models, aggregate demand equations, etc. However, there is no consensus in the literature on the optimal FCI estimation for the UK.

This study develops a new 'two-step' method that is based on the time-varying parameter regression with stochastic volatility (TVP-R-SV) method developed by Nakajima (2011a) to estimate the time-varying weights in an FCI. This study also uses a time-varying parameter factor-augmented VAR (TVP-FAVAR) with stochastic volatility (SV) model to estimate an FCI for the UK for the first time. The standard PCA that is used to construct an FCI assumes fixed factor loadings. This implies that the correlation between each pair of constituent financial variables remains constant. However, Hollo, Kremer and Duca (2012) and Contessi, Pace and Guidolin (2013) provide evidence against this. They discover that the correlations between each financial indicator examined are in fact time-varying. Using data for the US and the Euroarea, Breitung and Eickmeier (2011) also find unstable factor loadings. Given such concerns, Koop and Korobilis (2014) develop the TVP-FAVAR with SV model to estimate an FCI on the US data. The major advantage of using this specification is that it allows factor loadings to vary across the sample period. Although Koop and Korobilis (2014) argue that this model outperforms the traditional PCA to construct an FCI, this method has not yet been used to create an FCI for the UK.

This study then compares the forecasting performance of FCIs produced by different weighting methods in order to discover the optimal model for weighting variables in an FCI for the UK. The results explain why earlier studies, such as Castro (2011), fail to capture the changes in the financial sector in some specific periods. The results also point to the TVP-FAVAR with SV model as the optimal method for weighting constituents in an FCI for the UK. This finding will guide the future econometric exercise to examine the BOE's monetary policy's reaction to financial conditions indices.

The remainder of this study is organised as follows: Section 1.2 reviews the literature in the area of FCIs. Section 1.3 discusses data issues, and Section 1.4 introduces the methodologies used in this study. The empirical evidence is given in Section 1.5. In Section 1.6, this study outlines some concluding remarks.

1.2 Literature Review

Studies on FCIs date back to the early 2000s. In the existing literature including Good and Hofmann (2001), Mayes and Viren (2001), Angelopoulou, Balfoussia and Gibson (2013) and Wacker et al. (2014), the FCI is considered as a natural extension of the monetary condition index (MCI) which was initiated by the Bank of Canada.

In the 1990s, the Bank of Canada (Freedman, 1994) began working on the creation of an MCI that combined both the interest rate and the exchange rate. The main rationale for the development of an index that included the exchange rate as well is that under a flexible exchange rate regime, the interest rate set by a central bank may give an incomplete picture of the impact of a monetary policy change on the real economy. As illustrated in Freedman (1994, 1995), lowering the interest rate not only tends to make monetary conditions more supportive by lowering credit costs, but also the associated depreciation of the exchange rate will make domestic assets cheaper for international investors and domestic goods cheaper for foreign residents. Hence, there will be a positive impact on output but increased inflationary pressures through these two channels – the interest rate channel and the exchange rate channel. Freedman (1994) identifies another rationale for the inclusion of these two variables in an MCI. An exogenous movement in the exchange rate will affect aggregate demand. As the objective is to obtain the estimated effect of the movement in these two variables on aggregate demand over time, the measures used by the Bank of Canada are the impact on demand of a percentage point change in the interest rate and exchange rate with the latter weighted by a factor of 1/3.

Following Freedman (1994), theoretical studies continue to explore the transmission of monetary policy. Bernanke and Gertler (1995) and Kiyotaki and Moore (1997) find that the two previously identified channels are insufficient to describe that process.

Asset prices also play a role. In the UK, the Monetary Policy Committee (MPC) maintains that if other things are equal, the higher interest rate should lower asset prices as expected future returns are discounted by a larger factor. The spending decisions of individuals and firms would then respond to changes in the market rate, the exchange rate and asset prices in the same direction resulting in a shift in domestic and external demand (MPC, June 2012, p. 4-6).

Therefore, the transmission channels are widened to include the impact of monetary policy on asset prices. These broader measures are known as FCIs to distinguish them from MCIs (Hatzius et al., 2010). Essentially, these FCIs are designed to summarise information on the condition of the financial system in a single index. Gauthier, Graham and Liu (2004) compare the performance of an FCI and an MCI for Canada. They assess the correlation between these two indices and study their ability to forecast economic activity. Their results show that the MCI forecasts output incorrectly at all horizons. Compared with the FCI, the MCI has a larger mean squared forecasting error. Therefore, they conclude that the FCI outperforms the MCI for Canada.

This study considers (i) the usefulness of FCIs, (ii) the reasons behind the choice of variables included in an FCI and (iii) the methods used to construct FCIs. In Section 1.2.1, it explains what is meant by financial conditions and the reasons for assessing it. Section 1.2.2 discusses the definition of the Financial Conditions Index and the motivation for creating the index. Section 1.2.3 highlights the rationale for the inclusion of the six variables in the FCI in this study. Section 1.2.4 reviews the existing methods for constructing FCIs in the literature while emphasising the limitation and advantage of each method.

1.2.1 Definition of the Financial Condition

Financial conditions (FCs) refer to the state and functioning of financial markets that affect economic behaviour and consequently the current and future state of the economy. Wacker et al. (2014) introduce two benefits for both central bankers and market participants of assessing financial conditions.

Firstly, understanding financial conditions is crucial for making monetary policy decisions, because financial market changes will affect the transmission channels through which monetary policy affects the real economy. Shocks that affect the link between the policy instruments and aspects of the financial markets may alter the final outcome of a policy decision. Corsetti and Pesenti (2005), Kontonikas and Montagnoli (2006) and Lubik and Schorfheide (2007) all highlight the role of financial variables in the monetary authority's decisions.

Kontonikas and Montagnoli (2006) examine the theoretical motivation for the use of a Taylor rule augmented by asset prices. To do so, they construct a macro model where asset prices directly influence future inflation through the impact of wealth effects on aggregate demand. They show that if asset price changes cannot be explained by the fundamentals alone, optimal monetary policy will systematically respond to the non-fundamental component of asset prices. Although in 1993, Taylor (1993) omits the exchange rate in his monetary policy rule for the US Federal Reserve (Fed), he comments in his later work (Taylor, 1999) that a monetary rule similar to the Fed may not hold for other countries. Following Taylor (1999), Corsetti and Pesenti (2005) maintain that the standard policy objectives for a closed economy setting may not be appropriate for the design of an optimal monetary policy in an open economy. They establish a general equilibrium model of the optimal monetary policy among interdependent countries and discover that foreign exchange markets emerge as an important parameter in the conduct of optimal monetary policy by central bankers. Gali and Monacelli (2005) develop a theoretical model for small open economies based on the New Keynesian framework in Woodford (2003) which is used by Lubik and Schorfheide (2007) to estimate monetary policy for four open economies, i.e., Canada, Australia, New Zealand and the UK. They find that the central banks of Australia and New Zealand did not target the exchange rate over the last two decades whereas both the Bank of Canada and the BOE did.

Secondly, the financial market serves as a leading indicator of economic activity. Economic activity is affected by financing costs and credit availability for firms and households, both of which are broadly reflected in financial indicators. Kiyotaki and Moore (1997) derive a theoretical model that exhibits an important interaction between collateral values, asset prices, credit and real economic activity. At an empirical level, Goodhart and Hofmann (2000) introduce the linkage between the movement of asset prices, monetary aggregates and output growth in 17 advanced economies. Hatzius et al. (2010) discover that asset prices are useful in predicting economic growth in the US.

1.2.2 Definition of the Financial Conditions Index

Given the number and variety of financial indicators, financial conditions are often synthesised into one indicator for use by central bankers and market participants. Hatzius et al. (2010) define an FCI as a composite index that summarises the financial information on the future condition of an economy. Zheng and Wang (2014) maintain that an FCI should address the limitations of conventional measures, including the interest rate or money supplies, in assessing the financial state of a country/economy and in forecasting economic trends.

In the Eurozone, the European Central Bank (ECB, 2009) has a global index of financial turbulence to evaluate financial stress and to capture underlying market conditions. In the US, the St. Louis Fed (2015) updates its FCI regularly which is the first extracted principal component of the interest rate, yield spreads, bond indices and stock and bond market volatility.

1.2.3 Variables to be Included in a Financial Conditions Index

Drawing on extensive readings on FCIs, this study reviews a variety of indicators of financial conditions to construct an FCI. In PCA studies such as Hatzius et al. (2010), Paries et al. (2014) and Wacker et al. (2014), researchers extract factors from a large set of financial indicators. However, two shortcomings exist: first, they fail to give any theoretical or empirical reason for the choice of variables to be included; second, as shown in Boivin and Ng (2006) and Koop and Korobilis (2014), using all available data to extract factors is not always optimal in the PCA. Therefore, this study includes a small number of variables that are both theoretically and empirically sound.

Firstly, this study considers the two original MCI variables for inclusion, the interest rate and the exchange rate. As already mentioned, they have important information regarding the stance of policy and serve as channels for the delivery of monetary policy. The interest rate is sometimes considered as a measure of this stance itself because it is highly correlated with the instrument of monetary policy (Gauthier et al., 2004). Dudley and Hatzius (2000) argue that given the dominance of the capital market in the financial system, the focus should be on factors controlling the monetary policy transmission to the real economy such as the interest rate and the exchange value of the domestic currency. The MPC (June 2012) acknowledges the direct effects of the interest rate and the exchange rate on the spending behaviour of individuals and firms in the UK. Although an MCI does not outperform the FCIs in Gauthier et al. (2004), it has been widely agreed in the literature, including Zheng and Wang (2014), that an MCI with these two indicators is better to measure financial conditions than a single variable. For this reason, most FCIs (see, for instance, Good and Hofmann, 2001; Mayes and Viren, 2001; Montagnoli and Napolitano, 2005) have included these two variables in their series.

It is interesting to note that some indices include the short-term interest rate along with a set of financial variables. This is the case in Dudley and Hatzius (2000), Goodhart and Hofmann (2001), Mayes and Viren (2001), Montagnoli and Napolitano (2005) and Hatzius et al. (2010). However, others (including Castro, 2011) exclude the interest rate that is set by the central bank to maintain price and output stability (also mentioned in Paries et al., 2014). The latter focuses on the transmission of monetary policy not accounted for by the traditional channel. Hence, the financial conditions are determined endogenously by the financial sector in response to policy decisions. More recently, Wacker et al. (2014) discover that the decision to include or exclude the interest rate in FCIs does not matter for their results for the US. This study considers both cases in the later econometric exercises. It uses FCIs that include the interest rate to explore the most appropriate method for calculating an FCI. Then, it creates an FCI that excludes the interest rate in the selected optimal weighting method in order to examine whether the decision to include the interest rate in an FCI significantly alters the estimates of the FCI for the UK. In other words, this study also compares FCIs with an interest rate and without an interest rate in order to examine the sensitivity of estimation to the inclusion of the interest rate.

Secondly, a number of FCI studies, such as Dudley and Hatzius (2000) and Wacker et al. (2014), incorporate the wealth effect measure into the index. As in Hatzius et al. (2010), equity and house prices that affect wealth are the natural constituents of an FCI. As mentioned previously, Kiyotaki and Moore (1997) study the theoretical interaction between asset prices and the real economy and suggest that changes in asset prices should signal changes in the real Gross Domestic Product (GDP) and the inflation rate. Castro (2011) adds that if the interest rate and the exchange rate in FCIs measure the effect of monetary policy changes on domestic and external demand, the prices of stocks and properties should encapsulate the wealth effect on aggregate demand. The MPC (June 2012) also acknowledges that from the perspective of an individual, asset prices adjustments would change his/her financial wealth which in turn shifts his/her consumption. From the firm's perspective a rise (or fall) in a firm's asset prices should strengthen (or weaken) its borrowing capacity by increasing (or decreasing) the value of collateral. The growth in the volume of available funds tends to raise investment activities and *vice versa*. All of the above changes in the firms' and individuals' behaviour, when added up across the economy, generates the changes in aggregate spending. Total domestic spending plus the balance of trade and government expenditure reflects aggregate demand in the economy and is equal to GDP at market prices. Against this backdrop, FCI estimates, including Brave and Butter (2011) and Koop and Korobilis (2014), always consider equity and property prices in describing the condition of financial markets.

Hatzius et al. (2010) provide evidence on the predictive power of asset prices for real economic activity over two and four quarters ahead. To gauge their performance, they consider the ability of asset prices to predict the real GDP growth rate, payroll employment, the industrial production index and the unemployment rate. The results are very encouraging. They suggest that asset prices, measured as the S&P 500 Index and purged of the impact of inflation and the growth rate of real GDP, are useful to explain the variance in the two and four quarters ahead growth of the activity variables. Hence, this study follows the earlier FCI literature including Castro (2011) and adds real equity and real house prices to the set of financial variables to be included in the FCI for the UK.

Thirdly, studies such as Guichard et al. (2009), Hatzius et al. (2010) and Wacker et al. (2014) hold a view that an FCI should reflect the financial sector risk appropriately. As in Wacker et al. (2014), the rationale for including risk measures is straightforward: the spreads, i.e., the interest rate of one asset relative to another less profitable asset, measure the relative price at which the fund is available to certain market participants. Taylor (2008) argues that spreads, related to the concerns of the security default risks, add predictability at times of financial sector stress. Castro (2011) also maintains that credit spreads, calculated as the difference between the ten-year government bond yields and the return on corporate bonds, should be a good leading indicator of business cycles.

In addition, Castro (2011) adds that changes in the future interest rate spreads, i.e., the changes in the spreads between short-term interest rate futures contracts in the earlier quarter and the current short-term interest rate, should also signal the degree of volatility in agents' expectations that central banks aim to reduce. Prior to Castro (2011), Driffill, Rotondi, Savona and Zazzara (2006) identified the connection between monetary policy and targeting financial stability. Driffill et al. (2006) theoretically augment the analysis of determinacy of equilibrium in Bullard and Schaling (2002) and show the existence of a trade off between macroeconomic stabilisation and the movement on the futures market. At an empirical level, they focus on the Fed's response to interest rate futures. They discover that the component in the Fed's reaction function related to futures prices has the same degree of importance as the output component. Given the evidence in Driffill et al. (2006), Castro (2011) argues that from a central bank's perspective, the two aforementioned indicators, namely credit spreads and changes in futures interest rate spreads, should contain extra information regarding the markets stability and expectations. Therefore, these two additional variables used in Castro (2011) are also considered in this study.

1.2.4 The FCI Constructing Methods

As with the construction of MCIs, the methods for constructing FCIs tend to fall into two broad categories: the weighted-sum approach or the principal-component approach.

The idea behind the weighted-sum approach is reflected in Eq. (2.1):

$$FCI_t = \sum w_{i,t}\left(q_{i,t} - \tilde{q}_{i,t}\right) \tag{2.1}$$

where $q_{i,t}$ represents the value of the ith indicator, $\tilde{q}_{i,t}$ is the steady state of $q_{i,t}$, $w_{i,t}$ is the weight attached to each indicator, and t subscripts denote time. The weights, $w_{i,t}$, on each financial variable are assigned based on the estimate of the impact of a change in this variable on economic activity. There are three alternative methods in the use in the literature to obtain the estimated weights, $w_{i,t}$: (i) performing simulations with macro econometric models, (ii) employing impulse responses in a VAR and (iii) estimating an aggregate demand equation (i.e., a reduced form model). The aggregate demand equation is estimated with either ordinary least squares (OLS) or the Kalman filter algorithm (see, for instance, Gauthier et al., 2004; Castro, 2011).

The principal-component approaches extract common factors from a group of variables. They attempt to capture the greatest variation in the information set. Two alternatives exist in the literature: (i) the standard PCA and (ii) the TVP-FAVAR. The principal distinction between the two alternatives lies in the fact that the former assumes a fixed relationship between each pair of financial variables, whereas the latter allows for this relationship to change at each point of time.

More recently, Hatzius et al. (2010) emphasise that an FCI should measure the 'true' shocks in financial conditions. Therefore, the endogenous reaction of the financial variables in the FCI to current macroeconomic conditions should be removed in order to estimate the true shocks which are measured as the difference between the financial variables and its equilibrium value. As in Hatzius et al. (2010), if the only information contained in a financial variable about the future economy were of this endogenous variety, there is no reason to create such an index. Let $x_{i,t}$ be the ith financial indicator at time t and y_t a vector of macroeconomic variables such as the real output growth rate, the unemployment rate, etc. They suggest processing the data for financial variables using the following:

$$x_{i,t} = A_{i,t}(L)y_t + v_{i,t} \tag{2.2}$$

where $v_{i,t}$ is uncorrelated with y_t. Consequently, $v_{i,t}$ represents the i^{th} financial variable purged of its relation with current macroeconomic activity. Since Hatzius et al. (2010), purging the components of an FCI of current economic activity has been increasingly popular in the literature (see, for instance, Brave and Butters, 2011; Wacker et al., 2014; Koop and Korobilis, 2014).

This subsection begins with a review of the weighted-sum approaches. Section 1.2.4.1 reviews the use of large scale macro econometric models to estimate the weights.

Section 1.2.4.2 discusses the use of impulse responses in a VAR for estimating the weights in an FCI. Section 1.2.4.3 explores how the weights are calculated using an aggregate demand equation. Sections 1.2.4.4-1.2.4.5 then follow with a discussion of the two principal-component methods, respectively – the standard PCA and a TVP-FAVAR model.

1.2.4.1 Large Scale Macro Econometric Models

Large scale macro econometric models are normally used by monetary authorities, governmental organizations and some private institutions. They are structural models derived from optimising behaviour and are designed to gauge the structural features of an economy.

The US Goldman Sachs FCI (by Dudley and Hatzius, 2000) is constructed using the Fed's macro econometric models (i.e., the FRB/US model) combined with Goldman Sachs modelling. The FRB/US model of the US is maintained at the Fed for policy analysis and forecasting. In particular, individuals and firms are assumed to be forward-looking and optimise their welfare based on prices, sales, income and financial status. Individuals choose a path for the current and future consumption that maximises their lifetime utility subject to their budget constraints. Firms maximise expected profits by hiring workers, investing in capital goods and setting prices. The FRB/US model defines an explicit role for the financial market in the US economy. As emphasised in the Fed's bulletin (see, Reifschneider, Tetlow and Williams, 1999), changes in financial conditions whether driven by shifts in monetary policy or not are important factors in spending decisions of households and firms.

Reifschneider et al. (1999) derive the quantitative importance of financial effects for different categories of stocks and spending in the FRB/US model through estimating the response of stocks and private spending to changes in financial conditions including the interest rate, wealth and the exchange rate. They argue that this model provides a direct measure of the quantitative importance of each part of the monetary transmission mechanism.

Given the result in the Fed bulletin, Dudley and Hatzius (2000) determine the weights assigned to each constituent for the US Goldman Sachs FCI. In Dudley and Hatzius (2000), a rise in the FCI is associated with a tightening of financial conditions, whereas a fall indicates easing. They show that a one-point rise in the index is followed by a slowdown in the real GDP growth rate by roughly one percentage point. Compared with the real Fed funds rate and the M2 growth rate, the US Goldman Sachs FCI is found to be better at forecasting the growth rate of real GDP. This is attributed to the inclusion of additional financial variables such as the exchange rate and equity indicators that tighten the link between the FCI and the real economy.

The work of Guichard et al. (2009) is partially based on large-scale macroeconomic models. The weights used in their FCI for the US are estimated from a reduced form econometric model supplemented with the coefficients calibrated from the FRB/US model. They are calibrated so that a unit decline in the index indicates a one percent reduction in the GDP after 4-6 quarters. Although Guichard et al. (2009) choose the weights for the US FCI by a quantitative optimization procedure, they use their discretion or judgement to determine the weights for the FCIs for other economies (the Eurozone, Japan and the UK) using the existing US index as a reference point. For instance, drawing on wide readings, they believe that the strength of the interest rate transmission mechanism in the UK is stronger than that in the Eurozone. Therefore, they assume that the weights on the interest rate are identical in the UK and the US but one-third lower in the Eurozone.

However, the number of papers that use the macro econometric models is quite limited. As in Goodhart and Hofmann (2001), this model is superior to other methods including VARs and a reduced form model (i.e., an aggregate demand equation) in that it considers the structural features of an economy and the interaction of all macroeconomic variables. However, due to the lack of data availability, a macro econometric model with an explicit role for house prices is not available for many G7 countries. More recently, Price, Kapetanios and Young (2015), all economists with the Bank of England, acknowledge that although a well-defined structural model can make them understand the primitive shocks driving all aspects of the economy thus helping them to estimate an FCI, the BOE's models are not sufficiently robust enough to enable them to do this for the UK (see, Price et al., 2015). Thus, this study will go for other options.

In addition, Montagnoli and Napolitano (2005) argue that assuming fixed weights as in the US Goldman Sachs FCI is too restrictive and unnecessary in reality because the behaviour of firms and households may change through the business cycle or in reaction to certain events. Wacker et al. (2014) also argue that this assumption is too restrictive even for a developed country.

Paries et al. (2014) note that as a macro econometric model requires an estimation of the impacts of financial conditions on the macroeconomic outcomes (see, for instance, the US Goldman Sachs FCI produced by Dudley and Hatzius, 2000), the number of financial variables has to be kept to the minimum. For instance, there are only four variables in the US Goldman Sachs FCI. The three points above jointly illustrate why most studies choose other methods.

1.2.4.2 Impulse Responses in a VAR

The use of impulse response functions in a VAR has the advantage of taking into account the feedback between all variables as all are treated as endogenous. As in Asteriou and Hall (2011), a VAR is a system regression model in which all variables are treated in the same way. Gauthier et al. (2004) also note that compared with structural models and a reduced form aggregate equation, a VAR allows for more interaction between variables but imposes less economic theory.

With a VAR model, an FCI is created by weighting financial indicators based on their relative impacts on macroeconomic variables, such as output or inflation, in a specific period over which the monetary policy has its full impact on the real economy. In other words, FCI weights are derived according to the impulse response of macroeconomic variables to each constituent financial variables. However, it has an inherent limitation in that the weights on each index constituent are typically constant. Apart from that, Paries et al. (2014) notes that VAR studies usually keep the number of financial variable to a minimum in order to avoid making the VAR too heavy. For example, if there are ten variables in a VAR, a total of 100 impulse responses will be generated, which is too computationally intensive.

Goodhart and Hofmann (2001) estimate a six-variable VAR and then derive the relative weights for the endogenous variables in the FCI based on the average impulse response of the inflation rate to each financial variable over the next 12 quarters. Their result points to the interest rate as the most crucial indicator (as against real house prices, share prices and the exchange rate) in the financial market in the UK. As mentioned, their approach sets fixed weights for each index constituent. Although Primiceri (2005) introduces a time-varying parameter VAR (TVP-VAR) and the fixed-weight problem can be settled with that TVP-VAR, the number of variables is still required to be kept to a minimum to avoid the computational burden.

Gauthier et al. (2004) use an 18-order VAR and estimate their weights as cumulative impacts of a typical shock to each constituent on the output growth rate over two years. The resulting FCI for Canada incorporates six variables including the short and the long-term interest rate, the exchange rate, equity prices, house prices and the US credit spreads. While comparing the forecast performance of FCIs, they discover that the VAR-based FCI is good at predicting the long-term (i.e., 12, 18 and 24-month horizon) output but not good at short-term forecasting with six and nine months ahead.

1.2.4.3 Reduced Form Models

Gauthier et al. (2004) maintain that the advantage of deriving an FCI from a reduced form model is that the impact of each potential monetary transmission channel on the real economy can be identified given several restrictions. This study uses the reduced form model as one of the methods used to construct an FCI.

This empirical model for constructing an FCI is an augmented version of the standard inflation targeting specification proposed by Rudebusch and Svensson (1999). Their model consists of an aggregate supply equation (Phillips curve) which relates inflation (π_t) to an output gap (y_t) and an aggregate demand equation (IS curve):

$$\pi_{t+1} = \sum_{i=1}^{4} \alpha_{\pi i}\pi_{t+1-i} + \alpha_y y_t + \varepsilon_{t+1} \tag{2.3}$$

$$y_{t+1} = \sum_{k=1}^{2} \beta_{yk} y_{t+1-k} - \beta_r(\bar{\imath}_t - \bar{\pi}_t) + \eta_{t+1} \tag{2.4}$$

In order to avoid the appearance of a constant in the above equations, Rudebusch and Svensson (1999) de-mean all these variables prior to the estimation. They highlight at least two points that motivate the choice of Eq. (2.3) and Eq. (2.4) in modelling an economy:

Firstly, this system model consists of a Phillips curve and an IS curve. Hence, it glosses over some crucial features of the monetary transmission mechanism. The real interest rate ($\bar{\iota}_t - \bar{\pi}_t$) embedded in Eq. (2.4) represents the monetary policy transmission channel. In the spirit of this system, changes in monetary variables should shift the output gap which in turn affects the inflation rate. Secondly, it gauges the spirit of many practical policy-oriented macroeconomic models such as Gali and Monacelli (2005), Kontonikas and Montagnoli (2006) and Teranishi (2012).

In order to assess the role of financial variables in the conduct of monetary policy, Goodhart and Hofmann (2001) augment the above model in Rudebusch and Svensson (1999) by adding the effects of other financial variables:

$$\pi_{t+1} = \sum_{i=1}^{n_1} \alpha_{\pi i} \pi_{t+1-i} + \sum_{j=1}^{n_2} \alpha_{yj} y_{t+1-j} + \sum_{k=0}^{n_3} \alpha_{dk} dpo_{t+1-k} + \varepsilon_{t+1} \tag{2.5}$$

$$y_{t+1} = \sum_{i=1}^{m_1} \beta_{yk} y_{t+1-k} + \sum_{j=1}^{m_2} \beta_{rj} \left(\bar{\iota}_{t+1-j} - \bar{\pi}_{t+1-j} \right) + \sum_{h=1}^{m_3} \beta_{kh} x_{k,t+1-h} + \eta_{t+1} \tag{2.6}$$

where x_k includes the deviations of financial variables (e.g., the real exchange rate, real house prices and real share prices) from their long-run trends. These financial variables finally constitute an FCI. The term dpo_t denotes quarterly changes in the world prices of oil that acts as a proxy for supply shocks and helps to eliminate heteroskedasticity. The term $\bar{\iota}_{t+1} - \bar{\pi}_{t+1}$ is the de-trended real interest rate.

The main empirical rationale for this extension is from Goodhart and Hofmann (2000), who perform an econometric exercise for 17 industrialised countries including Belgium, Canada, Demark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Australia, Norway, Sweden, Spain, the UK and the US. Their results show that for all countries investigated, it is not possible to obtain a significant impact of the interest rate on the output gap unless the effect of financial variables are being controlled for. Not taking into account the impact of monetary policy on asset prices tends to lead to an increase in the central bank's loss function of 60%, as Goodhart and Hofmann (2002) show in their simulations. In other words, the deviation of the inflation rate and output from their target values will rise by 60%.

The report on 'The Transmission Mechanism of Monetary Policy' (MPC, June 2012) also argue that financial market plays an quite important role in transferring effect of monetary policy on real economic activity and inflation, which theoretically justifies the Goodhart and Hofmann (2000) extension as in Eq. (2.6).

Goodhart and Hofmann (2001) estimate Eq. (2.6) using OLS. They then evaluate the FCIs' out-of-sample forecasts in seven advanced countries, namely Canada, France, Germany, Italy, Japan, the UK and the US. They show that the FCIs based on the reduced form model generally perform better than those based on VARs. Their result is consistent with the later finding in Gauthier et al. (2004) who discover that an FCI based on the Eq. (2.6) is better for predicting near-term output growth in Canada.

As already mentioned, both a macro econometric model and a VAR assumes that the weight associated with each constituent is time-invariant. In Goodhart and Hofmann (2001), the estimates of the reduced form model also involves such an assumption. Montagnoli and Napolitano (2005) relax the fixed-weight assumption. Instead of employing OLS, they use the Kalman filter algorithm to estimate Eq. (2.6) for the Eurozone, the US and Canada. The time-varying weights are based on the coefficients and significance probability of variables at each point in time. Following Montagnoli and Napolitano, Castro (2011) completes a similar econometric exercise for the Eurozone, the UK and the US.

This study highlights two pitfalls in the existing literature using a reduced form model to estimate an FCI:

(i) As proposed by Hatzius et al. (2010), it is essential to purge financial indicators of the current macroeconomic activity. Their studies illustrate the effect of purging the FCIs of macroeconomic impacts in the case of the US. During the mid-1970s and early 1980s, the unpurged index is significantly more negative than the purged one. That is, the unpurged financial market indicators indicate severe disruption in the financial sector. However, much of this could be explained by the prevailing level of the real activity and the inflation rate. Even though the idea of Hatzius et al. (2010) is widely accepted in the PCA literature, none of the IS-curve based FCIs have considered this proposal.

(ii) Recall the simulation exercise in Nakajima (2011a) who empirically compares the regression performance of the Kalman filter algorithm against that of the TVP-R-SV model. The result implies that the Kalman filter algorithm that neglects the behaviour of stochastic volatility reduces the accuracy of estimates for time-varying parameters. As in Nakajima's experiment, the TVP-R-SV performs much better as the estimated parameters improve the tracing of the movement of the true values. Its 95% percent confidence intervals narrow overall and almost include true parameter values. Given this result, Nakajima (2011a) stresses the importance of incorporating stochastic volatility into a time-varying parameter regression (TVP-R), which supports the argument in Primiceri (2005) about shifting volatility. The findings that are given by Nakajima (2011a) also raise concerns regarding the accuracy of the time-varying parameter (TVP) estimates in Montagnoli and Napolitano (2005) and Castro (2011).

Given the above discussion, this study revises the conventional idea for estimating a reduced form model. It develops a 'two-step' method based on the TVP-R-SV algorithm. As a first step, this study employs this regression method to purge the current macroeconomic impacts on each financial variable. Hence, the term x_k in Eq. (2.6) should only consist of the series of purged financial variables. In the second step, it applies the TVP-R-SV again for Eq. (2.6) in order to obtain the time-varying parameters that will be used to calculate the weights. A larger parameter on a financial variable is associated with a greater weight attached to that constituent indicator and *vice versa*.

1.2.4.4 Conventional Principal Component Analysis

As in Brave and Butters (2011), the PCA is a statistical technique under which an FCI is regarded as the co-movement of the multiple indicators of the financial system and attempts to gauge the greatest variation in the information set. The surveyed literature uses between five (for instance, Zheng and Wang, 2014) and eighty financial variables.

More formally, it assumes that the financial variables ($x_{i,t}$) in an FCI can be thought of as comprising a common component consisting of a $k \times 1$ vector of unobserved financial factors (f_t) common to all indicators and a row vector of coefficients (λ_i) together with an idiosyncratic error term ($\varepsilon_{i,t}$):

$$x_{i,t} = \lambda_i' f_t + \varepsilon_{i,t} \tag{2.7}$$

where $\varepsilon_{i,t}$ captures the 'unique' variation in $x_{i,t}$. Under the assumption that $\varepsilon_{i,t}$ is not correlated across financial variables, the vector f_t denotes the co-variation among the financial indicators. The goal of the econometric exercise is to calculate f_t. In order to purge macroeconomic impacts, Hatzius et al. (2010) propose to use the residual in Eq. (2.2), $v_{i,t}$ (i.e., purged financial variables) instead of using unpurged variables, $x_{i,t}$.

Wacker et al. (2014) highlight the primary advantage of an index based on the PCA: it is purely data-driven and does not require any assumptions regarding the impacts of financial systems on the real economy. Brave and Butter (2011) stress the advantage that the PCA-based FCI is able to explain the interconnectedness within the financial market. This is a desirable characteristic that allows for the demonstration of the systematic importance of each financial indicator. The more correlated an indicator with its peers, the higher the weight it receives.

Drawing on extensive readings, this study discovers that the conventional estimates of FCIs based on the principal component analysis have followed two strands:

The first one summarises all variables with more than one single principal component ($k \geq 2$). Zheng and Wang (2014) construct an FCI for China by extracting five dynamic components from a group of financial indicators. They find that the first three components express the main trend of the financial markets. Hence, their FCI is weighted by the three components. In order to determine the weights, Zheng and Wang (2014) select the coefficients of variables as the weights in a linear regression model in which the dependent variable is output growth and the independent variables are the three major components.

On the other hand, to choose the number of factors to extract, Paries et al. (2014) set up several criteria depending on the trade-off between good fit and parsimony. The empirical results imply that the optimal number of components to retain is one for all the criteria and all of the economies under consideration including the Eurozone, France, Germany, Italy and Spain. In that case, the f_t is treated as an FCI and the weight that each financial indicator i has in the index is proportional to its lambda coefficient. In addition, Hatzius et al. (2010) consider 1-3 factors in their empirical work and disclose that the one-factor FCI performs at least as well as the two and three-factor versions. Wacker et al. (2014) also argue that their one-factor FCIs have explained the maximum variation in all the observed financial variables. Therefore, they focus on the one-factor version.

As in Wacker et al. (2014), there are criticisms of the PCA methodology as well. The conventional principal component analysis presumes that factor loadings (i.e., lambda coefficients) are time-invariant over the complete sample period. This means that the correlation structure between financial variables remains unchanged under the PCA. However, Hollo et al. (2012) and Contessi et al. (2013) discover evidence against this hypothesis. Hollo et al. (2012) aggregate five market-specific sub-indices (based on fifteen individual financial stress measures) and create a composite indicator of stress in the financial system. Their study clearly shows the time-varying correlations between each pair of sub-indexes. Focusing on the indicators of the fixed income yield spreads in the US, Contessi et al. (2013) discover that for almost half of the 55 pairs under investigation (particularly for liquidity and default-risk-related spreads), the 2008-9 crisis has left spreads much more correlated than they were previously. Hatzius et al. (2010) discuss why changes in weights used to average financial variables might be occurring. For instance, they mention that the role of the subprime housing market during the last financial crisis was a reason for raising the importance of house market indicators in an FCI. At an empirical level, Breitung and Eickmeier (2011) find some evidence of the dramatic changes in the US economy reflected in significant breaks in the factor loadings. For the Eurozone, they show that structural breaks in loadings coincide with the signing of the Maastricht treaty in 1992:II and the handover of monetary policy to the ECB in 1999:I. More recently, Bates, Plagborg-Moller, Stock and Watson (2013) also confirm the instability in the factor loadings.

1.2.4.5 Time-varying Parameter Factor-augmented VAR

Given the concerns about time-variation or structural breaks in loadings (discussed in Section 1.2.4.4), Koop and Korobilis (2014) introduce a new way of estimating FCIs. They work with a TVP-FAVAR with SV model which consists of two equations:

$$x_t = \lambda_t^y y_t + \lambda_t^f f_t + v_t \tag{2.8}$$

$$\begin{bmatrix} y_t \\ f_t \end{bmatrix} = c_t + B_{t,1} \begin{bmatrix} y_{t-1} \\ f_{t-1} \end{bmatrix} + \cdots + B_{t,p} \begin{bmatrix} y_{t-p} \\ f_{t-p} \end{bmatrix} + \varepsilon_t \tag{2.9}$$

where x_t is an $N \times 1$ vector of financial variables to be used in constructing FCIs. The term y_t represents an $s \times 1$ vector of macroeconomic variables of interest such as the real GDP growth rate, inflation and the unemployment rate. The inclusion of y_t in Eq. (2.8) is to purge the influences of business cycles as recommended in Hatzius et al. (2010). The f_t is a latent factor which is interpreted as an FCI. Based on the findings in Hatzius et al. (2010), Paries et al. (2014) and Wacker et al. (2014), Koop and Korobilis (2014) extract only one factor from x_t. The regression coefficients (λ_t^y) and factor loadings (λ_t^f) are both time-varying in the full sample period. This specification addresses the shortcoming of the fixed-loading assumption in the traditional PCA.

At this stage, it seems that Koop and Korobilis (2014) are working with a VAR to get FCIs. However, this is not the case. In order to illustrate a TVP-FAVAR based FCI in more detail, this study starts with the influential work by Bernanke, Boivin and Eliasz (2005) which extends a standard VAR to include a set of factors:

$$\begin{bmatrix} y_t \\ f_t \end{bmatrix} = \Phi(L) \begin{bmatrix} y_{t-1} \\ f_{t-1} \end{bmatrix} + \varepsilon_t' \tag{2.10}$$

where y_t is a vector of observed variables for modelling the dynamics of the economy in a standard VAR. However, Bernanke et al. (2005) argue that in many applications, additional information that is not fully captured by y_t could be relevant to modelling these series. Therefore, they introduce a vector of unobserved factors f_t so as to summarise the additional information. $\Phi(L)$ is a conformable lag polynomial of finite order d. If the terms of $\Phi(L)$ that relate y_t to f_{t-1} are all zero, this model reduces to a standard VAR in y_t.

Since Eq. (2.10) cannot be estimated independently without the unobservable factor f_t, Bernanke et al. (2005) introduce Eq. (2.11) to extract f_t from a group of informational time series, collectively denoted by the $N \times 1$ vector x_t:

$$x_t = \lambda^y y_t + \lambda^f f_t + v_t' \tag{2.11}$$

A two-step approach, analogous to that used in Stock and Watson (2002), is a popular procedure for estimating this factor-augmented VAR (FAVAR). As the first step, the space spanned by the factors is estimated using the principal components of x_t. With the weights determined by the standard PCA, the FAVAR (i.e., Eq. 2.10) is estimated using a standard regression method with f_t replaced by $\hat{f}_t$. Therefore, if one thinks of x_t as a large group of financial indicators and f_t as an FCI for modelling and predicting the macroeconomic variables in y_t, it is convenient to reach such a conclusion that the FAVAR based FCI is exactly the same as an FCI produced by the standard PCA.

The TVP-FAVAR with SV model used in Koop and Korobilis (2014) is an extension of the FAVAR in Bernanke et al. (2005). The primary improvement is to let parameters and volatility vary at each point in time. Though Koop and Korobilis (2014) develop a new two-step estimation algorithm to calculate the time-varying parameters and stochastic volatility, they still leave f_t (in Eq. 2.8) as a principal component estimate based on x_t. The FCI weights chosen are on the basis of the estimated time-varying loadings. As in Koop and Korobilis (2014), the purpose of modelling an FCI in a VAR structure instead of estimating it independently is to evaluate the forecasting performance of the index at each point in time. In addition, the use of VAR system enables them to better characterise the co-movement and the interdependence of financial variables. In answer to the question 'what makes a good FCI', Gauthier et al. (2004), Hatzius et al. (2010) and Koop and Korobilis (2014) provide an answer: 'it is the one that forecasts y_t (in Eq. 2.9) as well as possible'.

Koop and Korobilis (2014) focus on the US economy and compare the performance of a wide range of models including the TVP-FAVAR with SV model for forecasting the inflation rate, output growth and the unemployment rate. Their evaluation period starts in 1990:I and ends in 2013:III (which is an in-sample forecasting). The empirical result implies that a TVP-FAVAR with constant volatility model produces an FCI that has better predictive ability than an index based on the PCA. Adding stochastic volatility tends to improve forecasts substantially and then adding time-variation in parameters tends to improve them a bit more.

Apart from that, Koop and Korobilis (2014) compare their FCIs against the existing four Fed's financial conditions and stress indices for the US: (i) the St. Louis Financial Stress Index, (ii) the Kansas City Fed Financial Stress Index, (iii) the Cleveland Fed Financial Stress Index and (iv) the Chicago Fed National FCI. The result is encouraging. The FCI based on the TVP-FAVAR with SV model results in the lowest mean forecast errors of one to four quarters ahead. This finding indeed provides strong support for employing a TVP-FAVAR with SV to create FCIs in practice.

Furthermore, given the result that the algorithm in Koop and Korobilis (2014) yields better FCIs than the conventional PCA, it is quite interesting to compare the performance of an index based on their model against that produced by a weighted-sum approach such as the two-step method based on the TVP-R-SV algorithm proposed earlier.

1.3 Data

This study uses statistics published by the BOE and the Office for National Statistics (ONS) as the primary data source. The data used is quarterly. The sample period covers the years 1993:I-2013:II. During this time the MPC has been operating under inflation targeting and reporting its inflation forecasts on a quarterly basis. Though several earlier studies (e.g., Koop and Korobilis, 2014) use real-time/published data to construct FCIs, many FCIs (e.g., Goodhart and Hofmann, 2001; Castro, 2011) are estimated with *ex-post* data (i.e., subsequent revisions of published data). Adema (2004), Osterholm (2005) and Sauer and Strum (2007) argue that the use of real-time data (instead of *ex-post* data) would not lead to substantially different results. Kapetanios, Price and Young (2015) also maintain that *ex-post* financial data are still very useful in real time when attempting to understand and forecast macroeconomic development. Since the real-time data for house prices and output growth is difficult to access, this study uses *ex-post* data in the econometric exercise.

Appendix 4 shows the evolution of the primary variables used in the estimation of the FCIs for the UK. Prior to the estimation, this study considers several measures of the interest rate, the inflation rate and the output gap. In the estimation, it only chooses the ones that have been followed most closely by the BOE. A detailed description of the variables mentioned and their respective sources is presented in Appendix 1-3.

The alternative interest rate measures considered include the official central bank interest rate (OfficRate), the three-month inter-bank sterling lending rate (Libor3m) and the discount rate of three-month Treasury bills (TreasRate). This data is obtained from the statistics of the BOE. Nelson (2000) argues that the actual interest rate that was used by the BOE has changed over time including the bank rate, the minimum lending rate, the two-week repo rate, etc. In addition to Nelson (2000), Martin and Milas (2004) and Castro (2011) also argue that the TreasRate has a closer relationship with all the interest rate instruments used in the Bank's history. Following the literature, this study uses the TreasRate as the nominal interest rate (i_t) for the sample period analysed:

$$i_t = \frac{TreasRate_t}{100} \tag{3.1}$$

Following the BOE, the inflation rate (Infl_0) plotted in Appendix 4.2 is calculated as the annual rate of change in the Consumer Price Index (CPI). However, the CPI statistics only started in 1996. The historical estimate of inflation back to 1988 is calculated by the ONS based on the Retail Price Index (RPI). Following the existing literature in this field such as Martin and Milas (2014), this study calculates the inflation rate with the RPI for the period of 1993-1996. In order to ensure the stationarity of these variables, this study uses the inflation rate rather than the index:

$$\pi_t = ln\,(Price_t) - ln\,(Price_{t-4}) = ln\left[\frac{Infl\_0_t}{100} + 1\right] \tag{3.2}$$

There are some issues around estimating the output gap. It is calculated as the difference between real economic output and its potential level. However, potential output, defined as the highest level of real sustainable GDP, cannot be observed and is difficult to measure.

In order to measure the potential level of real GDP, Hodrick and Prescott (1997) note that the potential GDP has a smoothly varying trend. This trend is approximated well by passing the real GDP through the Hodrick-Prescott (HP) filter with a smoothness parameter, λ. Technically, the HP filter is a two-sided linear filter that calculates the smoothed series s of y by minimising the variance of y around s subject to a penalty that constrains the second difference of s. That is, the HP filter chooses s to minimise the result of the following equation:

$$\sum_{t=1}^{T}(y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1}[(s_{t+1} - s_t) - (s_t - s_{t-1})]^2 \tag{3.3}$$

The penalty parameter (λ) controls the smoothness of the series s_t. The larger the λ is the smoother the s_t will be. As λ approaches infinity, s_t approaches a linear trend.

Alternatively, Clarida, Gali and Gertler (2000) use the potential GDP estimated from a fitted quadratic function of time assuming that the trend GDP is deterministic as opposed to being stochastic. From a theoretic perspective, both the HP filter and the quadratic deterministic trend are variants of a detrended method that seeks to reduce the variability of a particular trend component. However, in practise the level of output is likely to behave differently in response to different real shocks such as shocks related to the changes in technology, productivity and consumer preferences. As argued in Chadha, Sarno and Valente (2004), the de-trended output may not gauge these situations in reality.

To overcome the disadvantages associated with measuring the potential level of real GDP, Orphanides and Williams (2002) and Ireland (2004, 2007) choose the real GDP growth rate as the output measure. Therefore, their output gap is defined as the difference between the real GDP growth rate and its own average.

As illustrated in Orphanides and Williams (2002), when the output gap is uncertain it may be better to relate monetary policy to changes in GDP rather than its level. That is because there is likely to be less uncertainty about changes in GDP than its starting level. Given that the supply capacity of the economy is increasing in line with its trend, the growth rate of real GDP provides an indication of changes in the size of output. Another advantage of using the real GDP growth rate is to avoid spurious regression results. As the real GDP level tends to inherit a unit root, Ireland (2007) indicates that central banks respond instead to the GDP growth rate as a stationary measure of the real economic activity. The sample average of the growth rate of the real GDP is taken by Ireland (2007) as the potential output in his estimation.

Therefore, this study uses the growth rate of real GDP as the unique indicator for the real economic output. The output gap ($\bar{y}_t$) is defined as the difference between the real GDP growth (gy_t) and its average ($\overline{gy}_t$):

$$gy_t = ln\ (Output_t) - ln\ (Output_{t-1}) \tag{3.4}$$

$$\bar{y}_t = gy_t - \overline{gy}_t \tag{3.5}$$

The data for the level of real GDP is sourced from the ONS. Appendix 4.3 plots the gy_t.

This study follows Castro (2011) and sources the real effective exchange rate index (REER) from the OECD database to measure the movement of sterling against the UK's primary trading partners. An increase in the REER indicates a sterling depreciation. Appendix 4.4 plots the evolution.

For the reasons discussed in Section 1.2.3, this study uses both the real house price index (RHPI) and the real share price index (RSPI) to assess asset price movements relative to goods price movements in the UK. The data for real house prices (RHP), obtained from the Nationwide Building Society, is used to derive the RHPI series. The RSPI is based on the quarterly average of the nominal FTSE 100 index (NSPI) that is obtained from the OECD statistics. Appendix 4.5 shows an upward trend in the RHPI prior to the 2008-9 subprime mortgage crisis. Affected by the spread of that crisis, the index drops considerably. Though it appears to rise again in 2009, the real house price level is still much lower than that before 2008. As plotted in Appendix 4.6, the RSPI looks quite different. It exhibits several spikes throughout the sample such as 1999-2000 and 2006-2007. More recently, it picks up significantly from the trough in 2009:I indicating a recovery of the UK equity market.

Given the rationale in the literature including Driffill et al. (2006), Teranishi (2012) and Castro (2011), this study considers two additional indicators, credit spreads (CredSprd) and changes in the future interest rate spreads (ΔFutSprd) in an FCI. The data for the three-month sterling future implied interest rate (FutIR) is plotted in Appendix 4.7. This study compares the ten-year UK government bond yields (Yield_10yr) against the yields of UK corporate bonds (CorpBond) in Appendix 4.8. Credit spreads are the difference between the two variables (see, Appendix 2). Similar to the other data, the difference between these two yields widens during the 2008-9 financial crisis.

As in Castro (2011), the real interest rate (rir_t) is the *ex-post* real interest rate obtained by subtracting the inflation rate from the nominal short-term interest rate:

$$rir_t = i_t - \pi_t \tag{3.6}$$

For the purpose of making the FCI interpretable, this study uses the deviation of each financial variable from its long-run trend level, which is consistent with much of the FCI literature (for instance, Castro, 2011). The name and the code of each constituent financial variable are presented in Appendix 3.

Table 1: Unit Root and Stationary Tests

	ADF	PP	KPSS
π_t	-1.2727	-1.9433	0.4658#
$\bar{y}_t$	-3.7689*	-3.8429*	0.5703#
rir_tGap	-0.6039	-0.5668	0.7743
REERGap	-3.3345*	-2.9621*	0.0378#
RSPIGap	-2.9945*	-2.7191*	0.0403#
RHPIGap	-3.9347*	-2.8855*	0.0731#
CredSprdGap	-5.1627*	-5.2267*	0.0247#
△FutSprdGap	-8.8033*	-10.485*	0.0918#

Note: *: Unit root is rejected at a significance level of 10%; #: The stationarity is not rejected at a significance level of 1%; all the test regressions here contain a constant.
Codes: rir_tGap: the difference between the three-month treasury bill discount rate and its steady-state level; REER Gap: the percentage deviation of the real effective exchange rate index from its HP (1997) filter; RSPIGap: the percentage deviation of the real share price index from its HP (1997) filter; RHPIGap: the percentage deviation of the real house price index from its HP (1997) filter; CredSprdGap: the percentage deviation of the credit spread from its HP (1997) filter; △FutSprdGap: the percentage deviation of changes in the sterling futures spread from its HP (1997) filter.

Table 1 reports the results of unit root and stationary tests for the variables used in the econometric exercise. Due to the low power and poor performance of unit root tests in small samples, this study follows the methodology used in Castro (2011). It reports the results of two unit root tests, i.e., augmented Dickey-Fuller (ADF, Dickey and Fuller, 1979) and Phillips-Perron (PP, Phillips and Perron, 1988) tests, and the KPSS stationarity (Kwiatkowski, Phillips, Schmidt and Shin, 1992) test results to see whether the power is an issue.

The test results in Table 1 imply that the power of unit root tests seems to be an issue for the UK. The ADF and PP tests are unable to reject the unit root in some variables such as π_t and rir_t Gap. However, according to the KPSS test π_t is stationary.

Although the evidence fails to support the stationarity hypothesis for rir_tGap given the sample period, if this study were to consider a longer time period it would expect to find evidence of stationarity for the real interest rate gap. Earlier studies, such as Petersen (2007) and Castro (2011), also give evidence showing that the real interest rate does not have a unit root. The unit root and stationarity tests for the additional variables in FCIs are also presented. These results are quite desirable. Both the ADF and the PP tests reject the hypothesis of a unit root in the times series of REERGap, RSPIGap, RHPIGap, CredSprdGap and ΔFutSprdGap. The KPSS test provides evidence of stationarity for all of these five variables.

1.4 Methodology

As earlier noted, methods for weighting financial variables in an FCI tend to fall into two categories: (i) a weighted sum approach or (ii) a principal-component approach. Section 1.4.1 introduces the former in which the weight on each financial variable is assigned based on the estimates of the relative impact of the changes in that variable on the output gap ($\bar{y}_t$). It applies the TVP-R-SV to a reduced form model to obtain the estimates. Section 1.4.2 discusses the TVP-FAVAR with SV model that shares many characteristics of a standard PCA. It also extracts an unobservable factor from a group of financial variables.

1.4.1 A Weighted-sum Approach – The 'Two-step' Method

This subsection aims to augment the traditional way of estimating a reduced form demand equation. It develops a 'two-step' method using the TVP-R-SV procedure but which is based on several earlier studies including Goodhart and Hofmann (2001), Hatzius et al. (2010) and Nakajima (2011a). The introduction of the two-step method using the TVP-R-SV procedure is an original contribution to literature.

1.4.1.1 The Reduced Form Aggregate Demand Equation

The reduced form model used in the two-step approach is produced by Rudebusch and Svensson (1999):

$$\bar{\pi}_t = \sum_{i=1}^{m} \beta_{1,i,t}\bar{\pi}_{t-i} + \beta_{2,t}\bar{y}_{t-1} + \varepsilon_{1,t} \tag{4.1}$$

$$\bar{y}_t = \sum_{k=1}^{n} \alpha_{1,k,t}\bar{y}_{t-k} + \alpha_{2,t}rirGap_{t-1} + \varepsilon_{2,t} \tag{4.2}$$

where $\bar{\pi}_t$ represents the de-meaned inflation rate, $\bar{y}_t$ is the real economic output gap and rir_t denotes the real interest rate. As in Rudebusch and Svensson (1999), the de-meaned real interest rate ($rirGap_t$) is used in this reduced form model. Eq. (4.1) is a backward-looking Phillips curve relating the de-meaned inflation rate ($\bar{\pi}_t$) to a lagged output gap ($\bar{y}_t$) and to lags of de-meaned inflation ($\bar{\pi}_{t-i}$). The i subscripts denote the lagging periods. Eq. (4.2) is a form of an IS curve which relates the real output gap to its own lags and to the lagged demeaned real interest rate. As in Rudebusch and Svensson (1999), the term $rirGap_t$ represents the monetary transmission channel which, in the view of central banks like the Bank of Canada (see, Freedman, 1994) and the BOE (see, MPC, June 2012), involves the interest rate, the exchange rate and possibly asset prices.

For the purpose of taking into account the roles of the FCI constituents in monetary policy setting, this study follows Goodhart and Hofmann (2001) and extends the above model as follows:

$$\bar{\pi}_{t+1} = \sum_{i=0}^{m} \beta_{1,i,t}\bar{\pi}_{t-i} + \beta_{2,t}\bar{y}_t + \varepsilon^S_{1,t+1} \tag{4.3}$$

$$\bar{y}_{t+1} = \alpha_{1,t}\bar{y}_t + \sum_{j=1}^{6} \alpha_{2,j,t}x_{j,t} + \varepsilon^D_{1,t+1} \tag{4.4}$$

Therefore, a system model consisting of Eq. (4.3) and Eq. (4.4) is the equivalent of a conventional backward-looking aggregate demand and supply (AD-AS) specification augmented with a financial market.

Based on the rationale given in Section 1.2.3, the information set x_t includes $rirGap_t$, REERGap, RSPIGap, RHPIGap, CredSprdGap and ΔFutSprdGap, all of which have useful information. Consistent with Castro (2011), this study uses the deviation of financial variables from their long-run trends to estimate an FCI. The definition of these six variables is available in Appendix 3.

Given the argument put forward by Hatzius et al. (2010) that an FCI measure should focus on the 'true' financial shocks, this study purges financial variables of their relation with the current macroeconomic conditions. The equation used below is similar to the one proposed in Hatzius et al. (2010) and Koop and Korobilis (2014):

$$z_{j,t} = \gamma_{j,t}\bar{y}_t + x_{j,t} \tag{4.5}$$

where $z_{j,t}$ denotes the j^{th} indicator at each point of time. Hatzius et al. (2010) purge nominal financial variables of both the inflation rate and the real GDP growth rate. As this study uses real financial variables, the only dependent variable employed in Eq. (4.5) is the de-meaned real GDP growth rate ($\bar{y}_t$). Therefore, the term $x_{j,t}$ denotes a real indicator uncorrelated with current macroeconomic activity. The $x_{j,t}$ that is obtained from Eq. (4.5) will then be used in Eq. (4.4).

The weights attached to each variable are obtained as:

$$w_{j,t} = \frac{|\alpha_{2,j,t}|}{\sum_{j=1}^{6} |\alpha_{2,j,t}|} \tag{4.6}$$

$$FCI_t = \sum_{j=1}^{6} \left(w_{j,t} \times x_{j,t}\right)$$

where $|\cdot|$ denotes the absolute value, and $\alpha_{2,j,t}$ is the parameter on $x_{j,t}$ in Eq. (4.4). An FCI is estimated as the internal product of the vector of weights and the vector of the purged financial variables as noted above. Drawing on the Nakajima (2011a) TVP-R-SV (discussed later in Section 1.4.1.2), this study develops a new method, the two-step method using the TVP-R-SV procedure, to estimate the unobserved FCI:

Step (1): Purge the financial variables of their relations with the de-meaned growth rate of real GDP. Unlike earlier studies such as Hatzius et al. (2010), Eq. (4.5) is estimated with the TVP-R-SV algorithm that is presented in the next section.

Although Hatzius et al. (2010) have disentangled the effect of current macroeconomic conditions on financial indicators, they fail to take into account the possible changes in the relationships between financial variables and macroeconomic conditions. Estimating Eq. (4.5) with the TVP-R-SV algorithm which allows for the possible changes in both parameters and volatilities corrects for this.

Step (2): Estimate a reduced form model with the purged financial variables obtained in the first step. This step also involves an application of the TVP-R-SV model. It estimates Eq. (4.4) with this time-varying parameter algorithm to obtain the coefficients at each point in time, t.

Allowing for the possibility of changes in the parameters, Castro (2011) employs the Kalman filter algorithm over the state-space form of Eq. (4.4). However, Nakajima (2011a) compares the regression performance of the TVP-R-SV algorithm against the Kalman filter. His result shows that the Kalman filter fails to take account of changes in volatility and lacks accuracy in estimating parameters. Using the TVP-R-SV (instead of the Kalman filter algorithm) is expected to reduce the estimation errors in Castro (2011) and also to recover the dynamic relationships between the de-meaned real GDP growth and its explanatory variables. The weight on each constituent of the FCI is obtained with Eq. (4.6).

The resulting FCI is expected to contain useful information on the monetary transmission mechanism. It produces valuable information regarding the financial health of the economy and future macroeconomic activity. Either a large positive or negative value of the FCI means that current financial market conditions have significantly deviated away from long-run trend and hence could be considered a warning signal. Furthermore, a rise in the FCI signals an improvement in the financial market and *vice versa*.

1.4.1.2 The TVP-R-SV Specification

Castro (2011) employs the Kalman filter to determine the weight assigned to each index constituent. He uses the TVP regression with constant volatility over the state-space form of Eq. (4.4):

$$\bar{y}_t = \alpha_t z_t' + \varepsilon_t \tag{4.7}$$

$$\alpha_t = L_t \alpha_{t-1} + \mu_t \tag{4.8}$$

where z_t' is a $p \times 1$ observation vector of seven elements, one lag of $\bar{y}_t$, rir_tGap, REERGap, RSPIGap, RHPIGap, CredSprdGap and ΔFutSprdGap, conforming to the description of Eq. (4.4). The $p \times p$ state vector of α_t contains all the slope parameters that are evolving over time, and p (= 7, in this case) therefore denotes the number of elements in the state vector. In this general formation, Eq. (4.7) is called an observation equation and Eq. (4.8) is a transition (or state) equation. Unpurged indicators are taken as explanatory variables for the real output gap. As in Castro (2011), this algorithm can estimate unobservable changes in any coefficient in α_t. It allows for recovering the dynamic relationships between the output gap and its explanatory variables.

Nakajima (2011a) highlights an important characteristic of the system in Eqs. (4.7-4.8) that the variances of the error terms, ε_t and μ_t are assumed to be time-invariant given by:

$$\varepsilon_t \sim N(0, \sigma^2), \mu_t \sim N(0, \Sigma) \qquad (4.9)$$

According to Nakajima (2011a), this algorithm is a reduced form of the TVP-R-SV in which the variance of ε_t (in Eq. 4.7) is constant. The simulation econometric exercise in Nakajima (2011a) shows that the application of the Kalman filter algorithm may result in biases in the estimated parameters. This motivates the use of the TVP-R-SV algorithm to construct an FCI in this study.

A basic TVP-R-SV is given as:

$$\bar{y}_t = \alpha_t z_t' + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_t^2) \qquad (4.10)$$

$$\alpha_t = I_p \alpha_{t-1} + \mu_t, \quad \mu_t \sim N(0, \Sigma) \qquad (4.11)$$

$$\sigma_t^2 = \gamma exp(h_t), \quad \gamma > 0; \quad h_{t+1} = \phi h_t + \eta_t, \quad |\phi| < 1 \qquad (4.12)$$

where α_t is a vector with p (equals to seven in this case) unknown parameters. Σ is a positive-definite matrix. Eq. (4.10) is a regression function, and the disturbance of the regression follows a normal distribution with a time-varying volatility, σ_t^2. The log-volatility (h_t) is formulated to follow the AR(1) process as in Eq. (4.12) – this is the principal difference between the TVP-R-SV and the Kalman filter algorithm. If ε_t is constant, the model forms a time-varying parameter regression with constant volatility which is equivalent to a standard Kalman filter algorithm. Letting I_p be an identity matrix, Eq. (4.11) sets parameters to follow the first-order random walk process thus allowing both a temporary and permanent shift in parameters. A drifting parameter is meant to capture possible nonlinearity such as a gradual change or structural break. The initial condition is set as: $\alpha_0 = 0$, $\mu_0 \sim N(0, \sum_0)$, $h_0 = 0$, $\eta_0 \sim N\left(0, \sigma_\eta^2/(1-\phi^2)\right)$.

Regarding the α_t and h_t as state variables, the TVP-R-SV then forms the state space model. As in Nakajima (2011a) and Malik and Banerjee (2013), a Bayesian approach using Markov chain Monte Carlo (MCMC) sampling is superior to the conventional maximum likelihood (ML) in this case. Given a time-variant σ_t^2, Eq. (4.10) forms a non-linear Gaussian state-space model. The traditional method using the ML requires a quite heavy computational burden to repeat the filtering many times to evaluate the likelihood function for each set of parameters until reaching the maximum. The MCMC allows for making some inference for the state variables with uncertainty of parameters (including α, $\sum$, γ, ϕ) with the samples drawn from the MCMC procedures. Thus, this study does not use the ML estimation and follows the Nakajima (2011a) algorithm. It uses the Bayesian inference and the MCMC sampling for the TVP-R-SV estimation.

1.4.1.3 The MCMC Algorithm for the TVP-R-SV

For carrying out the TVP-R-SV algorithm, this study specifies the prior distribution as $\pi(\theta)$. The purpose of this Section 1.4.1.3 is to explain the methodology for generating the joint posterior distribution, $\pi(\theta, \alpha, h \mid y)$. The functional form of $\pi(\theta, \alpha, h \mid y)$ derived by Nakajima (2011a) is given in Appendix 5. There are several ways to implement the MCMC algorithm to explore this distribution. This study follows the one taken by Nakajima (2011a) as follows:

Step (1): Initialize θ, α and h.

Step (2): Sample $\alpha \mid \sum, \gamma, h, \bar{y}$.

Step (3): Sample $\sum \mid \alpha$.

Step (4): Sample $h \mid \gamma, \phi, \sigma_\eta, \alpha, \bar{y}$.

Step (5): Sample $\phi \mid \sigma_\eta, h$.

Step (6): Sample $\sigma_\eta \mid \phi, h$.

Step (7): $\gamma \mid \alpha, h, \bar{y}$.

Step (8): Go to Step (2).

The details of this procedure are described in Appendix 6. This study adds value to the derivation of this procedure in Nakajima (2011a) by providing a much clearer interpretation drawing on readings on Bayesian inference.

1.4.2 A Principal-component Approach – The TVP-FAVAR Model

This section discusses a time-varying parameter factor augmented VAR in Koop and Korobilis (2014) and its variants. It uses the TVP-FAVAR model to estimate an FCI for the UK for the first time.

This methodology partly features a standard PCA by estimating an FCI as the co-movements of multiple variables. To distinguish it from a standard PCA in Hatzius et al. (2010), Koop and Korobilis (2014) allow the loadings to change over time. This specification builds on the findings in earlier studies including Hollo et al. (2012) and allows for changes in the correlations between financial variables. Their purpose in estimating an unobserved FCI in a VAR is to evaluate the forecasting performance of the index. It aims to answer questions such as 'what makes a good FCI'.

1.4.2.1 The TVP-FAVAR Models

Following Koop and Korobilis (2014), this study writes a p-lag TVP-FAVAR as:

$$x_t = \lambda_t^y \bar{y}_t + \lambda_t^f f_t + u_t, \quad u_t \sim N(0, V_t) \tag{4.13}$$

$$\begin{bmatrix} \bar{y}_t \\ f_t \end{bmatrix} = c_t + B_{t,1} \begin{bmatrix} \bar{y}_{t-1} \\ f_{t-1} \end{bmatrix} + \cdots + B_{t,p} \begin{bmatrix} \bar{y}_{t-p} \\ f_{t-p} \end{bmatrix} + \varepsilon_t, \quad \varepsilon_t \sim N(0, Q_t) \tag{4.14}$$

where x_t is an $n \times 1$ vector of unpurged variables (for instance, RSPIGap, RHPIGap, REERGap, CredSprdGap, ΔFutSprdGap and rir_tGap) for computing an FCI. The term $\bar{y}_t$ denoting the de-meaned growth rate of real GDP, enters Eq. (4.13) to purge the impact of the current macroeconomic conditions from the financial indicators. In order to compare the FCIs resulting from different weighting methods, the same financial indicators as used in Section 1.4.1 are employed in this subsection. The unobservable factor f_t is interpreted as the FCI. Following the suggestion in Hatzius et al. (2010), Wacker et al. (2014) and Paries et al. (2014), this study extracts only one factor from x_t. The terms u_t and ε_t are the zero-mean Gaussian errors with covariances V_t and Q_t. λ_t^y and λ_t^f respectively denote the regression coefficients and loadings. $B_{t,1}, \ldots B_{t,p}$ are VAR parameters. This model differs from the conventional PCA in that it allows for the time-variation in loadings. Negro and Otrok (2008) and Eickmeier, Lemke and Marcellino (2009) suggest a model where the loadings are set as random walks. Primiceri (2005) also assumes that the VAR parameters follow a random walk process. Following these papers, this study sets λ_t^y, λ_t^f and $B_{t,1}, \ldots B_{t,p}$ as:

$$\lambda_t = \lambda_{t-1} + v_t, \quad v_t \sim N(0, W_t) \tag{4.15}$$

$$\beta_t = \beta_{t-1} + \eta_t, \quad \eta_t \sim N(0, R_t) \tag{4.16}$$

where $\lambda_t = \left((\lambda_t^y)', (\lambda_t^f)'\right)'$, $\beta_t = \left(c_t', vec(B_{t,1})', \ldots, vec(B_{t,p})'\right)'$. Given Primiceri (2005) recommendation, this study considers the heteroskedasticity (i.e., when V_t and Q_t are time-variant). As in Primiceri (2005) and other factor literature including Koop and Korobilis (2014), the covariance matrix V_t is diagonal thus ensuring that u_t is a vector of idiosyncratic shocks.

Use the notation that $z_t = \begin{bmatrix} \bar{y}_t \\ f_t \end{bmatrix}$. Eq. (4.13) and Eq. (4.14) can be written in state-space form as:

$$x_t = z_t \Lambda_t + u_t, \quad u_t \sim N(0, V_t) \tag{4.17}$$

$$z_t = z_{t-1} \beta_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, Q_t) \tag{4.18}$$

where all the disturbance terms in the above equations are uncorrelated over time and with each other. A system of Eq. (4.15-4.18) now constitutes a TVP-FAVAR with SV. It takes into account the likely changes in parameters (λ_t^y and β_t) and loadings (λ_t^f). The disturbance of Eq. (4.17) and Eq. (4.18) follows the normal distribution with changes in volatilities (V_t and Q_t). Koop and Korobilis (2014) stress another characteristic of this system – unlike Stock and Watson (2002) who extract a factor and then use it in a separate forecasting regression (as described in Section 1.2.4.5), Koop and Korobilis (2014) use a multivariate system to forecast $\bar{y}_t$ and model all variables jointly.

As in Koop and Korobilis (2014), in order to investigate whether a TVP-FAVAR with SV model is superior to other factor models in estimating an FCI for the UK, this study considers several restrictions on this model and obtains two versions of the FAVAR model. The two restricted forms of the TVP-FAVAR with SV model are explained as follows:

(i) An FA-TVP-VAR with SV model with the restriction that λ_t^f is fixed. This specification assumes time-varying parameters (λ_t^y and β_t) and stochastic volatility but constant loadings (λ_t^f). It is close to a standard PCA. The objective of estimating an FA-TVP-VAR with SV model is to assess whether a system of Eq. (4.15-4.18) outperforms a standard PCA when taking into account the proposal in Hatzius et al. (2010).

(ii) A TVP-FAVAR with CV model with the restriction that V_t and Q_t are fixed. It assumes the time-varying parameters (λ_t^y and β_t) and loadings (λ_t^f) but constant volatility. The objective of estimating a TVP-FAVAR with CV model is to assess whether a heteroskedastic version is better than a homoskedastic specification while estimating an unobservable FCI.

The criterion for evaluating an FCI is developed in Hatzius et al. (2010) and Koop and Korobilis (2014). It is an FCI that forecasts macroeconomic activity as well as possible. Apart from that, Eq. (4.18) alone is able to assess the forecasts of existing FCIs. For a comparative purpose, this study also compares the forecasting ability of the FCIs based on TVP-FAVARs with the FCIs obtained from the weighted-sum method in Section 1.4.1

1.4.2.2 Estimation Methodology of a TVP-FAVAR

This subsection illustrates the full estimation procedures for a TVP-FAVAR with SV model and the econometric theories behind it. If researchers have selected a specification for V_t, Q_t, W_t and R_t and a prior for the initial conditions, the Bayesian inference can be taken in a straightforward fashion with the MCMC sampling. As in Nakajima (2011a), this algorithm works well with the TVP-Rs and small TVP-VARs. However in the case of an FAVAR model with TVPs and SVs, Koop and Korobilis (2013, 2014) maintain that this Bayesian simulation method is computationally intensive, as over tens of thousands of draws may be required to reach the proper convergence. For the goal of estimating a TVP-FAVAR with SV model, Koop and Korobilis (2013, 2014) use two models, the forgetting factor model and exponentially weighted moving average (EWMA) model. They then develop a fast two-step algorithm based on a dual Kalman filter method.

Forgetting factors (also known as discount factors) that are used with state space models such as Raftery, Karny and Ettler (2007) and Koop and Korobilis (2012, 2013), do not require the use of the MCMC sampling. To explain the role of these forgetting factors in the estimation of a TVP-FAVAR with SV model, this study considers a standard state-space model with forgetting for the adaptive estimation of the regression parameters. This is an essentially standard Kalman filter algorithm and the purpose of reviewing it is to fix ideas. Let z_t be a $1 \times m$ vector of explanatory variables for the dependent variable y_t for $t = 1, \dots, T$:

$$y_t = z_t \theta_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, H_t) \tag{4.19}$$

$$\theta_t = \theta_{t-1} + \eta_t, \quad \eta_t \sim N(0, Q_t) \tag{4.20}$$

where Eq. (4.19) is an observation equation and Eq. (4.20) is a state equation. θ_t is an $m \times 1$ vector of coefficients (i.e., states). The error terms ε_t and η_t are assumed to be mutually independent at all leads and lags. As in Raftery et al. (2007), for any given values of H_t and Q_t the Kalman filter algorithm can be used to carry out recursive inference. It begins with the result that:

$$\theta_{t-1} | Y^{t-1} \sim N\left(\hat{\theta}_{t-1}, \Sigma_{t-1|t-1}\right), \quad Y^{t-1} = \{y_1, y_2, \dots, y_{t-1}\} \tag{4.21}$$

The Kalman filter algorithm proceeds with:

$$\theta_t|Y^{t-1} \sim N\left(\hat{\theta}_{t-1}, \Sigma_{t|t-1}\right), \quad Y^{t-1} = \{y_1, y_2, \dots, y_{t-1}\} \tag{4.22}$$

where:

$$\Sigma_{t|t-1} = \Sigma_{t-1|t-1} + Q_t \tag{4.23}$$

The subscript '$t|t-1$' is read as the time $t-1$ forecasts given information up to and including time $t-1$. Eq. (4.22) is the prediction equation. Raftery et al. (2007) suggest that the $m \times m$ matrix Q_t is required for computation. However, there is little information available for doing so. Therefore, they recommend using a form of forgetting and replacing Eq. (4.23) by:

$$\Sigma_{t|t-1} = \frac{1}{\lambda}\Sigma_{t-1|t-1}, \quad 0 < \lambda \leq 1 \tag{4.24}$$

The resulting model is written as:

$$Q_t = \left(\frac{1}{\lambda} - 1\right)\Sigma_{t-1|t-1}, \quad 0 < \lambda \leq 1 \tag{4.25}$$

where the parameter, λ is referred to as a forgetting factor. As in Hannan, McDougall and Poskitt (1989) and Koop and Korobilis (2012), the name 'forgetting factor' means that in the above specification observation j periods in the past receives a weight of λ^j. For instance, for quarterly macroeconomic data, a factor of 0.99 suggests that an observation five years ago receives roughly 80% (i.e., 0.99^{20}) as much weight as last period's observation. In the state space literature including Raftery et al. (2007) and Koop and Korobilis (2012, 2014), it is common to use a value of λ close to 1.0. Raftery et al. (2007) complete some experiments in the area of factor specification and discover that this method's outcome (i.e., using a forgetting factor) is not sensitive to the reasonable changes in forgetting factors and the selection of a forgetting factor. Given Eq. (4.24), it is now unnecessary to estimate Q_t while estimating the prediction equation. What is required is an approach to estimate H_t.

Estimation in the system of Eq. (4.19-4.20) is completed by the updating equation:

$$\theta_t|Y^t \sim N\left(\hat{\theta}_t, \Sigma_{t|t}\right), \quad Y^t = \{y_1, y_2, \dots, y_t\} \tag{4.26}$$

where:

$$\hat{\theta}_{t|t} = \hat{\theta}_{t|t-1} + \Sigma_{t|t-1} z_t' \left(H_t + z_t \Sigma_{t|t-1} z_t'\right)^{-1} \left(y_t - z_t \hat{\theta}_{t-1}\right) \tag{4.27}$$

$$\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{t|t-1} z_t' \left(H_t + z_t \Sigma_{t|t-1} z_t'\right)^{-1} z_t \Sigma_{t|t-1} \tag{4.28}$$

As emphasised by Koop and Korobilis (2012), all the above results are analytical and does not require the MCMC algorithm. This feature should reduce the computational burden significantly.

Koop and Korobilis (2014) suggest using the EWMA model for calculating the time-varying variances, V_t and Q_t. To explain the general idea of this technique, this study refers to Risk-Metrics (1996, p. 78) emphasising two key advantages of the EWMA estimator (Eq. 4.30) over an equally weighted model (Eq. 4.29): *"first volatility reacts faster to shocks, as recent data carries more weight than the data in the past; second, following a shock, the volatility declines exponentially because the weight of the shock observation falls"*. Let σ^2 denote the variance of time series data, g_t. For a given set of T observations, the formulas used to compute equally and exponentially weighted standard deviations are written as:

$$\sigma = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(g_t - \bar{g})^2} \tag{4.29}$$

$$\sigma = \sqrt{(1-\lambda)\sum_{t=1}^{T}\lambda^{t-1}(g_t - \bar{g})^2} \tag{4.30}$$

In Eq. (4.30) the EWMA model depends on the parameter λ $(0 < \lambda < 1)$. It is a decay factor. Eq. (4.29) and Eq. (4.30) are equivalent in the limit, i.e., as $T \rightarrow \infty$. Using an example of the GMP/DEM exchange rate in the Autumn of 1992, Risk-Metrics (1996) favours Eq. (4.30), as it incorporates external shocks better than Eq. (4.29) and thus provides a more realistic measure of current volatility. Apart from that, Risk-Metrics highlights another attractive feature of an EWMA estimator which will be used in the econometric exercises in this study. Assume the mean $\bar{g}$ is zero, then the dynamic volatility σ_t^2 of the time series g_t can be written in a recursive form:

$$\sigma^2_{t+1|t} = (1-\lambda)\sum_{i=0}^{\infty}\lambda^i g^2_{t-i} = (1-\lambda)(g^2_t + \lambda g^2_{t-1} + \lambda^2 g^2_{t-2} + \cdots)$$
$$= (1-\lambda) g^2_t + \lambda(1-\lambda)(g^2_{t-1} + \lambda g^2_{t-2} + \cdots)$$
$$= \lambda\sigma^2_{t|t-1} + (1-\lambda) g^2_t \tag{4.31}$$

The Koop and Korobilis (2014) two-step estimation method for a TVP-FAVAR with SV model is built on the work of Doz, Giannone and Reichlin (2011). Since both the factor f_t and the loadings λ_t (in Eq. 4.13) are not observable, it is impossible to apply the Kalman filter algorithm for the state space model directly. Therefore, a dual linear Kalman filter is required in this situation: in the first step, update the parameter $\theta_t = (\lambda_t, \beta_t)$ with an estimated f_t. As noted in Doz et al. (2011), the principal component estimates of f_t based on $x_{1:t}$ can be used to estimate θ_t. Bates et al. (2013) give both theoretical and empirical support for this estimation algorithm. In the second step, update the factor f_t with the estimate of θ_t. Hence, this procedure requires the use of two distinct Kalman filters/smoothers, one for θ_t and the other for f_t. To be distinguished from the two-step method (as in Section 1.4.1) implemented in two separate steps, this algorithm models a system of Eq. (4.15-4.18) jointly. As highlighted in Koop and Korobilis (2014), this approach greatly improves the estimation efficiency for an FCI.

In summary, the estimates of the error covariance matrices in multivariate time series specifications require extremely computationally extensive methods. This study follows Koop and Korobilis (2014) to estimate the TVP-FAVAR with SV model. For matrices V_t and Q_t (i.e., the variance of u_t and ε_t in Eq. 4.17-4.18), it uses the EWMA estimator. For modelling matrices W_t and R_t (i.e., the variance of u_t and ε_t in Eq. 4.15-16), it uses the forgetting factors. Appendix 7 illustrates the structure of the estimation algorithm in more detail.

1.5 Empirical Evidence

This section estimates the reduced form model and the TVP-FAVARs as described in the methodology section. It uses the same variables for all the estimation models (i.e., rir_tGap, REERGap, RSPIGap, RHPIGap, CredSprdGap and ΔFutSprdGap). The aim is to find the most appropriate variable-weighting method for the UK. The sample runs from 1993:I to 2013:II. Section 1.5.1 discusses the FCIs produced by a reduced form model using the two-step estimation approach. Section 1.5.2 presents the FCIs produced by a TVP-FAVAR and its variants. For comparative purposes, this study examines the forecasting ability of each FCI at the end of Section 1.5.2.

1.5.1 Two-step Method – The TVP-R Estimation

As described in Section 1.4.1.1, the first step in implementing the two-step method is to purge the financial variables of their relationship with current macroeconomic activity:

$$z_{j,t} = \gamma_{j,t}\bar{y}_t + x_{j,t} \tag{5.1}$$

where $z_{j,t}$ denotes the i^{th} indicator at time t and $x_{j,t}$ is the purged financial variable. Therefore, the $x_{j,t}$s reflects the true financial shocks. For the reasons given in Section 1.3, this study uses the de-meaned growth rate of real GDP ($\bar{y}_t$) to measure current macroeconomic activity. The TVP-R-SV algorithm is used to estimate Eq. (5.1) for the six financial variables respectively: rir_tGap, REERGap, RSPIGap, RHPIGap, CredSprdGap and ΔFutSprdGap.

In the second step, the $x_{j,t}$ values will be substituted into the reduced form model:

$$\bar{y}_{t+1} = \alpha_{1,t}\bar{y}_t + \sum_{j=1}^{6}\alpha_{2,j,t}x_{j,t} + \varepsilon_{1,t+1}^{D} \tag{5.2}$$

Eq. (5.2) is then estimated with the TVP-R-SV algorithm.

To implement the TVP-R-SV model, this study draws 10,000 samples, after the initial 1,000 samples are discarded, by assuming the following prior distributions:

$$\textstyle\sum \sim IW(4,40\times I), \quad \alpha_1 \sim N(0,10\times I), \tag{5.3}$$

$$\frac{\phi+1}{2} \sim Beta(20,1.5), \quad \sigma_\eta^2 \sim IG(2,0.02), \quad \gamma \sim IG(2,0.02)$$

For comparative purpose, it re-estimates Eq. (5.1) and Eq. (5.2) using a time-varying parameter regression with time-invariant volatility (TVP-R-CV). The MCMC in the frame of Bayesian inference is employed for this estimation. The FCI produced by the two-step method, but estimated using the TVP-R-CV, is named as the CV-FCI.

Table 2 produces the estimates for the posterior means, standard deviations, credible intervals and convergence diagnostics (CD) (Geweke, 1992). To check the convergence of the Markov chain, Geweke (1992) suggests comparing the first n_0 draws with the last n_1 draws. The middle draws are not included. The CD statistics are computed by:

$$CD = (\bar{x}_0 - \bar{x}_1) / \sqrt{\frac{\hat{\sigma}_0^2}{n_0} + \frac{\hat{\sigma}_1^2}{n_1}} \tag{5.4}$$

where

$$\bar{x}_j = (1/n_j) \sum_{i=m_j}^{m_j+n_j-1} x^{(i)} \tag{5.5}$$

$x^{(i)}$ is the i^{th} draw. $\sqrt{\hat{\sigma}_j^2/n_j}$ is the standard error of $\bar{x}_j$ respectively for $j = 0, 1, 2, \ldots$ If the sequence of the MCMC sampling is stationary, then the distribution converges to a standard normal distribution. This study sets $m_0 = 1, n_0 = 1000, m_1 = 5001$ and $n_1 = 5000$, and the $\hat{\sigma}_j^2$ is calculated using a Parzen (1962) window with a bandwidth of $B_m = 500$.

To measure how well the MCMC chain mixes, Table 2 reports the inefficiency factor (Inef) for the selected parameters in the last column. Inef is defined as the ratio of the numerical variance of the posterior sample average to that of the sample average from uncorrelated draws:

$$Inef = 1 + 2 \sum_{s=1}^{B_m} \rho_s \tag{5.6}$$

where ρ_s is the sample autocorrelation at lag s. As in Nakajima (2011a), when the Inef is equal to M the exercise needs to draw the MCMC sample M times as many as the uncorrelated sample. The smaller the CD and inefficiency factors are, the stronger the convergence and the more efficient the MCMC estimation will be.

In the estimated result, the null hypothesis of convergence to the posterior distribution is not rejected for the parameters at the 5% significance level based on the CD statistics. The inefficiency factors are quite low (<100), which implies an efficient sampling for the parameters and the state variables in the time-varying parameter regressions. To assess whether there is stochastic volatility this study reports the estimated ϕ, σ_η and γ for the TVP-R-SV algorithm, all of which differ significantly from zero indicating that the time-invariant volatility hypothesis can be rejected in this case. In addition, it also finds that given the restriction of constant volatility some of the estimated parameters have changed. This motivates the further comparison of the estimation results between the two specifications.

Table 2: Estimation Results of the TVP-R Algorithm

Parameter	Mean	Stdev	95%L	95%U	Geweke	Inef
Panel 1: TVP-R-SV results						
Sig11 ($\sum_{11}$)	0.0547	0.0395	0.0107	0.1638	0.293	37.13
Sig22 ($\sum_{22}$)	0.0093	0.0055	0.0029	0.0241	0.086	31.57
Sig33 ($\sum_{33}$)	0.0066	0.0036	0.0023	0.0160	0.001	15.75
Sig44 ($\sum_{44}$)	0.0026	0.0009	0.0013	0.0047	0.764	10.02
Sig55 ($\sum_{55}$)	0.0081	0.0040	0.0029	0.0182	0.003	18.86
Sig66 ($\sum_{66}$)	0.0386	0.0473	0.0049	0.1546	0.454	62.49
Sig77 ($\sum_{77}$)	0.0253	0.0247	0.0040	0.0912	0.001	60.08
Phi (ϕ)	0.8639	0.0989	0.6170	0.9858	0.774	8.830
Siget (σ_η)	0.1233	0.0539	0.0583	0.2603	0.555	28.10
Gamma (γ)	0.0139	0.0107	0.0036	0.0414	0.098	23.76
Panel 2: TVP-R-CV results						
Sig11 ($\sum_{11}$)	0.0434	0.0371	0.0075	0.1348	0.722	34.38
Sig22 ($\sum_{22}$)	0.0099	0.0059	0.0028	0.0253	0.195	15.57
Sig33 ($\sum_{33}$)	0.0066	0.0038	0.0023	0.0162	0.605	20.88
Sig44 ($\sum_{44}$)	0.0026	0.0009	0.0013	0.0048	0.704	8.660
Sig55 ($\sum_{55}$)	0.0083	0.0041	0.0029	0.0186	0.074	15.30
Sig66 ($\sum_{66}$)	0.0314	0.0325	0.0048	0.1240	0.049	43.85
Sig77 ($\sum_{77}$)	0.0257	0.0247	0.0043	0.0889	0.796	39.42
Sigma (σ)	0.1190	0.0440	0.0602	0.2268	0.001	45.08

This study computes the sum of squared error (SSE) and the root mean squared error (RMSE) with the in-sample data for both the TVP-R-SV and TVP-R-CV algorithm. The in-sample errors are:

$$In\ sample\ error_t = \bar{y}_t - \hat{y}_t \tag{5.7}$$

where $\hat{y}_t$ is the estimated real output gap, and t is the time subscript. This study discovers that allowing for stochastic volatility tends to reduce the RMSE and SSE by roughly 11.27% and 21.27% respectively. This finding gives additional support to Nakajima (2011a) that a TVP-R-SV algorithm outperforms the conventional constant-volatility regression (the TVP-R-CV), which is equivalent to the Kalman filter algorithm, improves the accuracy of the estimation.

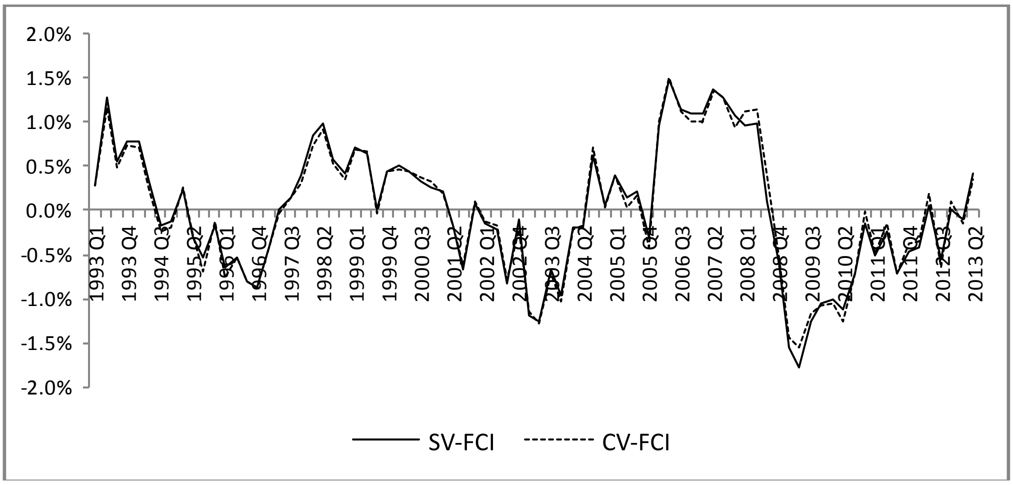


Figure 1: FCIs from the Weighted-sum Approach

Figure 1 plots the FCI using this two-step method. Estimates are provided using the two TVP algorithms. As already mentioned, the resulting FCI could be considered as a measure of the deviation of financial markets from long-term trend similar to Castro (2011) and Goodhart and Hofmann (2001). A rise signals an improvement in financial markets with growing asset prices, narrowing credit spreads, depreciating currency and *vice versa*.

As plotted in Figure 1, the TVP-R-SV based FCI (SV-FCI) rises from 0.278% (relative to the long-term trend) in the beginning of the sample period to reach a local peak around 1993:II. It then appears to decline reaching a trough in 1996:IV. Another three peaks exist in 1998:II, 2006:II and 2007:II. The decline after 2007:II shows the deterioration of the UK financial market following the beginning of the global financial crisis in August 2007 in the US. Affected by the spread of that crisis, the index drops dramatically and hits the lowest level in 2009:II. Since 2009:III, the SV-FCI has kept rising except for a slight fall in 2011:I, 2011:III and 2012:III. It becomes positive at the end of the sample and indicates the recovery of the UK financial market relative to the long-run trend.

Prior to 2002, the trend of the SV-FCI is consistent with the estimates of Castro (2011) for the UK. Both are using the same financial variables. As already mentioned, Castro (2011) uses unpurged data and applies the Kalman filter algorithm to Eq. (4.4) to derive an FCI. The FCI in Castro (2011) is found to be positive around 2002-2003 but negative between 2005 and 2007, which is far from the reality. The MPC's Inflation report published in August 2002 (p. 3-5) outlines how the corporate accounting irregularities in the US became increasingly apparent after the bankruptcy of Enron in December 2001 and triggered the decline in international equity prices. In the UK, equity prices fell to a six-year low in July 2002. Although house prices rose in 2002:III, the performance of the real estate market was not adequate enough to offset the deterioration of the domestic equity market. In 2003:I, the MPC (February 2003, p. 4) acknowledged both the continuation of the deterioration in the equity market and the further rise in the volatility of equity prices due to the greater perceived likelihood of a war in Iraq. Given the BOE's comments on the UK economy at that time it is reasonable to argue that the SV-FCI with negative values between 2002 and 2003 matches more closely the financial conditions in the UK at that time.

In addition, the 2005-2007 period is characterised by a significant rise in asset prices both in the UK and around the world. As displayed in Appendix 4, the credit spread narrows during this time and the real effective exchange rate remains at a high level. All these indicators are suggesting an improvement in the financial market relative to its long-run trend, which means that the estimated FCI in Castro (2011) fails to properly describe the development of the UK financial system for the period 2005-2007.

Although the TVP-R-CV algorithm results in a higher value of the RMSE and SSE, the two estimated indices based on the TVP-R-SV and TVP-R-CV respectively are quite similar in Figure 1. However, differences still exist in particular before and during the recent 2008-9 financial crisis. An important finding at this stage is that similar results are found for both the SV-FCI and CV-FCI for the 2002 and 2007 periods. It is crucial to note that the CV-FCI is obtained with the same model and variables as in Castro (2011). The only difference between both estimations is that this study estimates the CV-FCI with purged financial variables whereas Castro (2011) uses the raw unpurged data. This implies that use of unpurged financial data is likely to have distorted the estimate of such an index. This can be thought of as additional supportive evidence for the proposal in Hatzius et al. (2010).

1.5.2 The Principal Component Method – The TVP-FAVARs Estimation and Forecasting

For comparison purposes this subsection uses the same variables as in Section 1.5.1. It uses the de-meaned growth rate of real GDP ($\bar{y}_t$ in Eq. 3.5) to measure the macroeconomic activity and employs rir_t Gap, REERGap, RSPIGap, RHPIGap, CredSprdGap and ΔFutSprdGap to construct an FCI.

This study re-states Eq. (4.13) and Eq. (4.14) as:

$$x_t = \lambda_t^y \bar{y}_t + \lambda_t^f f_t + u_t, \quad u_t \sim N(0, V_t) \tag{5.8}$$

$$\begin{bmatrix} \bar{y}_t \\ f_t \end{bmatrix} = c_t + B_{t,1} \begin{bmatrix} \bar{y}_{t-1} \\ f_{t-1} \end{bmatrix} + \cdots + B_{t,p} \begin{bmatrix} \bar{y}_{t-p} \\ f_{t-p} \end{bmatrix} + \varepsilon_t, \quad \varepsilon_t \sim N(0, Q_t) \tag{5.9}$$

The result is estimated using the full sample period of data from 1993:I to 2013:II. As mentioned in Section 1.2.3, some FCIs include the short-term interest rate along with a set of financial variables and others focus on the monetary transmission mechanisms that are not accounted for by the traditional interest rate channel. This study considers both cases. To compare the FCIs' forecasting performance, it uses indices that contain the interest rate. Hence, the de-meaned real interest rate rir_tGap still enters as one of the x_t at this stage.

Figure 2 displays the factor estimates using the six financial variables. The estimates from the three versions of the TVP-FAVARs are very similar except for a small number of differences at certain times. This finding is consistent with the results in Koop and Korobilis (2014) for the US data. As maintained in Koop and Korobilis (2014), the stochastic volatility may partially explain why the TVP-FAVAR with SV model and TVP-FAVAR with CV estimates are on average similar but differ more substantially at some peaks and troughs.

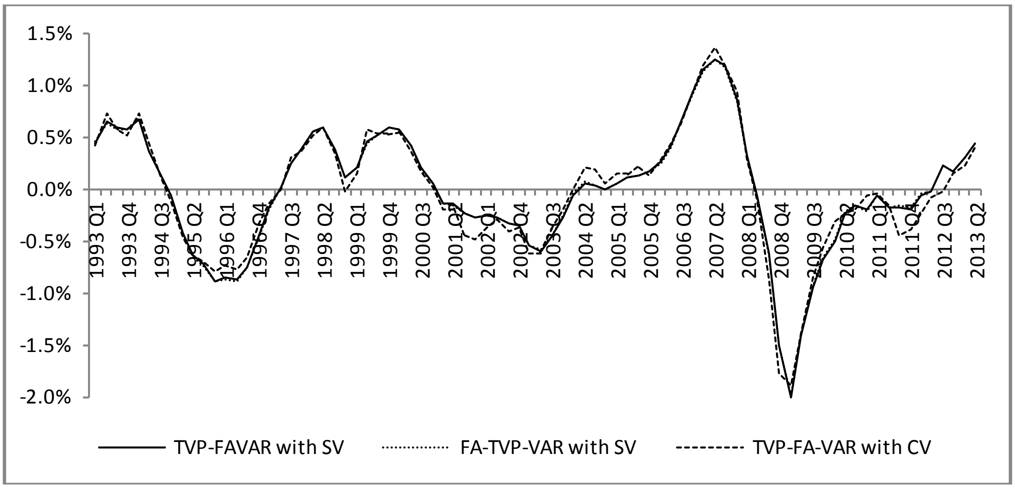


Figure 2: FCIs Produced by TVP-FAVARs:

Furthermore, performing a comparison of FCIs from the weighted-sum approach and PCA gives more findings. Although the broad patterns in the FCIs plotted in Figure 1 and 2 are similar, there are significant differences. The estimates produced by the factor models (in Figure 2) are smoother, and they lead both the SV-FCI and CV-FCI during the last financial crisis. It is interesting to note that the factor estimates usually achieve lower values at some peaks across the sample. As compared with the estimate from Koop and Korobilis (2014), all the estimated FCIs in the UK tend to lag the US index in the recent crisis in 2008-9, which is consistent with prior expectations.

At this moment, it is difficult to express any view on whether any FCI is better or worse than others. As in Hatzius et al. (2010) and Koop and Korobilis (2014), a good FCI is one that predicts macroeconomic activity as well as possible. Thus, this study follows Koop and Korobilis (2014) and examines the performance of the FCIs estimated for forecasting the de-meaned real GDP growth rate. Since the MPC (June 2012) estimates that it takes roughly 4-5 quarters for real GDP to feel the maximum effect of an interest rate change, this study includes four lags of fci_t in Eq. (5.9), i.e., setting $p = 4$. With the sample period running from 1993:I-2013:II, the evaluation period covers between 1994:I and 2013:II for $h = 1, 2, \ldots, 4$ quarters ahead. The selection of the evaluation period is designed to include the longest in-sample period for prediction.

The process of predicting macroeconomic activity with FCIs is done in Eq. (5.9). Re-writing the forecasting equation as:

$$\bar{y}_{t+h} = \left(b_{t,1}\bar{y}_{t+h-1} + \cdots + b_{t,p}\bar{y}_{t+h-p}\right) + \left(c_{t,1}fci_{t+h-1} + \cdots + c_{t,p}fci_{t+h-p}\right) + \varepsilon_t, \quad \varepsilon_t \sim N(0, Q_t) \tag{5.10}$$

where the FCIs in this equation are produced by various models including the two-step TVP-R-SV, two-step TVP-R-CV and the three versions of the TVP-FAVAR. The forecasting accuracy evaluation is completed with the following steps:

Step (1): Estimate the baseline model that is a TVP-VAR with stochastic volatility using three macroeconomic variables $\bar{y}_t$, rir_t and π_t that occur in the traditional monetary transmission mechanism.

Step (2): Use this baseline model to forecast output growth ($\bar{y}_t$) and calculate its mean squared forecast error (MSFE). Therefore, the output growth rate is predicted with their own lags, lagged inflation rates and interest rates (i.e., no FCIs).

Step (3): Forecast output growth with its own lags and lagged FCIs (see, Eq. 5.10). Then estimate the MSFE for each FCI used. This study evaluates an FCI's forecasting accuracy by comparing its MSFE with the MSFE of the baseline model. The purpose is to examine whether the inclusion of an FCI would improve the prediction of economic activities.

Table 3: Forecasting Performance of FCIs (MSFE)

	h=1	h=2	h=3	h=4
Panel 1: FCIs from the principal-component approach:				
Actual MSFEs of a TVP-VAR (no FCI)	0.4165	0.7106	0.8379	0.7232
TVP-FAVAR with SV model	*0.7307	*0.6648	*0.7242	*0.8462
FA-TVP-VAR with SV model	*0.7309	*0.6650	*0.7242	*0.8458
TVP-FAVAR with CV model	*0.7317	*0.6670	*0.7287	*0.8583
Panel 2: FCIs from the weighted-sum approach:				
Two-step method based on a TVP-R-SV	*0.7573	*0.6899	*0.7657	*0.8860
Two-step method based on a TVP-R-CV	*0.7593	*0.6975	*0.7666	*0.8906

Note: This study employs the Diebold-Mariano (1995) test to examine whether the forecast errors differ significantly from the benchmark's MSFEs. The test is developed by Diebold and Mariano (1995) and comprehensively described in Garratt, Koop, Mise and Vahey (2009). If an MSFE has a *, it means that method forecasts significantly different from the benchmark TVP-VAR.

Table 3 presents the forecasting performance for each FCI. For the benchmark (i.e., no FCI), it gives the actual MSFEs. The details of the choice of forgetting factors are presented in Appendix 7. Panel 1 is organised so that it begins with a TVP-VAR with stochastic volatility and then presents the MSFEs of a TVP-FAVAR with SV model relative to those of the benchmark. For instance, the number 0.7307 means that one-quarter ahead, the FCI based on the TVP-FAVAR with SV model can lower the MSFE relative to benchmark by roughly 27% ($\approx 1 - 0.7307$). Panel 1 Table 3 also compares the FCIs from two restricted versions. Panel 2 gives the forecasting results for both the SV-FCI and CV-FCI. Several observations stand out:

Firstly, in general Panel 1 shows a pattern where forecasts improve by adding in the extensions. Moving from the benchmark to the constant loading model (FA-TVP-VAR with SV) has large benefits for both the short-term ($h = 1,2$) and medium-term ($h = 3,4$) forecasts of the de-meaned real GDP growth rate. This result confirms the empirical findings in Hatzius et al. (2010) which are based on the US data. It indicates that the study of the financial system is helpful for forecasting future macroeconomic activity in the UK. Furthermore, moving from the FA-TVP-VAR with SV model or the TVP-FAVAR with CV model to the TVP-FAVAR with SV has much more benefits for the prediction of economic output. Adding the time-varying loadings only reduces predictive errors at the horizons $h = 1,2$, and including SV substantially increases forecasting accuracy at any leading horizons (i.e., $h = 1, \dots, 4$).

Secondly, also of interest is the performance of the principal-component methods relative to the FCIs produced by the weighted-sum methods. Panel 2 shows the result of applying the SV-FCI and CV-FCI respectively to Eq. (5.10). Including either of the two indices leads to fairly good forecasting performance as compared to the benchmark but rarely as good as the FCIs from the three FAVARs. Therefore, Table 3 clearly points to the TVP-FAVAR with SV model as the optimal methodology (i.e., the best weighing model) for estimating an FCI for the UK.

To examine whether the inclusion of the interest rate will alter the estimates of FCIs, this study constructs an index that excludes rir_tGap using the TVP-FAVAR with SV model. For the sake of simplicity the new index is called the Taylor rule Financial Conditions Index (TRFI). Re-specifying a TVP-FAVAR with SV for the TRFI as:

$$\Omega_t = \lambda_t^y \bar{y}_t + \lambda_t^f TRFI_t + u_t, \quad u_t \sim N(0, V_t) \tag{5.11}$$

$$\begin{bmatrix} \bar{y}_t \\ TRFI_t \end{bmatrix} = c_t + B_{t,1} \begin{bmatrix} \bar{y}_{t-1} \\ TRFI_{t-1} \end{bmatrix} + \cdots + B_{t,p} \begin{bmatrix} \bar{y}_{t-p} \\ TRFI_{t-p} \end{bmatrix} + \varepsilon_t, \quad \varepsilon_t \sim N(0, Q_t) \tag{5.12}$$

where Ω_t is a vector of 5 unpurged financial indicators including REERGap, RSPIGap, RHPIGap, CredSprdGap and ΔFutSprdGap. The aim of the above system is to extract a factor $TRFI_t$ from the information set Ω_t. Since the TRFI does not include rir_tGap, it tends to avoid the simultaneous reaction when used in a Taylor rule.

As plotted in Figure 3, the overall pattern of a TRFI is quite similar to the FCI estimated with the same model (i.e., the TVP-FAVAR with SV based FCI in Figure 2) but using six financial variables. This supports the findings of Wacker et al. (2014) for the US that the decision as to whether to include the interest rate or not does not affect the results of the factor analysis. Therefore, this study will explain the evolution of the UK financial market with Figure 3.

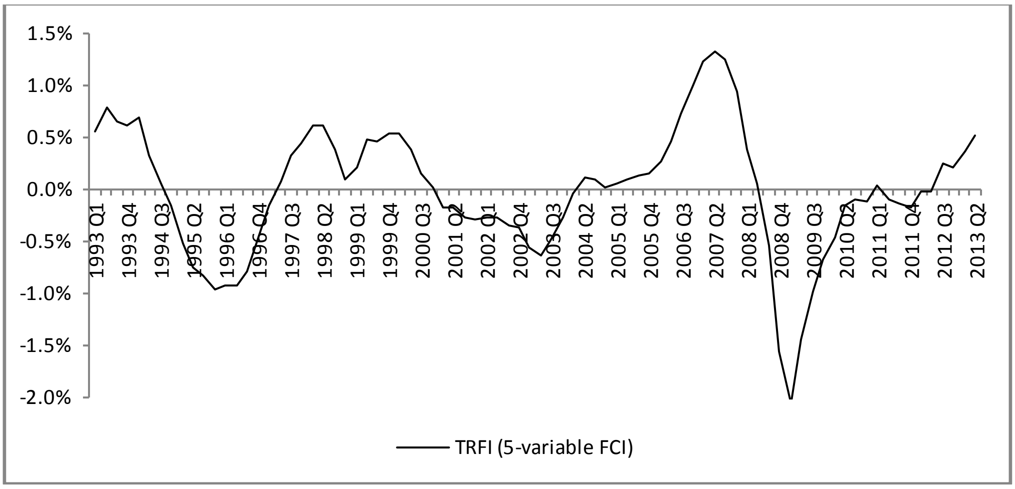


Figure 3: TRFI Produced by a TVP-FAVAR with SV

In general, the movement of the financial market in the UK exhibits the characteristics of business cycles throughout the sample. The index hits the first peak around 1993:II. Its composition is dominated by the foreign exchange market. As mentioned in the MPC's Inflation report in May 1993, the effective sterling exchange rate depreciates by 14.5% between September 1992 (the last month before suspension of sterling's membership of the EU's exchange rate mechanism) and February 1993. In the following Inflation report in August 1993, the MPC reports that sterling is further weakened by the political uncertainty. However, on the other hand, as shown in

Appendix 4 the bond market, equity market and housing market are relatively quiet with little fluctuations between 1993:I and 1994:II.

The subsequent decline in the TRFI after 1994:II reflects the expected response of the financial market to an increase in the BOE's official interest rate. It increases to 6.03% in December 1994 and remains at a level of 6.63% during most of 1995. The BOE raises the interest rate substantially in 1997 from 5.92% to 7.03%. The REER, RSPI and RHPI are all negatively affected by changes in the interest rate which results in the FCI falling below its long-term trend (as measured with the Hodrick-Prescott filter) from 1995:I to 1997:I.

As already mentioned, the financial recession between 2001 and 2003, reflected in the TFRI, is triggered by the corporate accounting irregularities in the US. It triggers the sharp decline in international equity prices. The S&P 500 falls by 16.3%, the Euro Stoxx decreases by 23.6% and a comparable Japanese index drops by 10.8% between May and July 2002. The FTSE-All-Share index reaches its six-year low in July 2002 in the UK. The share price index falls continually throughout 2003:I but appears to recover in the second quarter. The FTSE-All-Share index then increases by 8.9% between February 5th and May 7th 2003 indicating the recovery of the overall financial system in 2003:II.

The subsequent boom in the financial market is evident in the substantial increase in asset prices before the 2008-9 crisis. As noted earlier, all the five indicators show a more expensive financial market relative to its long-run-trend for this period.

The following period of 2008-9 shows the considerable turmoil in international financial markets. The TRFI falls from a 20-year high in 2007:II to a 20-year low in 2009:I. According to the MPC's Inflation report released in February 2008, the declines in equity prices reflects the growing pessimism among investors about growth prospects. In addition, the MPC notes in its 2009 report (February 2009, p.17-18) that the cyclical factors and the downgrading of expectations on the long-run growth for the UK accounts for some of the fall in sterling at this time, while indicators of housing market activity which has historically provided a very good guide to the near-term price trends largely remains very weak.

The continuation of the accommodative stance of the low interest rate and quantitative easing[1] brings the financial market back to or at least close to the long-run trend around 2011. In February 2011, the MPC announces that the FTSE index (all-share) had continued to recover from the trough in 2009 and credit spreads have fallen below the pre-crisis level. The announcement suggests that this in part reflects cyclical factors, but to large extent it reflects a persistent reassessment of the risks associated with holding corporate bonds. Therefore, all indicators contribute to the rise in the TRFI after 2009.

1.6 Conclusions

This study investigates alternative methods for constructing an FCI for the UK economy. It reviews the existing estimation methodologies and discusses the advantages and disadvantages of each (sub)approach. In the weighted-sum approach there are three models used in the literature, a macro econometric model, a VAR model and a reduced aggregated demand equation. Although the macro econometric model is superior to the VAR models and the reduced form model in that it takes the structural features of the economy into account, Price et al. (2015) who are the Bank of England's economists acknowledge that the Bank does not have a robust macro econometric model to use. In addition, both of the macro econometric models and the VARs impose fixed weights in the FCI. This is considered too restrictive. Although it would be appropriate to use a TVP-VAR to address this limitation, the VAR structure requires a small number of variables. Therefore, this study uses the reduced form model to represent the weighted-sum approach in the econometric exercise.

Employing the reduced form model, this study develops a new weighted-sum method, the two-step method using the TVP-R-SV procedure, to estimate FCIs. In the first-step, the TVP-R-SV algorithm is used to purge the current macroeconomic impact on the financial market. Then in the second step, the TVP-R-SV algorithm is used again to estimate a reduced form model with the purged data. The estimated parameters are taken to calculate the weight on each constituent in the FCI. This two-step process

[1] Joyce, Tong and Woods (2011) of the BOE's Macro Financial Analysis Division explain the quantitative easing policy: "*in response to the intensification of the financial crisis in Autumn 2008, the Bank of England, in common with other central banks, loosened monetary policy using both conventional and unconventional policy measures. In the UK, the principal element of these unconventional measures was the policy of asset purchases financed by central bank money, so-called quantitative easing.*" Also according to Joyce et al. (2011), between 2009 and 2010, the Bank purchased £200 billion of assets.

aims to overcome at least two shortcomings in the existing studies that use the reduced form approach to estimate FCIs. Existing studies (e.g., Castro, 2011) ignore both the role of stochastic volatility and the impact of macroeconomic activity on the financial system.

The two-step method is compared with the TVP-FAVAR model of Koop and Korobilis (2014) which is used as a representative of the principal-component approach. The TVP-FAVAR with SV model is likely to be a better method than a standard PCA, as this model allows the loading factors, i.e., the relations among a set of variables, to change at each point in time.

In the econometric exercise, this study compares the forecasting performance of FCIs produced by the various methodologies. As emphasised in Hatzius et al. (2010) and Koop and Korobilis (2014), the best FCI is one which predicts macroeconomic activity as well as possible. In order to compare the estimates in this study against those from the earlier literature in the UK such as Castro (2011), this study chooses the same set of financial variables.

The estimates of the mean squared forecast errors in the econometric exercise brings several important findings. Firstly, the TVP-R-SV algorithm in Nakajima (2011a) is superior to the traditional Kalman filter algorithm or TVP-R-CV for estimating a reduced form aggregate demand equation based FCI. When comparing the TVP-R based FCIs against the FCI in the existing literature such as Castro (2011), this study discovers that the estimate in Castro (2011) fails to capture the evolution of the financial system in some periods. The above results justify the use of the two-step method which is developed in this study. Secondly, the TVP-FAVAR with SV model produces an FCI with lower forecasting errors as compared to an index produced by the standard PCA. This supports the use of the time-varying loadings in the principal-component approach. Finally, comparing the FCIs produced by the weighted-sum approaches to those from the principal-component methods shows that in the UK the TVP-FAVAR with SV model is the optimal methodology for weighting financial variables in an FCI. However, this study acknowledges that the estimated FCI here may not be the most appropriate one. This is because in addition to the estimation method (i.e., variable weighting method) the selection of financial variables also affects the estimates of an FCI. The next chapter will extend the variables used to incorporate more financial indicators in order to determine the optimal FCI.

Appendices:

Appendix 1: Description of the Raw Variables and Respective Sources:

No.	Name	Description	Source	Sample
1	TreasRate	Three-month treasury bill discount rate (quarterly average)	BOE statistics	1993:I-2013:II
2	OfficRate	Official central bank interest rate (quarterly average)	BOE statistics	1993:I-2013:II
3	Libor3m	Three-month (Libor) interbank lending rate (quarterly average)	BOE statistics	1993:I-2013:II
4	REER	Real effective exchange rate index (2000=100)	OECD statistics	1993:I-2013:II
5	RHP	Real house price (CPI deflated, , seasonally adjusted)	Nationwide Building Society	1993:I-2013:II
6	NSPI	Nominal share price index (quarterly average of FTSE 100)	OECD statistics	1993:I-2013:II
7	CorpBond	UK commercial corporate bond index yields (quarterly average)	DataStream	1993:I-2013:II
8	Yield_10yr	Ten-year quarterly average yield from British Government Securities	BOE statistics	1993:I-2013:II
9	FutIR	Three-month sterling interest rate futures contracts (quarterly average)	DataStream	1993:I-2013:II
10	Real GDP	Domestic gross production (in millions of chained 2010 price)	ONS statistics	1993:I-2013:II
11	Infl_0	Inflation rate, seasonally adjusted	ONS statistics	1993:I-2013:II
12	CPI	Consumer price index, seasonally adjusted, quarterly average (2005=100)	ONS statistics	1993:I-2013:II

Appendix 2: Processed Financial Variables:

No.	Name	Description	Sample
1	rir_t	The difference between TreasRate and Infl_0	1993:I-2013:II
2	RSPI	Real share price index (CPI deflated)	1993:I-2013:II
3	RHPI	Real house price index (1993Q1=100)	1993:I-2013:II
4	CredSprd	The difference between Yield_10yr and CorpBond	1993:I-2013:II
5	FutSprd	The difference between FutIR in the earlier quarter and the current TreasRate	1993:I-2013:II
6	ΔFutSprd	The quarterly changes in FutSprd	1993:I-2013:II

Note: the calculation of all variables in this table is done by the author, based on the data collection of variables in Appendix 1.

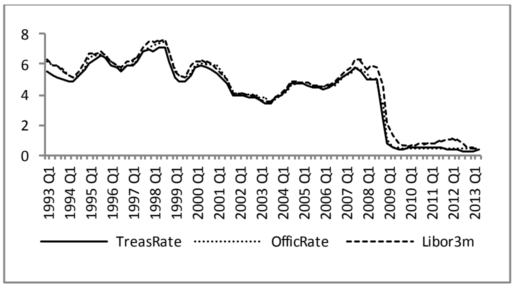
Appendix 3: Description of the FCI Constituents:

No.	Name	Description	Sample
1	rir_tGap	The difference between RTrea3m and its steady-state level	1993:I-2013:II
2	REERGap	The percentage deviation of REER from its equilibrium	1993:I-2013:II
3	RSPIGap	The percentage deviation of RSPI from its equilibrium	1993:I-2013:II
4	RHPIGap	The percentage deviation of RHPI from its equilibrium	1993:I-2013:II
5	CredSprdGap	The difference between CredSprd and its equilibrium	1993:I-2013:II
6	ΔFutSprdGap	The difference between ΔFutSprd and its equilibrium	1993:I-2013:II

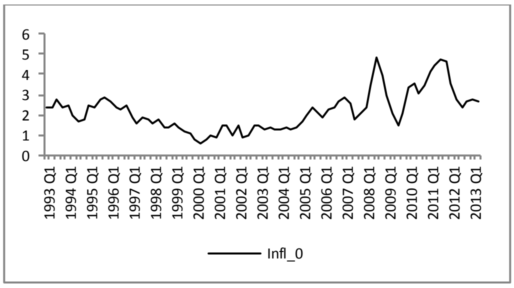
Note: while estimating an FCI, this study uses the deviation of the above 6 variables from their long-run trends. The long-run trend level of RTrea3m is defined as its average. Consistent with Castro (2011), the Hodrick-Prescott (1997) filter is taken to estimate the equilibriums of the remaining five variables.

Appendix 4: Variables for Estimating the FCIs for the UK, 1993-2013

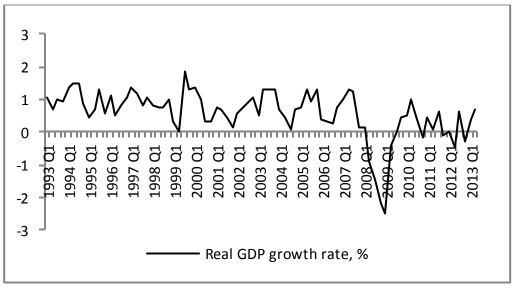
Appendix 4.1: Interest rates, %



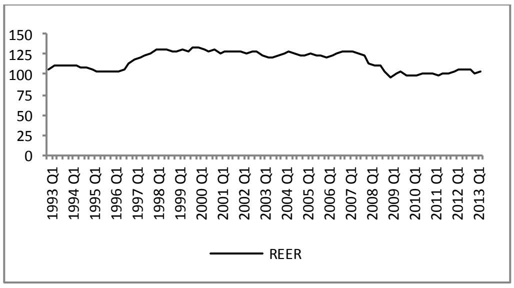
Appendix 4.2: The inflation rates, %



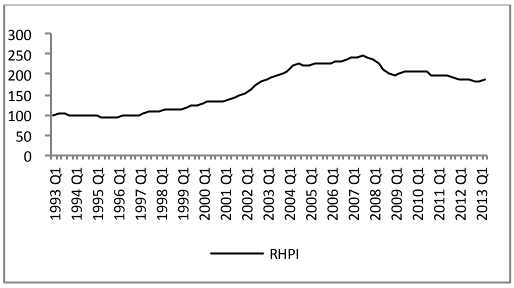
Appendix 4.3: Real gross domestic product



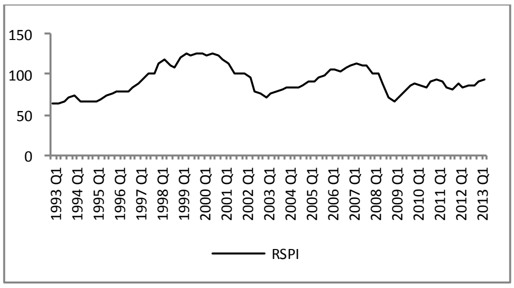
Appendix 4.4: Real effective exchange rate index



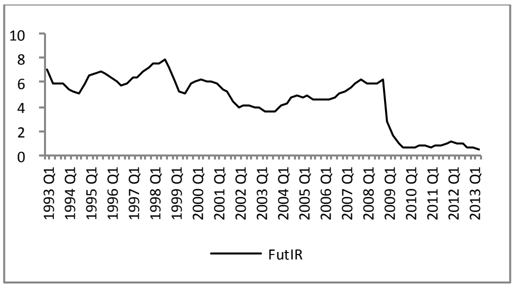
Appendix 4.5: Real house price index



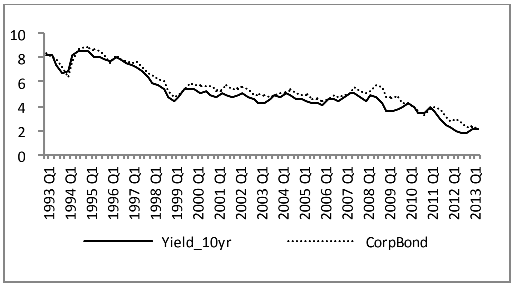
Appendix 4.6: Real share price index



Appendix 4.7: Sterling interest rate futures, %



Appendix 4.8: CorpBond vs. Yield_10yr, %



Appendix 5: Joint Posterior Distribution for the TVP-R-SV Model:

$$\pi(\theta,\alpha,h \mid y) \propto \pi(\theta) \times \prod_{t=1}^{n} \frac{1}{\sqrt{2\pi\gamma}e^{h_t/2}} exp\left\{\frac{(\bar{y}_t - \alpha_t z_t')^2}{2\gamma e^{h_t}}\right\} \tag{A1}$$

$$\times \prod_{t=1}^{n-1} \frac{1}{(2\pi)^{p/2}|\Sigma|^{0.5}} exp\left\{-\frac{1}{2}(\alpha_{t+1} - \alpha_t)'\Sigma^{-1}(\alpha_{t+1} - \alpha_t)\right\} \times \frac{1}{(2\pi)^{k/2}|\Sigma_0|^{0.5}} exp\left\{-\frac{1}{2}(\alpha_1'\Sigma_0^{-1}\alpha_1)\right\}$$

$$\times \prod_{t=1}^{n-1} \frac{1}{\sqrt{2\pi}\sigma_\eta} exp\left\{-\frac{(h_{t+1} - \phi h_t)^2}{2\sigma_\eta^2}\right\}$$

$$\times \frac{\sqrt{1-\phi^2}}{\sqrt{2\pi}\sigma_\eta} exp\left\{-\frac{(1-\phi^2)h_1^2}{2\sigma_\eta^2}\right\}$$

where $\theta \equiv \left(\Sigma, \phi, \sigma_\eta, \gamma\right)$

Appendix 6: The Details of the Procedures for Implementing the MCMC Algorithm:

i. Sample $\alpha \mid \sum, \gamma, h, \bar{y}$:

Following Primiceri (2005), Nakajima (2011a) and Nakajima, Kasuya and Watanable (2011), this study uses the simulation smoother in Jong and Shephard (1995) to sample the parameters (α) from the conditional posterior distribution, $\pi(\alpha \mid \sum, \gamma, h, \bar{y})$. Jong and Shephard (1995, p. 343) illustrates the algorithm of the simulation smoother on a state space model:

$$\bar{y}_t = Z_t \alpha_t + G_t u_t, \quad (t = 1, \dots, n) \tag{A2}$$

$$\alpha_{t+1} = T_t \alpha_t + H_t u_t, \quad (t = 0, \dots, n-1) \tag{A3}$$

where $\alpha_0 = 0$, $u_t \sim N(0, I)$, and $G_t H_t' = 0$. The lower letters denote column vectors; and the upper letters are for matrices.

The simulation smoother draws $\tau \sim \pi(\tau \mid \alpha, \bar{y})$, where $\tau = (\tau_0, \dots, \tau_t)$ with $\tau_t = H_t u_t$. Initially, run, for $t = 1, \dots, n$, the Kalman filter:

$$e_t = \bar{y}_t - Z_t a_t, \quad D_t = Z_t P_t Z_t' + G_t G_t', \quad K_t = T_t P_t Z_t' D_t^{-1}, \tag{A4}$$

$$L_t = T_t - K_t Z_t, \quad a_{t+1} = T_t a_t + K_t e_t, \quad P_{t+1} = T_t P_t L_t' + H_t H_t',$$

where $a_1 = 0$ and $P_1 = H_0 H_0'$. On this Kalman filter pass, the quantities e_t, D_t and K_t are stored. Then set $r_n = U_n = 0$ and run the simulation smoother for $t = n, \dots, 1$:

$$C_t = \Xi_t - \Xi_t U_t \Xi_t, \quad \epsilon_t \sim N(0, C_t), \quad V_t = \Xi_t U_t L_t, \tag{A5}$$

$$r_{t-1} = Z_t' D_t^{-1} e_t + L_t' r_t - V_t' C_t^{-1} \epsilon_t,$$

$$U_{t-1} = Z_t' D_t^{-1} Z_t + L_t' U_t L_t + V_t' C_t^{-1} V_t,$$

where, $\Xi_t = H_t H_t'$, and store $\tau_t = H_t H_t' r_t + \epsilon_t$. For the initial state, $\tau_0 = H_0 H_0' r_0 + \epsilon_0$, where $\epsilon_0 \sim N(0, C_0)$ with $C_0 = \Xi_0 - \Xi_0 U_0 \Xi_0$. Once τ is drawn, it is easy to obtain α_t using the state equation Eq. (A3), replacing $H_t u_t$ with τ_t.

Hence, in the case of the TVP-R-SV algorithm (i.e., Eq. 4.10 – Eq. 4.12) to sample α, the correspondence of the variables is written as:

$$Z_t = z_t', \quad G_t = \left(\sqrt{\gamma} e^{\frac{h_t}{2}}, 0_p'\right), \tag{A6}$$

$$T_t = I_p, \quad H_t = \left(0_p, \Sigma^{0.5}\right), \quad H_0 = \left(0_p, \Sigma_0^{0.5}\right),$$

where 0_p is a $p \times 1$ zero vector.

ii. Sample $\sum \mid \alpha$:

For sampling $\sum$ from $\pi(\sum \mid \alpha)$, specify the prior as $\sum \sim IW(\nu_0, S_0^{-1})$ density, where IW denotes the inverse-Wishart distribution. It is always positive, and the integral is infinite. The algorithm used here is similar to the one described in Hoff (2009, p. 110-112):

$$\pi(\Sigma) \propto |\Sigma|^{-(\nu_0+p+1)/2} exp\{-tr(S_0\Sigma^{-1})/2\} \tag{A7}$$

where the expression 'tr' stands for trace and for a square $p \times p$ matrix A, $tr(A) = \sum_{g=1}^{p} a_{g,g}$, the sum of the diagonal elements. According to Hoff (2009, p.111), write the conditional distribution posterior distribution as:

$$\begin{aligned} \pi(\Sigma \mid \alpha) &\propto \left(|\Sigma|^{-(\nu_0+p+1)/2} exp\{-tr(S_0\Sigma^{-1})/2\}\right) \\ &\quad \times \left(|\Sigma|^{-(n-1)/2} exp\{-tr(S_\alpha\Sigma^{-1})/2\}\right) \\ &= |\Sigma|^{-(\nu_0+n-1+p+1)/2} exp\{-tr([S_0 + S_\alpha]\Sigma^{-1})/2\} \end{aligned} \tag{A8}$$

where:

$$S_\alpha = \sum_{t=1}^{n-1} (\alpha_{t+1} - \alpha_t)(\alpha_{t+1} - \alpha_t)' \tag{A9}$$

Given Eq. (A8) and (A9), the posterior distribution for $\sum$ depends on only α. Then, it is easy to draw the sample with the distribution, $\sum \mid \alpha \sim IW(\nu_0 + n - 1, [S_0 + S_\alpha]^{-1})$. Hoff (2009) explains the term $\nu_0 + n - 1$ as the posterior sample size, being the sum of the 'prior sample size' (ν_0) and data sample size ($n - 1$). $S_0 + S_\alpha$ can be thought of as the 'prior' residual sum of squares plus the sum of squares from the data.

iii. Sample $h \mid \gamma, \phi, \sigma_\eta, \alpha, \bar{y}$:

Similar to the sampling methods used in a TVP-VAR, Nakajima (2011a) mentions two alternatives for sampling the stochastic volatility, the mixture sample and multi-move sample. According to Nakajima (2011a), the later is expected to be more to draw $h \mid \gamma, \phi, \sigma_\eta, \alpha, \bar{y}$. Hence, it is taken in this study.

The multi-move sample for sampling latent state variables (e.g., h in Eq. 4.12) in non-linear and non-Gaussian state-space models (e.g., the TVP-R-SV) from the posterior density given the parameters is proposed by Shephard and Pitt (1997). Watanabe and Omori (2004) summarise two key features of the multi-move sample: (i) in contrast to the single-move sample generating a single state variable at one time, this method requires to divide the state variables into several blocks, and it samples each block at a time. This reduces inefficiencies of the single-move sample. (ii) rather than sampling state variables (e.g., h in Eq. 4.12) directly, it samples sate errors (e.g., η in Eq. 4.12) by acceptance rejection the Metropolis-Hasting (AR-MH) algorithm. Generating the candidates for the state errors involves two steps: firstly, approximate a true posterior density for a typical block of state errors; and then sample the candidates using the simulation smoother in Jong and Shephard (1995). Finally, state variables can be obtained by running the state equation with the draw of the state errors. The full algorithm for sampling h in the TVP-R-SV model comes from Nakajima (2011a, p. 137-139).

Set $y_t^* = (\bar{y}_t - \alpha_t z_t')/\sqrt{\gamma}$, and re-write the TVP-R-SV specification as:

$$y_t^* = exp(h_t/2)e_t, \quad (t = 1, \dots, n) \tag{A10}$$

$$h_{t+1} = \phi h_t + \eta_t, \quad |\phi| < 1, \quad (t = 0, \dots, n-1) \tag{A11}$$

where $h_0 = 0$ and $\eta_0 \sim N\big(0, \sigma_\eta^2/(1-\phi^2)\big)$.

For sampling a general block $(h_r, \dots, h_{r+d})$, consider the draw of state errors:

$$(\eta_{r-1}, \dots, \eta_{r+d-1}) \sim \pi(\eta_{r-1}, \dots, \eta_{r+d-1} \mid \omega) \tag{A12}$$

$$\propto \prod_{t=r}^{r+d} \frac{1}{e^{h_t/2}} exp\left(-\frac{y_t^{*2}}{2e^{h_t}}\right) \times \prod_{t=r-1}^{r+d-1} f(\eta_t) \times f(h_{r+d})$$

$$f(\eta_t) = \begin{cases} exp\left\{-\dfrac{(1-\phi^2)\eta_0^2}{2\sigma_\eta^2}\right\}, & if\ t = 0 \\ exp\left(-\dfrac{\eta_t^2}{2\sigma_\eta^2}\right), & if\ t \geq 1 \end{cases} \tag{A13}$$

$$f(h_{r+d}) = \begin{cases} exp\left\{-\dfrac{(h_{r+d+1} - \phi h_{r+d})^2}{2\sigma_\eta^2}\right\}, & if\ r+d < n \\ 1, & if\ r+d = n \end{cases} \tag{A14}$$

where $r \geq 1$, $d \geq 1$, $r + d \leq n$, and $\omega = \left(h_{r-1}, h_{r+d+1}, \gamma, \phi, \sigma_\eta, \alpha, \bar{y}\right)$.

Then sample the state errors of $(\eta_{r-1}, \dots, \eta_{r+d-1})$ from Eq. (A12) using the AR-MH algorithm with the proposal distribution below:

To construct the proposed distribution, consider the second-order Taylor expansion of:

$$g(h_t) \equiv -\frac{h_t}{2} - \frac{y_t^{*2}}{2e^{h_t}} \tag{A15}$$

around a certain point of $\hat{h}_t$, namely,

$$g(h_t) \approx g(\hat{h}_t) + g'(\hat{h}_t)(h_t - \hat{h}_t) + \frac{1}{2} g''(\hat{h}_t)(h_t - \hat{h}_t)^2 \tag{A16}$$

$$\propto \frac{1}{2} g''(\hat{h}_t) \left\{h_t - \left(\hat{h}_t - \frac{g'(\hat{h}_t)}{g''(\hat{h}_t)}\right)\right\}^2$$

where:

$$g'(\hat{h}_t) = -\frac{1}{2} + \frac{y_t^{*2}}{2e^{h_t}} \tag{A17}$$

$$g''(\hat{h}_t) = -\frac{y_t^{*2}}{2e^{h_t}} \tag{A18}$$

and the discussion on the term of $\hat{h}_t$ is presented below.

Now, use the proposal density formed as:

$$\pi(\eta_{r-1}, \dots, \eta_{r+d-1} \mid \omega) \propto \prod_{t=r}^{r+d} exp\left(-\frac{(h_t^* - h_t)^2}{2\sigma_t^{*2}}\right) \times \prod_{t=r-1}^{r+d-1} f(\eta_t) \tag{A19}$$

where:

$$\sigma_t^{*2} = -\frac{1}{g''(\hat{h}_t)} \tag{A20}$$

$$h_t^* = \hat{h}_t + \sigma_t^{*2} g'(\hat{h}_t) \tag{A21}$$

when $t = r, \dots, r + d - 1$, and $t = r + d = n$. For $t = r + d < n$,

$$\sigma_{r+d}^{*\,2} = \frac{1}{-g''(\hat{h}_{r+d}) + \phi^2/\sigma_\eta^2} \tag{A22}$$

$$h_{r+d}^* = \sigma_{r+d}^{*\,2}\{g'(\hat{h}_{r+d}) - g''(\hat{h}_{r+d})\hat{h}_{r+d} + \phi h_{t+d+1}/\sigma_\eta^2\} \tag{A23}$$

According to Nakajima in 2011a, the above proposal distribution is derived from its correspondence to the following state space model:

$$h_t^* = h_t + \varpi_t, \quad t = r, \dots, r + d \tag{A24}$$

$$h_{t+1} = \phi h_t + \eta_t, \quad |\phi| < 1, \quad t = r - 1, \dots, r + d - 1$$

$$\begin{pmatrix} \varpi_t \\ \eta_t \end{pmatrix} \sim N\left(0, \begin{pmatrix} \sigma_t^{*2} & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix}\right), \quad t = r, \dots, r + d$$

with:

$$\eta_{r-1} \sim \begin{cases} N\left(0, \sigma_\eta^2/(1 - \phi^2)\right), & r = 1 \\ N(0, \sigma_\eta^2), & r > 1 \end{cases} \tag{A25}$$

With $\omega = (h_{r-1}, h_{r+d+1}, \gamma, \phi, \sigma_\eta, \alpha, \bar{y})$, take the simulation smoother over Eq. (A24) to draw the candidates of state errors $(\eta_{r-1}, \dots, \eta_{r+d-1})$. Then, run Eq. (A24) with the obtained candidates to generate the state variables.

As in Nakajima (2011a), for en efficient sampling it is desirable for $(\hat{h}_r, \dots, \hat{h}_{r+d})$ to be close to the mode of the posterior distribution. Hence, loop Step (iii.1) to Step (iii. 5) several times so as to reach the mode:

Step (iii.1): Initialise $(\hat{h}_r, \dots, \hat{h}_{r+d})$.

Step (iii.2): Calculate $(h_r^*, \dots, h_{r+d}^*)$, $({\sigma_r^*}^2, \dots, {\sigma_{r+d}^*}^2)$ by Eq. (A20) – Eq. (A23).

Step (iii.3): Run the moment smoother over Eq. (A24), using current $(h_r^*, \dots, h_{r+d}^*)$ and $({\sigma_r^*}^2, \dots, {\sigma_{r+d}^*}^2)$ to obtain $\hat{h}_t^*$.

Step (iii.4): Replace $(\hat{h}_r, \dots, \hat{h}_{r+d})$ by $(\hat{h}_r^*, \dots, \hat{h}_{r+d}^*)$.

Step (iii.5): Go to Step (iii.2).

iv. Sample $\phi \mid \sigma_\eta, h$:

Given the restriction that ϕ is bounded by a negative and a positive 1.0, the quantity $(\phi + 1)/2$ lies between 0 and 1. As in Hoff (2009, p. 34), one family of density that supports $(0, 1)$ is the beta distribution. Thus, Nakajima (2011a) assumes that:

$$\frac{(\phi + 1)}{2} \sim beta(\alpha_{\phi_0}, \beta_{\phi_0}) \tag{A26}$$

Specifying the prior distribution as $\pi(\phi)$, Nakajima derived the conditional posterior distribution of ϕ:

$$\pi(\phi \mid \sigma_\eta, h) \propto \pi(\phi) \times \sqrt{1 - \phi^2} \times exp\left\{-\frac{(1 - \phi^2)h_1^2}{2\sigma_\eta^2}\right\} \times exp\left\{-\frac{\sum_{t=1}^{n-1}(h_{t+1} - \phi h_t)^2}{2\sigma_\eta^2}\right\} \tag{A27}$$

which yields:

$$\pi(\phi \mid \sigma_\eta, h) \propto \pi(\phi) \times \sqrt{1 - \phi^2} \times exp\left\{-\frac{\sum_{t=2}^{n-1} {h_t}^2}{2\sigma_\eta^2}\left(\phi - \frac{\sum_{t=1}^{n-1} h_t h_{t+1}}{\sum_{t=2}^{n-1} {h_t}^2}\right)^2\right\} \tag{A28}$$

where the last term forms a kernel of the normal distribution. Therefore, he suggests taking the Metropolis-Hasting (MH) algorithm for sampling ϕ conditional on σ_η and h. Gilks, Richardson and Spiegelhalter (1996, p. 5-8) and Chib and Greenberg (1995) explain the general MH methodology: at each time t, the next state U_{t+1} is chosen by sampling a candidate point V from a proposal distribution $\pi(.\,|U_t)$ as the first step; the candidate point V is then accepted with probability $\alpha(U,V)$:

$$\alpha(U,V) = min\left(1, \frac{\pi(V)\pi(U \mid V)}{\pi(U)\pi(V \mid U)}\right) \tag{A29}$$

The result of Eq. (A29) is called the acceptance rate. If the candidate point is accepted, the next state becomes $U_{t+1} = V$. If the candidate is rejected, then $U_{t+1} = U_t$. Hence, the MH algorithm is to repeat Step (iv.2) – Step (iv.3) according to the length of the sample:

Step (iv.1): Initialise U_0, and set $t = 0$.

Step (iv.2): Sample a candidate point V from $\pi(.\,|U_t)$.

Step (iv.3): Sample a Uniform$(0, 1)$ random variable Z: if $Z \leq \alpha(U,V)$, set $U_{t+1} = V$, otherwise set $U_{t+1} = U_t$.

Step (iv.4): Increment t, and go to Step (iv.2).

For sampling ϕ in the TVP-R-SV algorithm, let ϕ^* denote the candidate point. Recall that the term in Eq. (A28) corresponds to the normal distribution, and that the term ϕ lies between -1.0 and +1.0. Hence, Nakajima (2011a) proposes that the candidate will follow the truncated normal (TN) distribution on the domain $|\phi| < 1$:

$$\phi^* \sim TN_{[-1,1]}\left(\mu_\phi, \sigma_\phi^2\right) \tag{A30}$$

where:

$$\mu_\phi = \frac{\sum_{t=1}^{n-1} h_t h_{t+1}}{\sum_{t=2}^{n-1} {h_t}^2} \tag{A31}$$

$$\sigma_{\phi}^{2} = \frac{\sigma_{\eta}^{2}}{\sum_{t=2}^{n-1} {h_t}^{2}} \tag{A32}$$

Re-write the acceptance rate (Eq. A29) for the TVP-R-SV model:

$$\alpha(\phi_t, \phi^*) = min\left(1, \frac{\pi(\phi^* \mid \sigma_\eta, h)\pi(\phi_t)}{\pi(\phi_t \mid \sigma_\eta, h)\pi(\phi^*)}\right) = min\left(1, \frac{\pi(\phi^*)\sqrt{1-{\phi^*}^2}}{\pi(\phi_t)\sqrt{1-\phi_t^2}}\right) \tag{A33}$$

where ϕ_t represents the current point drawn in the previous iteration. The acceptance step can be implemented according to Step (iv.3): draw a uniform random number Z ~ Uniform$(0, 1)$, and then compare it with the obtained $\alpha(\phi_t, \phi^*)$.

v. Sample $\sigma_\eta \mid \phi$, h:

For σ_η^2 the Bayesian estimation needs a family of prior distribution that has support on $(0, \infty)$. One such family of distribution is the gamma family. However, according to Hoff (2009, p. 74), this family is not conjugate for the normal variance. The gamma family turns out to be a conjugate class of densities for $1/\sigma_\eta^2$.

The above statement motivates to specify the prior of σ_η as:

$$\sigma_\eta^2 \sim IG(v_0/2, V_0/2) \tag{A34}$$

where IG denotes the inverse-Gamma distribution.

This gives:

$$\pi(\sigma_\eta) \propto \sigma_\eta^{-\left(\frac{v_0}{2}+1\right)} \times exp\left(-\frac{V_0}{2\sigma_\eta}\right) \tag{A35}$$

Then the condition posterior distribution for σ_η is obtained as:

$$\pi(\sigma_\eta \mid \phi, h) \propto \left[\sigma_\eta^{-\left(\frac{\nu_0}{2}+1\right)} \times exp\left(-\frac{V_0}{2\sigma_\eta}\right)\right] \times \left\{\frac{1}{\sigma_\eta} \times exp\left[-\frac{(1-\phi^2)h_1^2}{2\sigma_\eta^2}\right]\right\} \quad \text{(A36)}$$

$$\times \prod_{t=1}^{n-1}\left\{\frac{1}{\sigma_\eta} exp\left[-\frac{(h_{t+1}-\phi h_t)^2}{2\sigma_\eta^2}\right]\right\}$$

$$\propto \sigma_\eta^{-\left(\frac{\hat{\nu}_0}{2}+1\right)} \times exp\left\{\frac{\hat{V}_0}{2\sigma_\eta}\right\}$$

where:

$$\hat{\nu}_0 = \nu_0 + n \quad \text{(A37)}$$

$$\hat{V}_0 = V_0 + (1-\phi^2)h_1^2 + \sum_{t=1}^{n-1}(h_{t+1}-\phi h_t)^2 \quad \text{(A38)}$$

Thus the sample can be drawn as:

$$(\sigma_\eta^2 \mid \phi, h) \sim IG(\hat{\nu}_0/2, \hat{V}_0/2) \quad \text{(A39)}$$

vi. Sample $\gamma \mid \alpha, h, \bar{y}$:

Similar to the way of sampling σ_η, the prior of γ is assumed to follow the IG density:

$$\gamma \sim IG(w_0/2, W_0/2) \quad \text{(A40)}$$

Nakajima (2011a, p. 116) then derives the condition posterior density:

$$\gamma \sim IG(\widehat{w}_0/2, \widehat{W}_0/2) \quad \text{(A41)}$$

where:

$$\widehat{w}_0 = w_0 + n \quad \text{(A42)}$$

$$\widehat{W}_0 = W_0 + \sum_{t=1}^{n}\frac{(\bar{y}_t - \alpha_t z_t')^2}{e^{h_t}} \quad \text{(A43)}$$

Appendix 7: The Detailed Algorithm for Estimating a TVP-FAVAR with SV:

i. Initialise all system unknown parameters:

$$f_0 \sim N(0, \Sigma_{0|0}^{f}) \tag{A44}$$

$$\lambda_0 \sim N(0, \Sigma_{0|0}^{\lambda}) \tag{A45}$$

$$\beta_0 \sim N(0, \Sigma_{0|0}^{\beta}) \tag{A46}$$

$$V_0 \equiv 1 \times I_n \tag{A47}$$

$$Q_0 \equiv 1 \times I_{s+1} \tag{A48}$$

and obtain the principal component estimates of the factors, $\tilde{f}_t$ based on data up to period t. As in Koop and Korobilis (2014), the prior specification is based on the non--informative choices which are always appealing and numerically stable.

An alternative to such a prior is to choose the prior hyperparameters using a training sample data, as what Primiceri (2005) has done in a TVP-VAR. Koop and Korobilis (2014) also conduct a test to compare the results based on various prior specifications (i.e., non-informative versus training sample priors). Their sensitivity analysis shows that estimates are consistent in those two cases. They finally pursue the above settings for a factor augmented VAR due to the computational simplicity.

ii. With the above initial conditions, obtain the filtered estimates of λ_t, β_t, V_t and Q_t, using the following recursion for $t = 1, \ldots, T$.

(a). Similar to the procedures for calculating θ_t in Eq. (4.21-4.23), the Kalman filter tells:

$$\lambda_t | Data_{1:t-1} \sim N(\lambda_{t|t-1}, \Sigma_{t|t-1}^{\lambda}) \tag{A49}$$

$$\beta_t | Data_{1:t-1} \sim N(\beta_{t|t-1}, \Sigma_{t|t-1}^{\beta}) \tag{A50}$$

where:

$$\lambda_{t|t-1} = \lambda_{t-1|t-1}, \quad \Sigma^{\lambda}_{t|t-1} = \Sigma^{\lambda}_{t-1|t-1} + \widehat{W}_t \tag{A51}$$

$$\beta_{t|t-1} = \beta_{t-1|t-1}, \quad \Sigma^{\beta}_{t|t-1} = \Sigma^{\beta}_{t-1|t-1} + \hat{R}_t \tag{A52}$$

This is the only place where W_t and R_t enters into the Kalman filtering formula. As mentioned before and also in Raftery et al. (2007), there is little information available for specifying, computing or simulating W_t and R_t. Thus this study follows Koop and Korobilis (2014) and uses the form of Eq. (4.24) to substitute into the above equations:

$$\Sigma^{\lambda}_{t|t-1} = \frac{1}{k_3}\Sigma^{\lambda}_{t-1|t-1} \tag{A53}$$

$$\Sigma^{\beta}_{t|t-1} = \frac{1}{k_4}\Sigma^{\beta}_{t-1|t-1} \tag{A54}$$

The resulting model is a properly defined state space model:

$$\widehat{W}_t = \left(\frac{1}{k_3} - 1\right)\Sigma^{\lambda}_{t-1|t-1} \tag{A55}$$

$$\hat{R}_t = \left(\frac{1}{k_4} - 1\right)\Sigma^{\beta}_{t-1|t-1} \tag{A56}$$

The terms of k_3 and k_4 are forgetting factors and are typically slightly below one. As mentioned earlier, the estimation in the method is essentially age-weighted. The data j time points old has weights $(k_3)^j$ in the filtered estimates of λ_t and $(k_4)^j$ in β_t. Note also that factor loadings and parameters become time-invariant if $k_3 = k_4 = 1$. Koop and Korobilis (2014) accept the approach of the business as usual prior in Cogley and Sargent (2005) and assume that the changes in each period were quite slow and stable. To achieve this slow time variation, they set $k_3 = k_4 = 0.99$. Based on the findings in Raftery et al. (2007), this study is convinced that the output under this settings is reliable.

(b). Following Koop and Korobilis (2012, 2014), the EWMA is employed to remove the need for a posterior simulation algorithm for multivariate stochastic volatilise in the measurement equation. V_t and Q_t are calculated with the EWMA model:

$$\hat{V}_{i,t} = k_1 V_{i,t-1|t-1} + (1-k_1)\hat{u}_{i,t}\hat{u}'_{i,t}, \quad i = 1, \dots, n \tag{A57}$$

$$\hat{Q}_t = k_2 Q_{t-1|t-1} + (1-k_2)\hat{\varepsilon}_t\hat{\varepsilon}'_t \tag{A58}$$

where $\tilde{z}_t = \begin{bmatrix} y_t \\ \tilde{f}_t \end{bmatrix}$. $\hat{u}_{i,t} = x_{i,t} - \tilde{z}_t\lambda_{i,t|t-1}$ and $\hat{\varepsilon}_t = \tilde{z}_t - \tilde{z}_{t-1}\beta_{t|t-1}$ are produced by the Kalman filter. As already shown in Eq. (4.31), the decay factors k_1 and k_2 control the variation in these two matrices. Using the EWMA estimator needs to determine the k_1 and k_2. Risk-Metrics (1996) recommends values for k_1 and k_2 in the region of (0.94, 0.98). Koop and Korobilis (2014) set these to the value 0.96. They maintain that such values should provide volatility estimates that were quite close to the ones expected by integrated stochastic volatility models which have already been used in Bayesian VAR and FAVAR literature such as Primiceri (2005).

(c). Given information at time t, update λ_t and β_t according to the rules in Eq. (4.26-4.28). λ_t is updated using:

$$\lambda_{i,t}|Data_{1:t} \sim N(\lambda_{i,t|t}, \Sigma^{\lambda}_{ii,t|t}), \quad i = 1, \dots, n \tag{A59}$$

where

$$\lambda_{i,t|t} = \lambda_{i,t|t-1} + \Sigma^{\lambda}_{ii,t|t-1}\tilde{z}'_t\left(\hat{V}_{ii,t} + \tilde{z}_t\Sigma^{\lambda}_{ii,t|t-1}\tilde{z}'_t\right)^{-1}\left(x_t - \tilde{z}_t\lambda_{t|t-1}\right) \tag{A60}$$

$$\Sigma^{\lambda}_{ii,t|t} = \Sigma^{\lambda}_{ii,t|t-1} - \Sigma^{\lambda}_{ii,t|t-1}\tilde{z}'_t\left(\hat{V}_{ii,t} + \tilde{z}_t\Sigma^{\lambda}_{ii,t|t-1}\tilde{z}'_t\right)^{-1}\tilde{z}_t\Sigma^{\lambda}_{ii,t|t-1} \tag{A61}$$

and β_t is updated using:

$$\beta_t|Data_{1:t} \sim N(\beta_{t|t}, \Sigma^{\beta}_{t|t}) \tag{A62}$$

where

$$\beta_{t|t} = \beta_{t|t-1} + \Sigma^{\beta}_{t|t-1}\tilde{z}'_{t-1}\left(\hat{Q}_t + \tilde{z}_{t-1}\Sigma^{\beta}_{t|t-1}\tilde{z}'_{t-1}\right)^{-1}\left(\tilde{z}_t - \tilde{z}_{t-1}\hat{\beta}_{t|t-1}\right) \tag{A63}$$

$$\Sigma^{\beta}_{t|t} = \Sigma^{\beta}_{t|t-1} - \Sigma^{\beta}_{t|t-1}\tilde{z}'_{t-1}\left(\hat{Q}_t + \tilde{z}_{t-1}\Sigma^{\beta}_{t|t-1}\tilde{z}'_{t-1}\right)^{-1}\tilde{z}_{t-1}\hat{\beta}_{t|t-1} \tag{A64}$$

(d). Given information at t, update V_t and Q_t, using the EWMA estimator as follows:

$$V_{i,t|t} = k_1 V_{i,t-1|t-1} + (1-k_1)\hat{u}_{i,t|t}\hat{u}'_{i,t|t}, \quad i = 1, \dots, n \tag{A65}$$

$$Q_{t|t} = k_2 Q_{t-1|t-1} + (1-k_2)\hat{\varepsilon}_{t|t}\hat{\varepsilon}'_{t|t} \tag{A66}$$

where $\hat{u}_{i,t|t} = x_{i,t} - \tilde{z}_t \lambda_{i,t|t}$ and $\hat{\varepsilon}_{t|t} = \tilde{z}_t - \tilde{z}_{t-1}\beta_{t|t}$.

iii. Obtain the smoothed estimates of λ_t, β_t, V_t and Q_t, with the following recursions for $t = T-1, \dots, 1$.

(a). Given the information at $t+1$, update λ_t and β_t with the fixed interval smoother. λ_t is updated using:

$$\lambda_{i,t}|Data_{1:T} \sim N(\lambda_{i,t|t+1}, \Sigma^{\lambda}_{ii,t|t+1}), \quad i = 1, \dots, n \tag{A67}$$

where

$$\lambda_{i,t|t+1} = \lambda_{i,t|t} + \left[\Sigma^{\lambda}_{ii,t|t}\left(\Sigma^{\lambda}_{ii,t+1|t}\right)^{-1}\right]\left(\lambda_{i,t+1|t+1} - \lambda_{i,t+1|t}\right) \tag{A68}$$

$$\begin{aligned} \Sigma^{\lambda}_{ii,t|t+1} = \Sigma^{\lambda}_{ii,t|t} & \\ & + \left[\Sigma^{\lambda}_{ii,t|t}\left(\Sigma^{\lambda}_{ii,t+1|t}\right)^{-1}\right]\left(\Sigma^{\lambda}_{ii,t+1|t+1}\right. \\ & \left. - \Sigma^{\lambda}_{ii,t+1|t}\right)\left[\Sigma^{\lambda}_{ii,t|t}\left(\Sigma^{\lambda}_{ii,t+1|t}\right)^{-1}\right]' \end{aligned} \tag{A69}$$

and β_t is updated using:

$$\beta_t|Data_{1:T} \sim N(\beta_{t|t+1}, \Sigma^{\beta}_{t|t+1}) \tag{A70}$$

where

$$\beta_{t|t+1} = \beta_{t|t} + \left[\Sigma^{\beta}_{t|t}\left(\Sigma^{\beta}_{t+1|t}\right)^{-1}\right]\left(\beta_{t+1|t+1} - \beta_{t+1|t}\right) \tag{A71}$$

$$\Sigma^{\beta}_{t|t+1} = \Sigma^{\beta}_{t|t} + \left[\Sigma^{\beta}_{t|t}\left(\Sigma^{\beta}_{t+1|t}\right)^{-1}\right]\left(\Sigma^{\beta}_{t+1|t+1} - \Sigma^{\beta}_{t+1|t}\right)\left[\Sigma^{\beta}_{t|t}\left(\Sigma^{\beta}_{t+1|t}\right)^{-1}\right]' \tag{A72}$$

(b). Given information at $t+1$, update V_t and Q_t, using the following equations:

$$V_{t|t+1}^{-1} = k_1 V_{t|t}^{-1} + (1 - k_1) V_{t+1|t+1}^{-1} \tag{A73}$$

$$Q_{t|t+1}^{-1} = k_1 Q_{t|t}^{-1} + (1 - k_1) Q_{t+1|t+1}^{-1} \tag{A74}$$

iv. Means and variances of f_t given estimates of λ_t, β_t, V_t and Q_t in Step (i) to Step (iii) can be obtained using the standard Kalman filter and smoother.

With the above algorithm, the TVP-FAVAR with SV can be calculated by choosing value of k_1, k_2, k_3 and $k_4 < 1$. Given the rationale mentioned earlier, k_1 and k_2 are set to 0.96, k_3 and k_4 to 0.99 in Koop and Korobilis (2014). This study follows the settings in Koop and Korobilis (2014). In addition, the restricted case of a TVP-FAVAR with SV can be obtained by setting forgetting factors and decay factors to particular values: $k_3 = 1$ yields a FA-TVP-VAR with SV, and $k_3 = k_4 = 1$ results in a TVP-FAVAR with constant volatility.

CHAPTER 2

CONSTRUCTING FINANCIAL CONDITIONS INDICES FOR THE UNITED KINGDOM: THE CHOICE OF INDICATORS

2.1 Introduction

There exists a growing interest in the literature around estimating a financial conditions index (FCI) that summarises information on the current state of financial markets and serves as a good leading indictor of economic activity (Hatzius, Hooper, Mishkin, Schoenholtz and Watson, 2010; Paries, Maurin and Moccero, 2014; Wacker, Lodge and Nicoletti, 2014).

The purpose of this chapter is to construct an optimal financial conditions index for the United Kingdom (UK). Following Hatzius et al. (2010) and Koop and Korobilis (2014), the optimal FCI is chosen based on its ability to forecast economic activity. Drawing on extensive readings on FCIs, there are three decisions involved in the construction of an FCI, (i) variable inclusion, (ii) variable weighting and (iii) index rebalancing. The first involves the financial indicators that should be incorporated in an FCI. The second relates to the choice of methodology for weighting index constituents. The third focuses on further adjustments that are necessary to correctly track a specific financial market. Index rebalancing refers to whether there are any new constituents that should be included in an optimal FCI and/or whether there are any existing variables that should be removed from the index at each point in time. This is an important consideration for policy makers because it attempts to answer questions like which financial variables should central bankers use to assess a financial market during specific time periods, e.g., financial crisis.

Chapter 1 focuses on the second decision that is faced in the construction of the FCI – variable weighting. It compares various models for weighting financial variables in an FCI for the UK and discovers that a time-varying parameter factor-augmented VAR (TVP-FAVAR) with stochastic volatility (SV) model is the optimal weighting method given the choice of variables. However, Chapter 1 does not take into account the questions behind the first and the third choices such as which variables should be included in an FCI and how to rebalance index constituents at each point in time. Rather, it uses a small set of six variables which have been studied in the existing literature in order to compare results from using different weighting methods with previous studies.

There are limitations involved in the use of such a small data set. However, in many other principal-component studies like Hatzius et al. (2010), Paries et al. (2014) and Koop and Korobilis (2014), a large data set is used. Then FCIs are calculated as the co-movements of multiple financial variables. Focusing on the United States (US), Hatzius et al. (2010) argue that the narrowness of the underlying series in an FCI is likely to result in the exclusion of potentially important financial conditions. Therefore, they advocate developing a broader index of financial conditions in order to overcome this limitation. In the econometric exercise, they select 45 variables to fully represent the financial system. Since several financial indicators used in Hatzius et al. (2010) are not available in the UK, Wacker et al. (2014) study the financial market of the UK using the standard principal component analysis (PCA) and use only 16 variables. Focusing on other industrialised countries, Paries et al. (2014) use a panel of 62 indicators for the Eurozone (EU). Therefore, it is reasonable to conclude that just the six variables, as used in Chapter 1, may not be sufficient when constructing an optimal FCI. As maintained in Hatzius et al. (2010), the best FCI should have a larger data set than the coverage in any of the existing FCIs for the specific market.

This study builds on the findings in Chapter 1 on the optimal variable weighting method and employs the TVP-FAVAR with SV model as the optimal method to weight financial indicators. Therefore, the focus of this chapter is primarily on the other two decisions in FCI construction.

This study is the first to use Dynamic Model Averaging (DMA) to investigate the choice of financial indicators and which indicators should be included at each point in time for the FCI of the UK. A joint model of the DMA model and the TVP-FAVAR with SV model (henceforth, DMA-TVP-FAVAR model) is employed to address the above three choices simultaneously. Since the DMA-TVP-FAVAR model to be used in this study is purely data-driven and the DMA procedure is able to discover the most important constituents and then assign them the highest weights, it is crucial to incorporate as many relevant financial variables as possible. This study includes a much wider range of financial variables (21 indicators) compared to the existing literature (for instance, six variables in Guichard et al. 2009, six variables in Castro 2011, 16 variables in Wacker et al. 2014) for the UK. This study uses a similar period to Chapter 1, i.e., 1993:I to 2013:II.

The remainder of this chapter is organised as follows: Section 2.2 reviews the literature in the area of FCIs. Section 2.3 discusses data issues while Section 2.4 introduces the methodologies used in this study. The empirical evidence is given in Section 2.5. Section 2.6 concludes.

2.2 Literature Review

In this literature review, this chapter first considers the results obtained in Chapter 1. Then it discusses the development and application of the DMA method in order to provide an overall picture of how to create an optimal FCI for the UK.

As in Hatzius et al. (2010) and Chapter 1, all FCI estimation methods fall into two broad categories: (i) the weighted-sum approach and (ii) the principal-component approach. A weighted-sum method determines the weight on each financial variable based on the estimates of the impact of changes in this variable on economic activity. A principal-component method extracts common factors from a group of variables.

The weighted-sum approach has three alternatives including (i) performing simulations with a macro econometric model, (ii) employing impulse responses in a VAR model and (iii) estimating a reduced form model. Chapter 1 highlights several drawbacks in the existing weighted-sum methods. First, in either a macro econometric model or a VAR model the weights assigned to financial indicators are always fixed. Although a time-varying parameter VAR (TVP-VAR) model can be used to address this problem, VAR studies usually keep the number of financial variables to a minimum in order to avoid making the VAR too heavy. Second, large scale macro econometric models are not available in many countries. Price, Kapetanios and Young (2015) who are economists with the Bank of England (BOE) acknowledge that they do not have robust enough models available to enable them to study the UK economy. Third, all FCI studies using weighted-sum methods fail to take the Primiceri (2005) findings into account, namely that stochastic volatility should be considered in a regression analysis. To overcome these three limitations, Chapter 1 develops a 'two-step' procedure. Then it compares the forecasting ability of an FCI based on the procedure to indices created by various principal-component methods. As in Hatzius et al. (2010) and Koop and Korobilis (2014), a good FCI is one which forecasts economic activity. Chapter 1 indicates that the TVP-FAVAR with SV model that allows loadings to vary over time is the most appropriate method for weighting financial variables in an FCI for the UK.

In Chapter 1, all FCIs are still estimated *ex-post* with the entire data set. There are two primary limitations in the estimation in Chapter 1.

Firstly, it uses the same set of six variables as employed in the existing literature such as Castro (2011) in constructing the FCI. The purpose of this is to make a valid comparison between that and previous studies. However, as already mentioned, Hatzius et al. (2010), Paries et al. (2014) and Koop and Korobilis (2014) consider large data sets for the objective of including all potentially important financial conditions.

Secondly, as agreed by Hatzius et al. (2010) and Koop and Korobilis (2014), the other trait of an optimal FCI is to predict economic activity as accurately as possible. From an econometric or statistical point of view, there is growing evidence suggesting that using all the available data to extract co-movements in a principal-component method is not always optimal. This issue is also mentioned in Boivin and Ng (2006). In a forecasting exercise of real time data, they discover that factors extracted from as few as 40 series often yield satisfactory or even better result than using their entire set of 147 series. Therefore, a technique that is able to determine the best combination of constituents in an FCI is required. For example, if the information set has n variables, a maximum of $2^n - 1$ combinations of financial indicators can be used to construct an FCI. It is also necessary to decide at each point in time which combination has the greatest forecasting ability.

Koop and Korobilis (2014) adapt the Raftery, Karny and Ettler (2010) DMA model and develop a joint model combining both the DMA and the TVP-FAVAR with SV models for the US financial system. As already mentioned, the major advantage of a

DMA-TVP-FAVAR model is to construct the 'best' FCI at different points in time by considering different combinations of variables.

Prior to Raftery et al. (2010), the method used in the literature to select the different variables to make up an FCI at each point in time is the dynamic model selection (DMS) method. However, there is great uncertainty in the variable selection. As in Clyde and George (2004), in cases where no single combination of variables stands out, it would be preferable to a set of possible combinations. Instead of selecting a single optimal model at each point in time (as in the DMS method), the DMA-FAVAR-SV model constructs an FCI by averaging over various FCIs created using different financial variables and weights. The idea of averaging all the possible models addresses the expected risk (i.e., the problem where no single FCI stands out) of the final forecast in the DMS method.

Because Chapter 1 has already determined that the TVP-FAVAR with SV model is the best method for weighting index constituents in an FCI for the UK, this study follows Koop and Korobilis (2014) closely and employs the DMA-TVP-FAVAR model to determine (i) which variables should be included in the UK's optimal FCI and (ii) how to rebalance the combination of index constituents at each point in time – i.e., the two choices mentioned earlier. In order to ensure that the estimated FCI is the optimal index summarising financial market conditions in the UK, this study also includes a larger set of financial indicators than the coverage in any of the existing FCIs for the UK. It draws on extensive readings of the UK financial system including Goodhart and Hofmann (2001), Batini and Turnbull (2002), Guichard, Haugh and Turner (2009), Castro (2011) and Wacker et al. (2014) and attempts to include all the relevant indicators in the FCIs for the UK.

The DMA technique could be considered as an extension of the Bayesian model averaging (BMA). As in Raftery, Madigan and Hoeting (1997) and Raftery et al. (2010), both the BMA and the DMA methods address the problem of uncertainty about variable selection in a regression by averaging over all the possible combination of regressors. The primary difference between these two methods is that the former deals with model uncertainties in a static linear regression while the latter focuses on the similar problems in a state-space model. The reader is referred to Section 2.4 for a detailed discussion on the background and development of the DMA and the joint model of the DMA and TVP-FAVAR with SV models (i.e., the DMA-TVP-FAVAR model).

Applying the DMA-TVP-FAVAR model to the US data, Koop and Korobilis (2014) construct an FCI with 17 financial variables which results in $2^{17} - 1$ or 131,071 different combinations of variables. Then they compare the forecasting ability of the FCI created by the DMA-TVP-FAVAR model with that from a single TVP-FAVAR with SV model. The evaluation of prediction accuracy is based on the mean squared forecast errors (MSFEs) and the average predictive likelihood (APL). Their results show that using the DMA method with 131,071 TVP-FAVAR with SV models leads to substantial improvements in the FCIs' forecasting performance. For comparative purposes, they also create another FCI by applying the DMS method to the TVP-FAVAR with SV model (called DMS-TVP-FAVAR). The estimated APL implies that compared to the FCI from a DMA-TVP-FAVAR model the DMS-TVP-FAVAR model based FCI yields lower predictive likelihood. Therefore, Koop and Korobilis (2014) conclude that the DMA-TVP-FAVAR model is among the best forecasting models in the US.

However, the Koop and Korobilis (2014) work is limited to US data. To the knowledge of the author, few if any other studies have used the DMA-TVP-FAVAR model for creating FCIs in other countries. Thus, it would be particularly interesting to employ the DMA technique to further improve the estimation of FCIs in the UK.

2.3 Data

This study uses statistics published by the BOE, the Office for National Statistics (ONS) and DataStream as the primary data sources. The data used is quarterly. The sample period covers 1993:I-2013:II. During this time the Monetary Policy Committee (MPC) has been operating an inflation targeting approach and reporting its inflation forecasts on a quarterly basis. Although several earlier studies (e.g., Koop and Korobilis, 2014) use real-time data to construct FCIs, many FCIs are estimated with *ex-post* data (e.g., Goodhart and Hofmann, 2001; Castro, 2011). Adema (2004), Osterholm (2005) and Sauer and Strum (2007) argue that the use of real-time data (instead of *ex-post* data) would not lead to substantially different results. Since the real-time data for several important indicators, such as house prices and output growth, is very difficult to access, this study uses *ex-post* data in the econometric estimation.

Table 1: Description of Financial Variables

Name	Description
RTrea3m	Real three-month treasury bill discount rate (Trea3m), CPI deflated
REER	Real effective exchange rate index, CPI deflated
RSPI	Real share price index, quarterly average of FTSE 100, CPI deflated
RHPI	Real house price index, CPI deflated
CredSprd	Spread between Ten-year government bond yield and corporate bond yield
ΔFutSprd	Changes in spread between future interest rate last quarter and current Trea3m
ComSprd	Spread between three-month gilt rate and three-month commercial paper yield
BFinSprd	Spread between corporate bond yield and financial corporate bond yield
BQualSprd	Spread between the yields of AA-rated- and BBB-rated- corporate bond
LiborSprd	Spread between 3m gilt rate and the 3m London inter bank offered rate (Libor3m)
TedSprd	Spread between Trea3m and Libor3m
SoniaSprd	Spread between Sonia and Sterling one-year mean interbank lending rate
UnsecSprd	Spread between fixed mortgage rate and unsecured lending rate for personal loans
NFCLSprd	Spread between three-month gilt rate and private NFC interest rate on new loan
MktCap	MSCI UK equity market capitalisation
AllPER	FTSE all share P/E ratio
Writeoffs	UK write-offs and other revaluations of loans by banks
HouseLoan	Household Credit Market Debt Outstanding
TotalM	Total mortgages outstanding
CommIX	Reuters commodity Index, quarterly average
VolIX	FTSE 100 volatility index

Note: while estimating an FCI, this study uses the deviation of the above 21 variables from their long-run trend levels. As in Chapter 1, the long-run trend of RTrea3m is defined as its mean level. The Hodrick-Prescott (HP, 1997) filter is taken to estimate the long-term trend of the remaining 20 variables.

In addition to measuring conditions in the interest rate, the exchange rate and asset markets as in Chapter 1, this study also considers variables to account for safe spreads[1], private sector spreads, lending markets and equity markets. As in Wacker et al. (2014), the rationale for including risk measures such as safe spreads and spreads in the private sector is straightforward. Spreads measure the relative prices at which finance is available to certain market participants. The inclusion of lending variables is in line with the literature including Hatzius et al. (2010) and Koop and Korobilis (2014) in order to reflect how easily investors can access finance. Additional equity market indicators not only assess the performance of stocks overall but also measure market risks. This study employs 21 financial variables covering a wider range of

[1] Safe spread securities are those offering higher yields without taking much default risk. Therefore, safe spread is defined as the difference between a safe spread security and an appropriate benchmark (usually a risk-free return).

financial indicators than most existing FCIs in the UK. A detailed list of the variables is given in Table 1. Table 2 summarises the 21 variables under seven categories. Appendix 2 shows the evolution of all variables involved in this study.

Table 2: Categories of Financial Variables and Respective Covers

No.	Name	Sample	Source
Interest rates:			
1	RTrea3m	1993:I-2013:II	BOE statistics
Foreign exchange rate markets:			
2	REER	1993:I-2013:II	OECD statistics
Asset prices:			
3	RSPI	1993:I-2013:II	OECD statistics
4	RHPI	1993:I-2013:II	Nationwide Building Society
5	CommIX	1993:I-2013:II	DataStream
Safe spreads:			
6	LiborSprd	1996:I-2013:II	BOE statistics
7	TedSprd	1993:I-2013:II	BOE statistics
8	SoniaSprd	1997:I-2013:II	BOE statistics
Private sector spreads			
9	CredSprd	1993:I-2013:II	DataStream and BOE statistics
10	BFinSprd	2004:II-2013:II	DataStream
11	BQualSprd	2004:II-2013:II	DataStream
12	ΔFutSprd	1993:I-2013:II	DataStream and BOE statistics
13	ComSprd	2003:II-2013:I	DataStream and BOE statistics
14	UnsecSprd	1995:I-2013:II	BOE statistics
15	NFCLSprd	2004:I-2013:II	BOE statistics
Lending:			
16	Writeoffs	1993:I-2013:I	DataStream
17	HouseLoan	1993:II-2013:II	DataStream
18	TotalM	1997:III-2013:II	DataStream
Other equity market indicators:			
19	MktCap	1993:I-2013:II	DataStream
20	AllPER	1993:II-2013:II	DataStream
21	VolIX	2000:II-2013:II	DataStream

Since the model to be used in this study is purely data-driven and the DMA technique is expected to decide which variables are important, it seems quite important to include as many relevant financial variables in the information set as possible in creating an optimal FCI. Because of the data availability, the existing FCI literature (see, Wacker et al., 2014) uses fewer financial variables to construct an FCI for the UK than for the US. This study draws on extensive readings and attempts to include all financial indicators used in the existing literature for the UK. The DMA-TVP-FAVAR model is then employed to: (i) find the optimal combination of index constituents at each point in time, (ii) weight each possible index and also (iii) construct an optimal FCI.

Table 3: Description of the FCI Constituents

No.	Name	Description
1	RTrea3mGap	The difference between RTrea3m and its steady-state level
2	REERGap	The percentage deviation of REER from its long-term trend
3	RSPIGap	The percentage deviation of RSPI from its long-term trend
4	RHPIGap	The percentage deviation of RHPI from its long-term trend
5	CredSprdGap	The difference between CredSprd and its long-term trend
6	ΔFutSprdGap	The difference between ΔFutSprd and its long-term trend
7	ComSprdGap	The difference between ComSprd and its long-term trend
8	BFinSprdGap	The difference between BFinSprd and its long-term trend
9	BQualSprdGap	The difference between BQualSprd and its long-term trend
10	LiborSprdGap	The difference between LiborSprd and its long-term trend
11	TedSprdGap	The difference between TedSprd and its long-term trend
12	SoniaSprdGap	The difference between SoniaSprd and its long-term trend
13	UnsecSprdGap	The difference between UnsecSprd and its long-term trend
14	NFCLSprdGap	The difference between NFCLSprd and its long-term trend
15	MktCapGap	The percentage deviation of MktCap from its long-term trend
16	AllPERGap	The percentage deviation of AllPER from its long-term trend
17	WriteoffsGap	The (negative) percentage deviation of Writeoffs from its long-term trend
18	HouseLoanGap	The percentage deviation of HouseLoan from its long-term trend
19	TotalMGap	The percentage deviation of TotalM from its long-term trend
20	CommIXGap	The percentage deviation of CommIX from its long-term trend
21	VolIXGap	The (negative) percentage deviation of VolIX from its long-term trend

Note: the mnemonics for each financial variable in this table (e.g., RTrea3m, REER) are provided above in Table 1.

Prior to estimating an FCI, it is important to de-trend all the financial variables in Table 1 and Table 2. Hence, the resulting index could be explained as the deviation of the financial market from its long-term trend. As defined in the notes of Table 1, the trend level of each financial variable, except for real three-month Treasury bill discount rate (RTrea3m), is estimated with the HP filter (Hodrick and Prescott, 1997). In order to be consistent with the existing literature (e.g., Castro, 2011), this study defines the long-run trend of RTrea3m as its average. Table 3 summarises the de-trended variables applied to the DMA-TVP-FAVAR model. Although several studies argue that the HP filter may not be a good algorithm to estimate trends, a recent study of Guerrero (2008) provides evidence that it is a reasonable approach for extracting an unobservable trend. This study conducted a preliminary analysis by examining the movement of each de-trended financial variable. It discovers that asset prices were below their long-run trends during the financial crisis and the spreads widened in the sample period. All the indicators improve with the recovery of the UK financial market. Therefore, the preliminary analysis concluded that the general trends of the variables in Table 3 are consistent with prior expectations.

As in Chapter 1, this study uses the three-month Treasury bill discount rate (Trea3m), Consumer Price Index (CPI) inflation rate [2] and the Gross Domestic Product (in millions of chained 2010 price) growth rate to measure the short-term interest rate, inflation rate and real economic output in the UK respectively. To ensure stationarity, it transforms the CPI as follows (π_t denotes the annual inflation rate):

$$\pi_t = ln\ (CPI_t) - ln\ (CPI_{t-4}) = ln\left(\frac{CPI_t}{CPI_{t-4}}\right) \tag{3.1}$$

The output gap (y_t) is defined as the difference between the real GDP growth (g_t^y) and its sample average ($\bar{g}_t^y$):

$$g_t^y = ln\ (real\ GDP_t) - ln\ (real\ GDP_{t-1}) \tag{3.2}$$

$$y_t = g_t^y - \bar{g}_t^y \tag{3.3}$$

Another important issue in the econometric exercise is the treatment of missing values. As shown in Table 2, the sample of financial variables is not balanced. Although the sample period covers 1993:I-2013:II, some constituents have missing values in that they do not begin until 1997 or even later. In the case of using the DMA-TVP-FAVAR model to extract an FCI, there is a risk that the value of the FCI between 1993 and 1997 has to be extracted with financial variables which all have missing values during that period. To prevent such estimation issues, Koop and Korobilis (2014) let the equity index always be included in the list of financial indicators. However, selecting one financial variable to be always included in each TVP-FAVAR model (as in Koop and Korobilis, 2014) is quite arbitrary. This study chooses at least one financial indicator from each main category.

In the case of the UK, Chapter 1 reviews much relevant literature and concludes that the short-term interest rate, the real effective exchange rate index, real house prices

[2] Following the BOE, the inflation rate is calculated as the annual rate of change in the Consumer Price Index.

and the real share price index are the four most relevant indictors and are always employed in the FCI literature (see, for instance, Goodhart and Hofmann, 2001; Castro, 2011). Hence, this study differs from Koop and Korobilis (2014) in that it allows for the inclusion of RTrea3mGap, REERGap, RSPIGap and RHPIGap (see, Table 3 for the description of these four variables) in each model's information set. Therefore, this restriction would satisfy the statistical requirement of the DMA-TVP-FAVAR model[3]. This means that RTrea3mGap, REERGap, RSPIGap and RHPIGap are not subject to model averaging and the DMA technique is performed using the remaining 17 variables. Following Koop and Korobilis (2014), this study also makes the assumption that the FCI is estimated with observed data only. It replaces missing values of the remaining 17 variables with zeros. Therefore, at a minimum the FCI will be extracted using the RTrea3mGap, REERGap, RSPIGap and RHPIGap. To distinguish from Koop and Korobilis (2014) which assesses $2^n - 1$ factor models, this study tests 2^{n-4} factor models where n is the number of variables included in the information set.

Table 4 reports the results of unit root and stationary tests for the variables used in this study. Due to the low power and poor performance of unit root tests in small samples, this study follows the methodology used in Castro (2011). It reports the results of two unit root tests, i.e., augmented Dickey and Fuller (1979) test (ADF) and Phillips and Perron (1988) test (PP) to investigate whether test power is an issue. It also reports the Kwiatkowski, Phillips, Schmidt and Shin (1992) stationarity test (KPSS) results for robust purposes.

The test results displayed in Table 4 indicate that the power of unit root tests seems to be an issue for the UK. The ADF and the PP tests are unable to reject the unit root in RTrea3mGap. Although the evidence fails to support the stationarity hypothesis for RTrea3mGap given the sample period, if this study were to consider a longer time period it would expect to find evidence of stationarity for RTrea3mGap. The KPSS test produces the evidence of stationarity for the remaining 20 variables.

[3] This study also completes a robustness test by choosing another four financial variables (REERGap, TedSprdGap, MktCapGap and HouseLoanGap) from the four main categories. The results indicate that the general trend of the FCI is insensitive to this change in the restriction.

Table 4: Unit Root and Stationary Tests

	ADF	PP	KPSS
RTrea3mGap	-0.6039	-0.5668	0.7743
REERGap	-3.3345*	-2.9621*	0.0378#
RSPIGap	-2.9945*	-2.7191*	0.0403#
RHPIGap	-3.9347*	-2.8855*	0.0731#
CredSprdGap	-5.1627*	-5.2267*	0.0247#
Δ FutSprdGap	-8.8033*	-10.485*	0.0918#
ComSprdGap	-3.5669*	-3.2439*	0.0448#
BFinSprdGap	-4.0380*	-4.8224*	0.1202#
BQualSprdGap	-4.9176*	-4.9161*	0.0467#
LiborSprdGap	-4.9513*	-4.5812*	0.0254#
TedSprdGap	-5.5179*	-5.0141*	0.0208#
SoniaSprdGap	-6.3221*	-3.0188*	0.0293#
UnsecSprdGap	-4.7876*	-4.4222*	0.0267#
NFCLSprdGap	-4.3705*	-4.3561*	0.0506#
MktCapGap	-5.4137*	-5.0165*	0.0317#
AllPERGap	-5.6223*	-3.9395*	0.0249#
WriteoffsGap	-9.5520*	-10.431*	0.0691#
HouseLoanGap	-3.5028*	-5.3129*	0.0810#
TotalMGap	-3.1155*	-3.1201*	0.1363#
CommIXGap	-4.6011*	-3.1983*	0.0479#
VolIXGap	-4.1518*	-4.1378*	0.0673#

Note: * Unit root is rejected at a significance level of 10%; # The stationarity is not rejected at a significance level of 1%; all the test regressions here contain a constant.

2.4 Methodology

As already mentioned, the DMA method is an extension of the BMA technique which deals with model uncertainties in a static linear regression. This section begins with a discussion on the development of the DMA method and then applies it to the TVP-FAVAR with SV model to explain how the DMA-TVP-FAVAR model works.

In addition to the BMA and the DMA methods, other model selection techniques (e.g., the DMS method) are available to discover the single 'best' model – in other words, the single 'optimal' combination of financial variables (which is based on their forecasting ability). However, studies such as Leamer (1978), Draper (1995) and Raftery et al. (1997) argue that basing inference on a single model as if the single selected specification were the optimal one fails to address the uncertainty in the model selection, which may underestimate the uncertainty about quantities of interest. In order to solve this problem Leamer (1978) proposes a standard Bayesian solution.

As in Leamer (1978) and Raftery et al. (1997), selecting subsets of input variables is a basic part of a regression. In the case of a linear regression, the objective of data selection or the BMA method is to determine the best model of the form:

$$Y = \theta_0 + \sum_{j=1}^{n} \theta_j X_j + \varepsilon \tag{4.1}$$

where $X_1, X_2, \ldots, X_m$ is a subset of candidate inputs, $X_1, X_2, \ldots, X_n$. The best model is defined as the one that provides the most accurate prediction of the output variable, Y. Let $M = \{M_1, M_2, \ldots, M_K\}$ denote all the models being considered, M_k the correct model and Δ the quantity of interest. Express the posterior distribution of Δ given data as:

$$\pi(\Delta|\text{data}) = \sum_{k=1}^{K} \pi(\Delta|M_k, \text{data})\, \pi(M_k|\text{data}) \tag{4.2}$$

where the term $\pi(\Delta|\text{data})$ is an average of the posterior distribution under each model which is weighted by the corresponding posterior model probabilities, i.e., the BMA method. The posterior distribution of M_k is written as:

$$\pi(M_k|\text{data}) = \frac{\pi(\text{data}|M_k)\pi(M_k)}{\sum_{i=1}^{K} \pi(\text{data}|M_i)\pi(M_i)} \tag{4.3}$$

where:

$$\pi(\text{data}|M_k) = \int \pi(\text{data}|\theta^k, M_k)\, \pi(\theta^k|M_k) d\theta^k \tag{4.4}$$

Eq. (4.4) is the marginal likelihood of the model derived by integrating the product of the likelihood $\pi(\text{data}|\theta^k, M_k)$ and the prior $\pi(\theta^k|M_k)$ over the parameter space. The term θ^k is the parameter of the correct model M_k but is assumed to be constant. It is worth noting that in the BMA method all probabilities mentioned are implicitly conditional on M – the set of all models being taken into account. As analytically proven in Raftery et al. (1997), averaging over all models of interest in the BMA method results in better predictive ability than using a single best model. Madigan and Raftery (1994) also investigate this issue by employing three examples. In their experiments, the model averaging approach is found to have better out-of-sample predictive performance than any single model that may be reasonably selected.

However, as a static method the BMA method cannot deal with a situation where data arises sequentially and the form of generating models can change (Onorante and Raftery, 2014). For example, the BMA method may work poorly with a state space model. To address this problem, the DMA method considers a state space model. It allows model form and model parameters to evolve over time and tracks both recursively. In other words, the DMA method investigates a case where multiple state space models $(M_1, M_2, \dots, M_K)$ are considered, however there is uncertainty about which one of them is the best at each point in time. As a special case when parameters and model form do not change, the DMA method is reduced to a recursive implementation of the standard BMA method. The general form of a state space model which is used in the DMA method is written as:

$$y_t = \theta_t x_t + \varepsilon_t; \quad \varepsilon_t \sim iid\ N(0, V) \tag{4.5}$$

$$\theta_t = \theta_{t-1} + \delta_t; \quad \delta_t \sim ind\ N(0, H_t) \tag{4.6}$$

where Eq. (4.5) is an observation equation in which the regression parameters θ_t are allowed to evolve according to the state equation, Eq. (4.6). Since $M_1, M_2, \dots, M_K$ are K models of interest, the parameters θ_t and the predictors x_t for each candidate model are different. Let quantities specific to model M_k be denoted by a superscript (k). Then the model M_k takes the form:

$$y_t = \theta_t^{(k)} x_t^{(k)} + \varepsilon_t^{(k)}; \quad \varepsilon_t \sim iid\ N\left(0, V^{(k)}\right) \tag{4.7}$$

where $x_t^{(k)}$ contains a subset of the potential explanatory variables x_t. As mentioned, if there are n variables in x_t, there are $2^n - 1$ possible state-space models removing the model with no variables. The DMA method averages across all models with a recursive updating scheme. Its goal is to estimate the probability that model M_k applies at time t given information through time $t-1$. Let $\pi_{t|t-1,k}$ represent that probability. Once $\pi_{t|t-1,k}$ for $k = 1, 2, \dots, K$ are obtained, they can be employed to do model averaging. Those probabilities can also be thought of as the weights of M_k in predicting y_t with the data available at time t. As mentioned earlier, the DMS method arises if the model with the highest value for $\pi_{t|t-1,k}$ is selected. However, in the DMA method the model averaging is done at time t using $\pi_{t|t-1,k}$ for $k = 1,2,\dots,K$ as weights. As stressed in Onorante and Raftery (2014), the use of the word 'dynamic' in the term of the DMA means that these weights can vary over time.

Raftery et al. (2010) derive the updating equation for the DMA method:

$$\pi_{t|t,k} = \frac{\pi_{t|t-1,k} L_k(Data_t|Data_{1:t-1})}{\sum_{j=1}^{K} \pi_{t|t-1,j}\, L_j(Data_t|Data_{1:t-1})} \tag{4.8}$$

where $\pi_{t|t,k}$ is the update of $\pi_{t|t-1,k}$ with the data available at time t. The term $L_i(Data_t|Data_{1:t-1})$ is the predictive likelihood or a measure of fit for the model M_k. This algorithm estimates the weights to be taken in the next period with a forgetting factor α (Eq. 4.9). The name forgetting factor means that in the observation j periods in the past receives a weight of α^j. The reader is referred to Raftery et al. (2010) and Chapter 1 for an explanation of the forgetting factor.

$$\pi_{t+1|t,k} = \frac{\pi_{t|t,k}^{\alpha}}{\sum_{j=1}^{K} \pi_{t|t,j}^{\alpha}} \tag{4.9}$$

where the value of α is specified by users.

Although there is much evidence supporting the BMA approach in a static model, for the purpose of robustness Raftery et al. (2010) compare the predictive ability of the DMA against the single best performing model. Several results stand out. In the case of a small number of state space models where one of them is clearly best (i.e., the quantity of interest is one), the DMA method performs slightly better than the DMS. However, if there is uncertainty in selecting the best model and the number of models of interest is large (i.e., the quantity of interest Δ is high), the DMA method achieved significantly better performance. As previously noted, the better model is one which gives the most accurate prediction of the dependent variable. Considering FCI studies tend to include a large number of index constituents, this study opts to choose the DMA method instead of the DMS model in the following exercise.

Following Koop and Korobilis (2014) and Chapter 1, this study re-writes the p-lagged TVP-FAVAR with SV model as:

$$x_t = \lambda_t^y y_t + \lambda_t^f f_t + u_t, \quad u_t \sim N(0, V_t) \tag{4.10}$$

$$\begin{bmatrix} y_t \\ f_t \end{bmatrix} = c_t + B_{t,1} \begin{bmatrix} y_{t-1} \\ f_{t-1} \end{bmatrix} + \cdots + B_{t,p} \begin{bmatrix} y_{t-p} \\ f_{t-p} \end{bmatrix} + \varepsilon_t, \quad \varepsilon_t \sim N(0, Q_t) \tag{4.11}$$

where x_t is an $n \times 1$ vector of un-purged financial variables as constituents in computing an FCI. The term y_t denotes the de-meaned real GDP growth rate which enters Eq. (4.10) to purge the financial indicators of the impact of economic activity. The term f_t is a latent factor which is interpreted as an FCI. Both the regression coefficients λ_t^y and the factor loadings λ_t^f evolve throughout the full sample period. This model includes two equations. The former (Eq. 4.10) extracts a latent factor f_t from the information set x_t, and the latter (Eq. 4.11) models the dynamic interaction of the factors with y_t. Chapter 1 stresses that the TVP-FAVAR with SV model is an extension of the factor-augmented VAR (FAVAR) model developed in Bernanke, Boivin and Eliasz (2005). One of the primary purposes of estimating an FCI in a VAR model instead of computing it independently is to assess its ability to forecast y_t, i.e., to provide an answer to questions such as what makes a good FCI. Although earlier literature such as Hatzius et al. (2010) and Koop and Korobilis (2014) use the extracted FCI to forecast both the inflation rate and the output gap, this study only considers its forecasting ability for the real output gap. This is motivated by the fact that changes in financial markets impact real economic output more directly than the inflation rate. As explained by the MPC (June 2012, p. 3), the financial system affects output in the first round which in turn alters the inflation rate.

In the system of Eqs. (4.10-4.11), u_t and ε_t are zero-mean Gaussian errors with covariances V_t and Q_t and $B_{t,1}, \ldots B_{t,p}$ are VAR parameters. Negro and Otrok (2008) and Eickmeier, Lemke and Marcellino (2011) suggest a model where the factor loadings are set as random walks. Primiceri (2005) and Nakajima (2011a) also assume that VAR parameters follow a random walk process. Following these papers, this study sets λ_t^y, λ_t^f and $B_{t,1}, \ldots B_{t,p}$ as:

$$\lambda_t = \lambda_{t-1} + v_t, \quad v_t \sim N(0, W_t) \tag{4.12}$$

$$\beta_t = \beta_{t-1} + \eta_t, \quad \eta_t \sim N(0, R_t) \tag{4.13}$$

where $\lambda_t = \left((\lambda_t^y)', (\lambda_t^f)'\right)'$, $\beta_t = \left(c_t', vec(B_{t,1})', \ldots, vec(B_{t,p})'\right)'$. Given Primiceri's (2005) recommendation regarding heteroskedasticity, this study lets V_t and Q_t be time-variant. As in Primiceri (2005) and Koop and Korobilis (2014), the covariance matrix V_t is diagonal thus ensuring that u_t is a vector of idiosyncratic shocks.

While considering the DMA method with the TVP-FAVAR with SV model as in Koop and Korobilis (2014), let the quantities specific to the DMA-TVP-FAVAR model M_k be denoted by a superscript (k). Then $x_t^{(k)}$ is a subset of x_t. The term $f_t^{(k)}$ denotes an FCI implied by M_k:

$$x_t^{(k)} = \lambda_t^y y_t + \lambda_t^f f_t^{(k)} + v_t \tag{4.14}$$

$$\begin{bmatrix} y_t \\ f_t^{(k)} \end{bmatrix} = c_t + B_{t,1} \begin{bmatrix} y_{t-1} \\ f_{t-1}^{(k)} \end{bmatrix} + \cdots + B_{t,p} \begin{bmatrix} y_{t-p} \\ f_{t-p}^{(k)} \end{bmatrix} + \varepsilon_t \tag{4.15}$$

The DMA method estimates the final FCI by averaging over various $f_t^{(k)}$ (for $k = 1, 2, \ldots, K$). The weight on each individual $f_t^{(k)}$ is estimated with Eq. (4.9). The reader is referred to Chapter 1 or Koop and Korobilis (2014) for the full algorithm used to calculate a single TVP-FAVAR with SV model. This section provides a general description of the estimation process for a single TVP-FAVAR with SV model and then explains how to solve a DMA-TVP-FAVAR specification.

As demonstrated by Primiceri (2005), if a researcher has selected a specification for V_t, Q_t, W_t and R_t and priors for initial conditions, Bayesian statistical inference can be used in a very straightforward fashion with the Markov chain Monte Carlo (MCMC) method. Primiceri (2005) and Nakajima (2011a) show that this algorithm works quite well with state-space models while considering stochastic volatilities. However, in the case of a factor augmented VAR (FAVAR), Koop and Korobilis (2013, 2014) argue that this Bayesian approach is computationally intensive. Hence, they use two models, the forgetting factor model and the exponentially weighted moving average (EWMA) model. Then they develop a fast two-step algorithm based on a dual Kalman filter algorithm. With the algorithm developed in Koop and Korobilis (2014), this study estimates a single TVP-FAVAR with SV model by choosing values for k_1, k_2, k_3 and k_4. Parameters k_1 and k_2 control the expected amounts of time-variation in the volatility (V_t and Q_t), while the last two parameters (k_3 and k_4) indicate the expected amounts of time-variation in factor loadings (λ_t) and regression parameters (β_t) respectively. As in Koop and Korobilis (2014), this study sets $k_1 = k_2 = 0.96$ and $k_3 = k_4 = 0.99$ to estimate a single standard TVP-FAVAR with SV model.

To estimate a DMA-TVP-FAVAR model, this study runs the Koop and Korobilis (2014) algorithm for each of the 131,072 ($= 2^{21-4}$) TVP-FAVAR with SV models. As discussed earlier, the DMA uses $\pi_{t|t-1,k}$, the probability that model k applies at time t given the information through time $t-1$ as its weight in the averaging process. Raftery et al. (2010) introduce a model prediction equation with the forgetting factor method (as in Eq. 4.9):

$$\pi_{t|t-1,k} = \frac{\pi_{t-1|t-1,k}^{\alpha}}{\sum_{j=1}^{K} \pi_{t-1|t-1,j}^{\alpha}} \quad (4.16)$$[4]

where the exponent α is a forgetting factor. The above equation is equivalent to Eq. (4.9). Therefore, the associated model updating equation is Eq. (4.8). With Eq. (4.16) this study re-writes the weight used in the DMA method and attached to model k as follows:

$$\begin{aligned} \pi_{t|t-1,k} &\propto \left[\pi_{t-1|t-2,k} \times L_k(Data_{t-1}|Data_{1:t-2})\right]^{\alpha} \\ &= \prod_{i=1}^{t-1} [L_k(Data_{t-i}|Data_{1:t-i-1})]^{\alpha^i} \end{aligned} \quad (4.17)$$

where $\propto$ means proportionality.

Note that the focus in this study is on the ability of the FCI to forecast y_t. Therefore, this study sets the measure of fit as predictive likelihood for the real economic output gap $L_k(y_t|Data_{1:t-1})$. Eq. (4.17) is then rewritten as:

$$\pi_{t|t-1,k} \propto \prod_{i=1}^{t-1} [L_k(y_{t-i}|Data_{1:t-i-1})]^{\alpha^i} \quad (4.18)$$

[4] Eq. (4.16) requires to set the initial condition for $\pi_{0|0,k}$. This study lets $\pi_{0|0,k} = 1/K$ which is also done in many DMA studies including Raftery et al. (2010) and Koop and Korobilis (2012, 2013, 2014).

Therefore, model k tends to receive more weight at time t if it forecasts well in the recent periods. As in Koop and Korobilis (2012, 2013, 2014) and Chapter 1, the interpretation of the recent period is governed by the forecasting factor (α). Raftery et al. (2010) suggest that this forecasting factor (α) should be slightly less than but close to 1.0. Eq. (4.17) implies that forecast performance i periods ago receives α^i times as much weight as the last prediction period. Most existing DMA literature including Raftery et al. (2010) and Koop and Korobilis (2013, 2014) uses a benchmark value of $\alpha = 0.99$. This means that when using quarterly data the forecast three years ago has 88% as much weight as the forecast in the last quarter. This study follows Koop and Korobilis (2014) and uses a forgetting factor value of 0.99 in the econometric estimation. Finally, the DMA prediction can be done by averaging over the predictive results for each model using the weight, $\pi_{t|t-1,k}$. The model-average one-step-ahead prediction of the real output gap (y_t) is:

$$\hat{y}_t{}^{DMA} = \sum_{k=1}^{K} \pi_{t|t-1,k}\, \hat{y}_t^k \tag{4.19}$$

where $\hat{y}_t^k$ denotes the estimated output by model k.

2.5 Empirical Evidence

Before proceeding to the forecasting exercise, it is important to highlight the objective of this study which is to construct an optimal FCI for the UK. This study uses the same method (the TVP-FAVAR with SV model) as used in Chapter 1 to weight index constituents. It is particularly interesting to compare the estimates in this chapter with that in Chapter 1 to examine if adding the DMA method will improve the quality of an estimated FCI. The sample period runs from 1993:I to 2013:II.

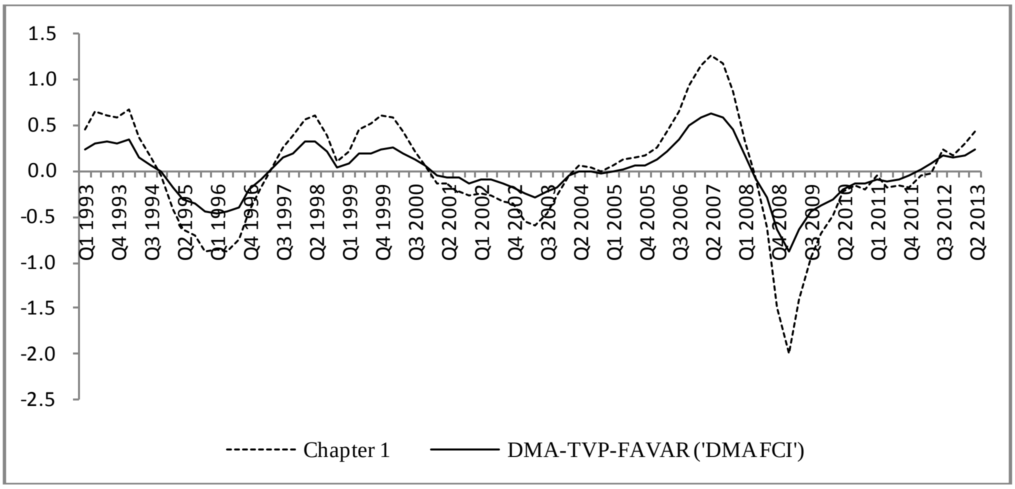


Figure 1: FCIs Constructed with the DMA-TVP-FAVAR Model; for comparison, the estimate in Chapter 1 is also plotted

Figure 1 displays the factor estimates with the six financial variables in Chapter 1 (i.e., the de-trended real interest rate, the de-trended real effective exchange rate, the de-trended real house price and the de-trended real share price) and the FCI implied by the DMA-TVP-FAVAR model. In general, the two indices exhibit similar tendencies, however the DMA-TVP-FAVAR based FCI is smoother. One possible explanation is that these two FCIs are estimated with the same principal variables (i.e., RTrea3mGap, REERGap, RSPIGap and RHPIGap), but the DMA-TVP-FAVAR model also considers a wider range of indicators which may explain the smoother index.

As shown in Figure 1, the movement of the 'DMA FCI' exhibits the characteristics of business cycles over the sample period. It hits the first peak around 1993:III which is primarily caused by the substantial depreciation of sterling. The decline of the index in 1994 reflects the response of financial markets to the rise in the BOE's official interest rate. As illustrated in Chapter 1, the real effective exchange rate, real equity prices and real house prices all dropped down below their long-run trend between 1995:I and 1997:I.

It is worth noting that the DMA-TVP-FAVAR based FCI confirms the recession in the financial markets during 2001-2003. This evidence points to the inadequacy of the FCIs in some of the earlier literature for the UK such as Castro (2011). As already discussed in Chapter 1 and the MPC's inflation reports, the 2001-3 recession is triggered by the corporate accounting irregularities in the US which causes sharp decreases in equity prices around the world. Financial markets then reflect prosperity between 2004 and 2007 when almost all indicators improve and are above their long-run trend. The recent financial crisis that is evident as a sudden decline in the DMA FCI around 2007:IV leads to considerable turmoil in the international financial system. Although the DMA FCI is relatively smooth compared to the estimates in Chapter 1, it also shows that the index falls from a 20-year high in 2007:II to a 20-year low in 2009:I. The MPC's Inflation reports in February 2008 and February 2009 both acknowledge that the fall in asset prices reflects growing pessimism among investors about the future. The housing market indicator which historically provides a good guide to near-term inflation trends largely remains very weak. In order to stimulate the domestic financial market and the real economy, the BOE has maintained its bank rate at a historical low of 0.5% and continued its asset purchases since 2009:II. This largely explains the subsequent recovery of the FCIs as plotted in Figure 1.

In addition, Figure 1 also confirms that the inclusion of the DMA method tends to yield a different FCI estimate. However, at this stage it is difficult to express any view on whether any FCI is better or worse than the other.

To examine the results improved by the DMA technique, this study calculates the time series average predictive likelihood (APL), i.e., the mean of $L_k(y_t|Data_{1:t-1})$ of the DMA-TVP-FAVAR model and that of the TVP-FAVAR with SV model. It is encouraging to find that the FCI based on the DMA technique has an APL of 0.5734, while the APL of an FCI estimated without any model averaging is 0.5693. This result indeed motivates the use of the DMA-TVP-FAVAR model in the UK. Furthermore, it could also be considered as evidence supporting Boivin and Ng (2006) who argue that using all available data to extract factors is not always optimal. As in concluded by Boivin and Ng (2006), sample size alone does not determine the properties of the principal component estimates and the quality of the data must be taken into account. For robustness purposes, this study reports the forecasting errors of these two FCIs later to examine whether adding the DMA method significantly improves the FCI's forecasting ability.

Figure 2 provides the evidence on which variables receive the highest weights in the DMA procedure. Following Koop and Korobilis (2014), the numbers in each panel of Figure 2 denote the probability that the DMA method assigns to models which contain the variable named in the title on the panel. Zero probabilities are assigned to periods of missing observations at the start of the sample period for the commercial paper spread (ComSprd), financial corporate bond spread (BFinSprd), low-quality corporate bond spread (BQualSprd), Libor spread (LiborSprd), sterling overnight lending rate spread (SoniaSprd), unsecured personal loan lending rate spread (UnsecSprd), non-financial corporations lending rate spread (NFCLSprd), all-share P/E ratio (AllPER), net outstanding debt in the housing market (HouseLoan), total mortgages outstanding (TotalM) and the FTSE 100 volatility index (VolIX). For example, the sample period of the sterling overnight lending rate spread starts from 1997:I. Thus, it is assigned zero probabilities during 1993:I-1996:IV.

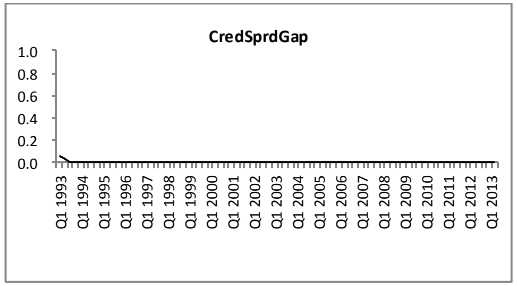
Figure 2 shows that the DMA weights for the UK can change rapidly over time. The patterns displayed also differ from the results of Koop and Korobilis (2014) for the US. Koop and Korobilis (2014) argue that many financial variables (such as the exchange rate index and housing market loans) became important during/after the 2008-9 recent financial crisis. In contrast, for the UK, indicators such as commercial paper spread (ComSprd), quality corporate bond spread (BQualSprd), non-financial corporations' lending rate spread (NFCLSprd) and FTSE 100 volatility index (VolIX) are given much lower weights during the 2008-9 crisis. Many indicators received zero weights after the Bank rate hit the effective zero lower bound (ZLB) in 2009:II.[5] The only indicator that receives an increasing weight in the recession is the sterling overnight lending rate spread (SoniaSprd). This indicates that (i) the UK reacted to the 2008-9 financial crisis differently and (ii) the sterling interbank lending cost and the liquidity risk of financial institutions became an important concern during the last financial crisis in the UK. Even in other periods, the financial market in the UK is dominated by a few indicators in the information set including ComSprd, BFinSprd, BQualSprd, SoniaSprd, TotalM and VolIX. Variables like the commercial price index (CommIX) which are important in the Koop and Korobilis (2014) US index have little relevance for the UK.

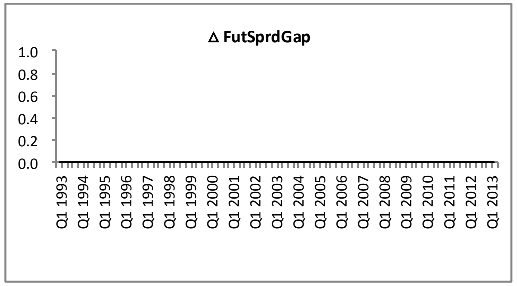
Regarding the weight on each category (as in Table 2) for the remaining 17 variables, it is particular interesting to find that the variables for the private sector (except for

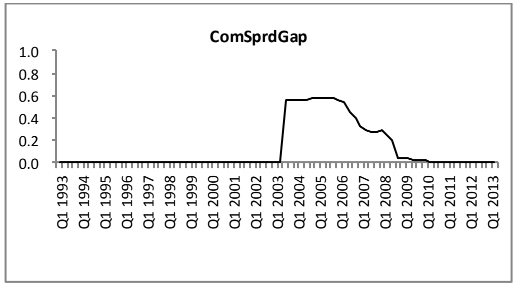
[5] The website of the BOE reports that "quantitative easing was first used by the MPC in March 2009. The official interest rate had been reduced to 0.5% and the MPC judged that it could not practically be reduced below that level". It is interesting to note that in August 2016 the BOE cut its bank rate to 0.2841%. However, given the Bank's previous reports regarding the ZLB (as mentioned), it is still reasonable for this study to use the 0.5% as the effective ZLB for the period before August 2016.

CredSprd and ΔFutSprd) usually receive considerable weights. This indicates that the private sector (especially the corporate bond market) plays an important role in the financial system of the UK and most of the private sector indicators (such as the UnsecSprd, the BFinSprd, ComSprd) contribute to the prediction of output. The lending sector which is considered important in the US receives the lowest weight. The DMA estimation results suggest that it is optimal to exclude both net debt outstanding in the house market (HouseLoan) and write-offs by banks (Writeoffs) when estimating an FCI. Even though the total mortgage outstanding (TotalM) enters into the DMA procedure, its weight is low. Since this study assumes that the real share price index (RSPI) is always included in the information set, it is not surprising to find that another two similar indicators, equity market capitalisation (MktCap) and all-share P/E ratio (AllPER) are given zero weights. This is because the information contained in MktCap and AllPER has already been captured in the RSPI.

Since the APL comparison shows the improvement of FCI estimates by the DMA method, it is also necessary to compare the forecasting ability of the DMA FCI and the indices based on the six key variables used in Chapter 1 for robustness purposes. The empirical results could be considered as further evidence to show whether the use of more financial information variables and using the DMA method is essential or not.


CredSprdGap


Δ FutSprdGap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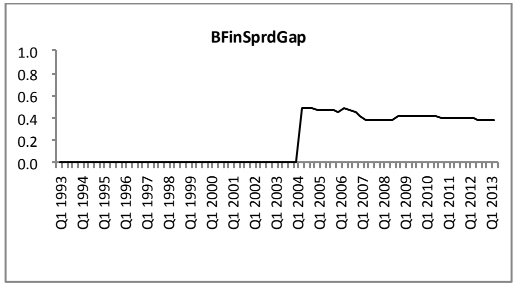
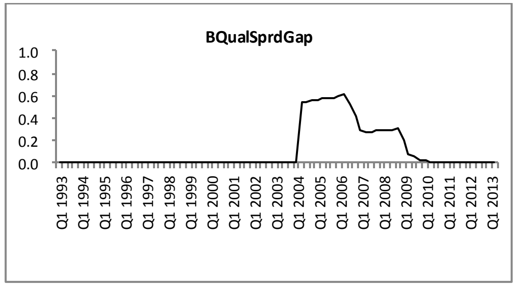
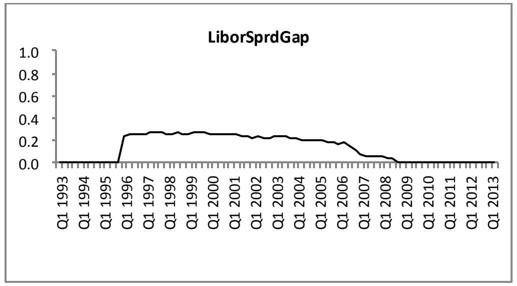
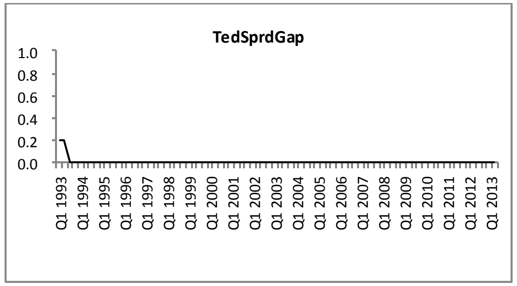
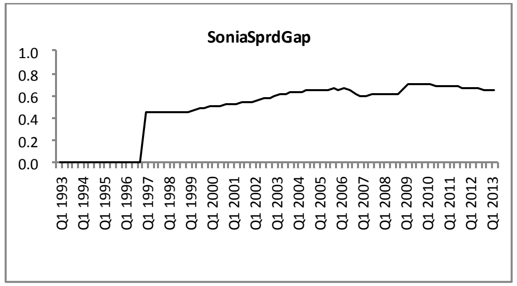
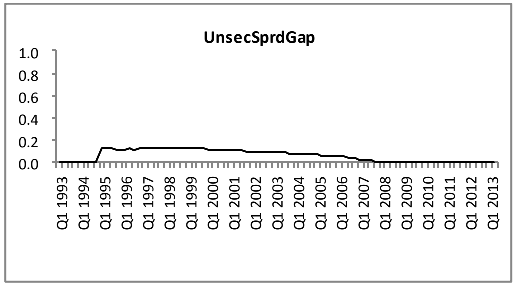

BFinSprdGap


BQualSprdGap


LiborSprdGap


TedSprdGap


SoniaSprdGap


UnsecSprdGap

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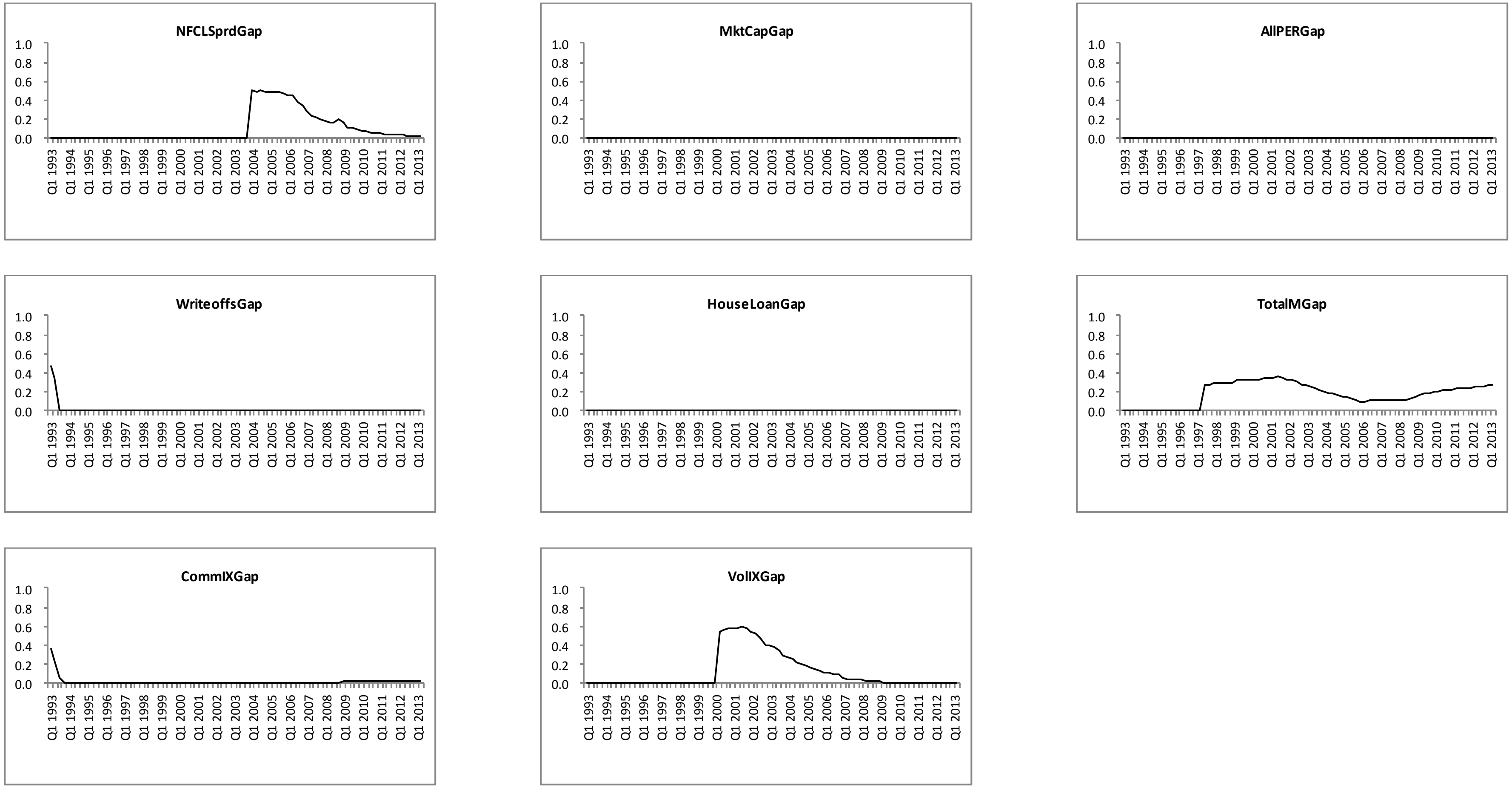


Figure 2: Time-varying Probabilities of Inclusion to the Final FCI for Each of the 17 Financial Variables

As noted earlier, a good FCI is one that forecasts economic activity well. This study follows Chapter 1 and investigates the forecasting performance of FCIs for the de-meaned real GDP growth rate. Given that the estimation period runs from 1993:I to 2013:II the evaluation period covers the period from 1994:I to 2013:II for $h = 1, \ldots, 4$ quarters ahead. This selection is designed to include the longest in-sample period for forecasting. Table 5 presents the MSFEs for each FCI divided by the MSFE of the benchmark which is a TVP-VAR with stochastic volatility using three macroeconomic variables ($\bar{y}_t$, rir_t and π_t) that occur in the traditional monetary transmission mechanism (the same benchmark used in Chapter 1). Table 5 gives the actual MSFEs for the benchmark.

Table 5: Forecasting Performance of FCIs (MSFE)

	h=1	h=2	h=3	h=4
Actual MSFEs of a TVP-VAR (no FCIs)	0.4165	0.7106	0.8379	0.7232
FCI 1 (created by TVP-FAVAR with SV in Ch.1)	*0.7307	*0.6648	*0.7242	*0.8462
FCI 2 (created by FA-TVP-VAR with SV in Ch.1)	*0.7309	*0.6650	*0.7242	*0.8458
FCI 3 (created by TVP-FAVAR with CV in Ch.1)	*0.7317	*0.6670	*0.7287	*0.8583
FCI created by a DMA-TVP-FAVAR	*0.7218	*0.6418	*0.6848	*0.7744

Note: in Chapter 1 FCI 1, FCI 2 and FCI 3 are all estimated with the first 6 variables listed in Table 1. Following Koop and Korobilis (2014), the exercise presented in this table uses a 4-lag VAR to estimate the predictive ability of an FCI. As in Chapter 1, this study takes the Diebold-Mariano (1995) test to examine whether a method's MSFEs differ significantly from the benchmark's MSFEs. If an MSFE has a *, it means that approach forecasts significantly different from the benchmark TVP-VAR with stochastic volatility.

Several results stand out: firstly, Table 5 shows that the forecast for short horizons (i.e., $h = 1$) improves slightly as a result of augmenting the index constituents and including the model averaging method. Secondly, when looking at a relatively longer forecasting horizon ($h = 3, 4$), it is encouraging to find that the MSFEs in the bottom line tend to be much smaller than those in Chapter 1. This finding supports the use of a DMA-TVP-FAVAR model relative to a factor model using several key variables.

2.6 Conclusions

This chapter aims to construct an optimal FCI for the financial market of the UK. The purpose of this is to improve the estimates in Chapter 1 by considering a larger information set and using the DMA technique to address two further questions, which index constituents should be included in an FCI and whether the constituents of the index should be adjusted at each point in time. This chapter has two main contributions: (i) the DMA-TVP-FAVAR model is applied to the UK data for the first time, and (ii) it includes a much wider range of financial variables than most existing FCIs in order to ensure that the DMA model has ample candidates to evaluate.

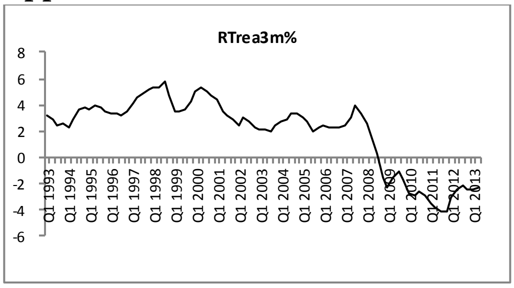
In the econometric estimation, this study compares the APLs of an FCI produced by the DMA-TVP-FAVAR model against an FCI without considering model averaging (i.e., using all available data). It obtains a result consistent with Boivin and Ng's (2006) findings that using all available information to extract an FCI is not always optimal. The 'DMA FCI' exhibits a relatively higher APL in this study. To examine the estimates in this chapter against those in Chapter 1, this study estimates the MSFEs of all FCIs. It shows that the forecasting ability of an FCI has been improved significantly by the DMA technique especially for a relatively long forecasting horizon. Therefore, this study is confident in arguing that under the condition that the best FCI predicts macroeconomic activity as well as possible, the FCI estimated in this chapter should be the most accurate index summarising all financial information in the UK market.

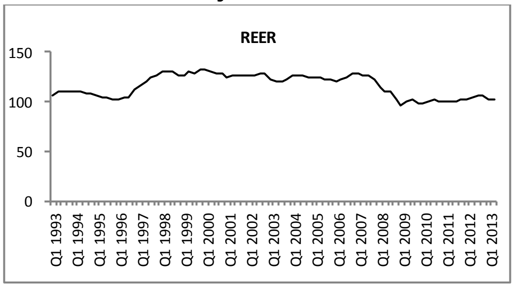
Appendices:

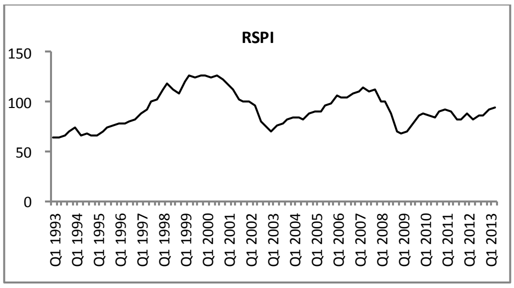
Appendix 1: Description of Other Relevant Variables and Respective Sources

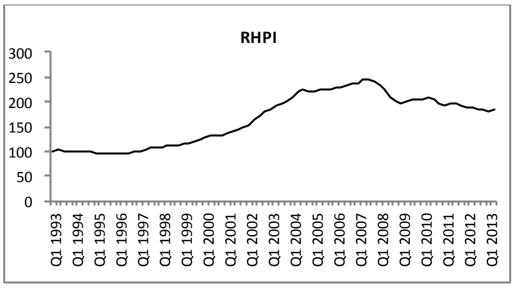
No.	Name	Description	Source	Sample
1	Real GDP	Domestic gross production (in millions of chained 2010 price)	Office for National Statistics	1993:I-2013:II
2	CPI	Consumer price index, seasonally adjusted, quarterly average (2005=100)	Office for National Statistics	1993:I-2013:II
3	Trea3m	Three-month treasury bill discount rate, quarterly average	BOE statistics	1993:I-2013:II
4	Libor3m	Three-month London inter bank offered rate, quarterly average	BOE statistics	1993:I-2013:II
5	Sonia	Sterling overnight index average lending rate, quarterly average	BOE statistics	1997:I-2013:II

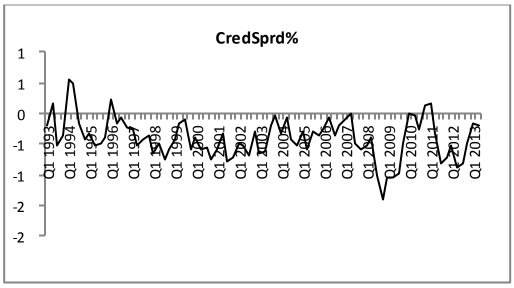
Appendix 2: Evolution of All Variables Used in This Study:

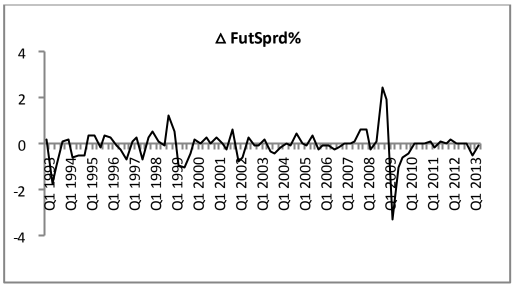


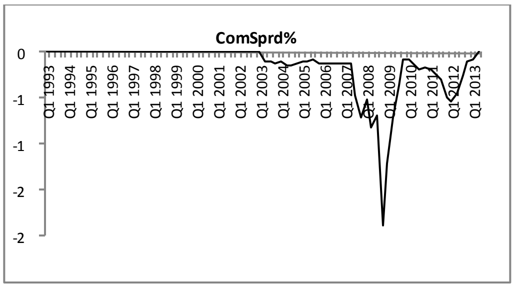


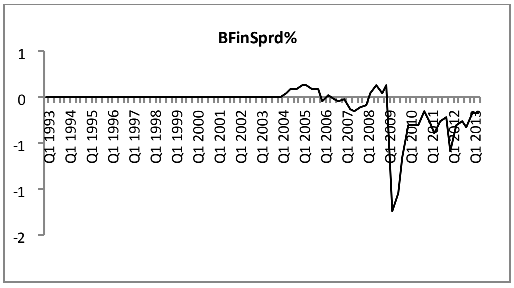


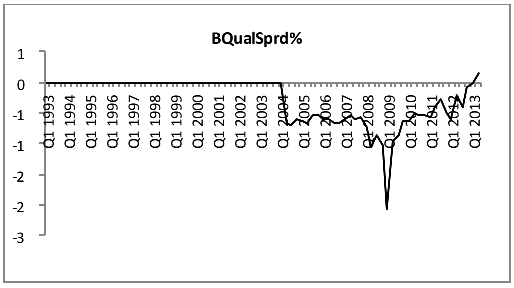


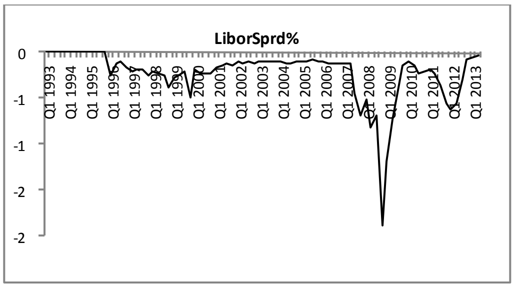












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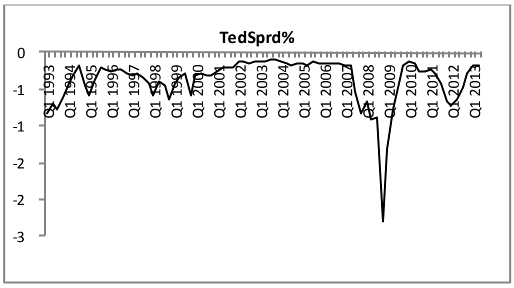

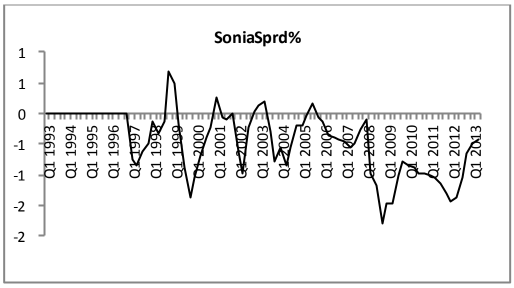

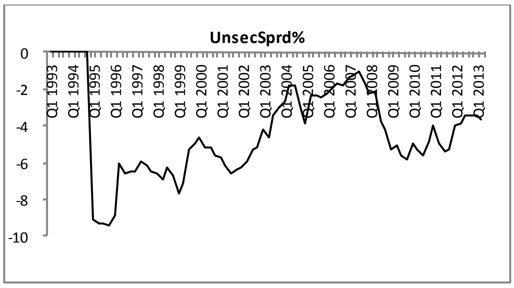

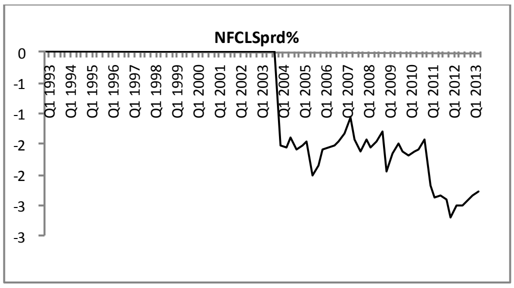

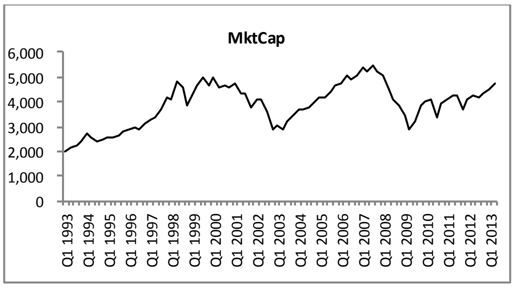

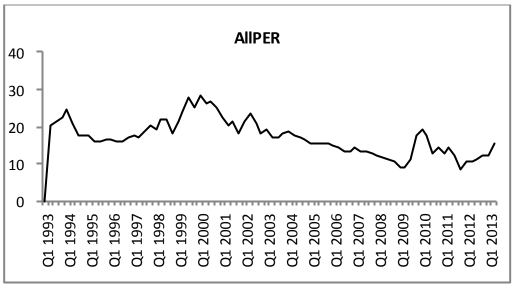

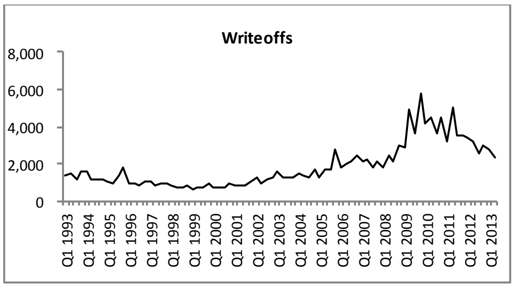

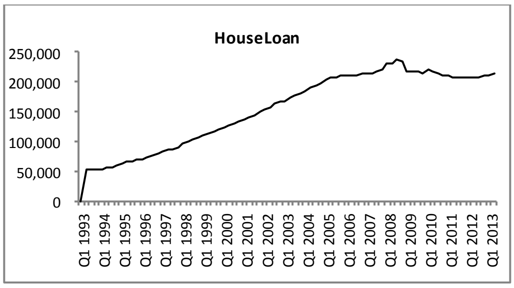

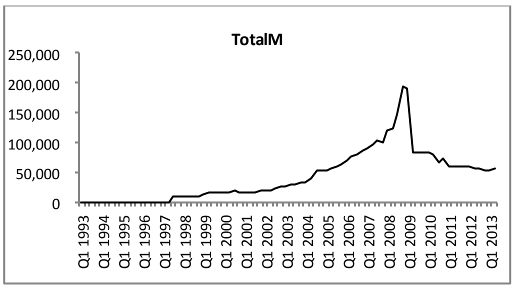

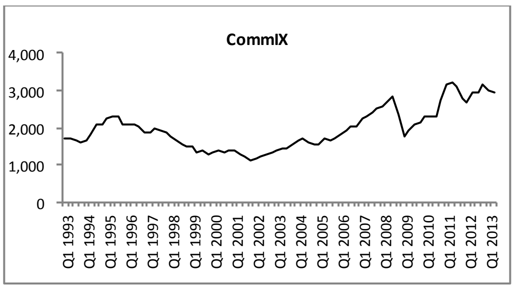

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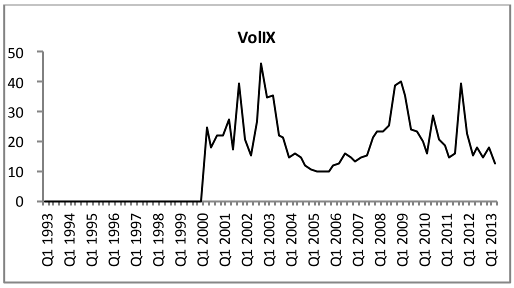
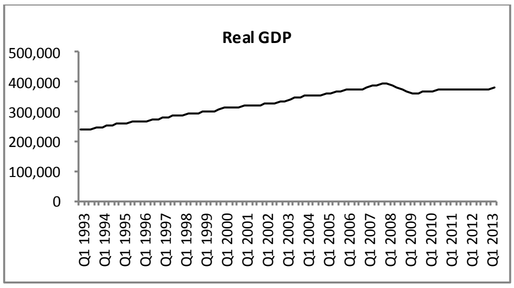




CHAPTER 3

DOES THE BANK OF ENGLAND HAVE AN IMPLICIT TIME-VARYING INFLATION TARGET?

3.1 Introduction

The Bank of England Act 1998 states that the objectives of the Bank of England (BOE) in relation to monetary policy include: (i) maintaining price stability and (ii) subject to that, supporting the economic policy of the government of the United Kingdom (UK). The BOE subsequently qualified the meaning of price stability in October 1992 when they defined it as achieving annual price inflation, as measured by the retail price index (RPI), in the range of 1-4%. In June 1995, the inflation objective was modified to achieve an inflation rate in the RPI of 2.5%. In November 2003, the target was re-defined as 2% in the Consumer Price Index (CPI). The use of the inflation target by the BOE reflects the primacy of price stability in its monetary policy framework.

The purpose of this chapter is to explore whether an inflation targeting country like the UK strictly followed its announced inflation target in the past and whether the BOE has an implicit (unannounced) short-term time-varying inflation target. The Monetary Policy Committee (MPC) acknowledges that the announced inflation objective reflects its long-run goals. The MPC brings inflation to the official target gradually, because the attempt to keep the inflation rate at the announced target may cause undesirable volatility in economic activity (see, MPC, March 2013). This reflects the short-run trade-offs that need to be made between the inflation rate and output variability when making monetary policy decisions. Consequently the hypothesis is that the BOE has an implicit target for inflation for its short-run purpose, which is different from the reference objective. It is worth emphasising that this hypothesis does not presume that the BOE is chasing short-term goals instead of maintaining long-run price stability. The aim of testing this hypothesis is to ascertain whether the BOE sets short-term objectives while adjusting inflation gradually to its long-run target.

This chapter uses monthly data, as the MPC met to set the policy rate every month. The sample period begins in October 1992 when the BOE started to target inflation and ends in June 2013. This chapter attempts to estimate the BOE's monetary policy objective from inflation data.

This is the first attempt to investigate whether the BOE has an implicit short-term inflation target. The existing studies in this topic (e.g., Ireland, 2007; Leigh, 2008) focus on an economy that does not have an explicit inflation objective such as the United States (US). Although some studies for the UK (see, Martin and Milas, 2004; Castro, 2011) discover that the BOE is attempting to keep the inflation rate within a particular range instead of pursuing a target of 2%, their analysis focuses on response parameters on inflation and economic activity. This means that they fail to provide any explanation for their findings such as which factors have dominated changes in the inflation target. To explain the shifts in the BOE's implicit short-term inflation target, this study employs a New Keynesian structural model (henceforth, structural model) from Ireland (2007) which is further extended in this study. For simplicity, the extension to the Ireland (2007) specification which allows an implicit short-term inflation target to react to both supply and demand-side shocks is referred to as the augmented structural model in this chapter.

The remainder of this chapter is organised as follows: Section 3.2 reviews the literature on inflation targeting. Section 3.3 discusses data issues while Section 3.4 introduces the methodology to be used in this study. The empirical evidence is given in Section 3.5. Section 3.6 concludes.

3.2 Literature Review

This review outlines the relevant theories behind the augmented structural model in order to provide a comprehensive background to the model. It is organised as follows: Section 3.2.1 briefly introduces the generalised Taylor rule. Section 3.2.2 discusses the New Keynesian model of Woodford (2003) and Gali (2015). Section 3.2.3 presents the existing literature on the estimation of policy goals. This study then proceeds in Section 3.2.4 with a short discussion on the literature on estimating price rigidity as it needs to specify the degree of price rigidity for the augmented structure model. Section 3.2.5 reviews the monetary policy framework of the BOE emphasising the motivation of this study.

3.2.1 The Taylor Rule Development

Over the last couple of decades, economic researchers have made a great number of proposals for monetary policy rules. In 1993, Taylor found that a very simple reaction function of only inflation and the output gap is best able to describe the behaviour of the Federal Reserve (Fed) over the period 1987-1992:

$$i = \pi + 0.5\bar{y} + 0.5(\pi - 2) + 2 \qquad (2.1.1)$$

where the output gap ($\bar{y}$, also called output bias) is measured by the percent deviation of real output (y) from a target (known as the potential output, y^*). The policy rule in Eq. (2.1.1) has the feature that the nominal federal interest rate (i) rises if the inflation rate (π) rises above the target of 2% and/or real output is above its potential level (y^*). If both inflation and real output remain at their target levels, the real federal rate would equal to 2%.

By linking interest rate adjustments directly to the inflation rate and the output gap, the Taylor rule provides a convenient and effective tool for studying monetary policy. Following his lead, the Taylor rule has been augmented by many others (see, for instance, Clarida et al., 1998, 2000; Orphanides, 2003) by allowing for both forward-looking behaviour and interest rate smoothing.

Clarida et al. (1998, 2000) propose a forward-looking version of the Taylor rule where central banks target both expectations of inflation and output biases instead of contemporaneous or past value of these two variables:

$$i - i^* = \gamma_\pi (E(\pi) - \pi^*) + \gamma_y E(y - y^*) \qquad (2.1.2)$$

where E(.) denotes expectations. This function allows central banks to consider a broad array of information to form their views on future inflation and the output gap. Subsequent studies like Fourcans and Vranceanu (2004) and Sauer and Sturm (2007) also highlight the importance of using a forward-looking Taylor rule in monetary policy analysis. Castro (2011) compares the performance of a simple Taylor rule (i.e., an interest rate reaction function of only past inflation and economic activity) with a forward-looking specification. His results indicate that the simple Taylor rule using past data cannot capture the reaction of the European Central Bank (ECB) to inflation. In the UK, the BOE also behaves in a forward-looking manner (see, Castro, 2011).

However, Clarida et al. (1998, 2000) argue that Eq. (2.1.2) is still too restrictive to explain changes in the interest rate because it assumes an immediate adjustment of the interest rate to its desired level. They thus propose to control for the observed autocorrelation in the interest rate. This is done by introducing interest rate smoothing into the Taylor rule, i.e., central banks do not adjust their interest rates immediately but gradually acclimatise to the target level, which is considered a partial adjustment mechanism:

$$i_t = \left(1 - \sum_{j=1}^{n} \rho_j\right)\left[i^* + \gamma_\pi (E(\pi) - \pi^*) + \gamma_y E(y - y^*)\right] + \sum_{j=1}^{n} \rho_j \, i_{t-j} \qquad (2.1.3)$$

where the sum of ρ_j captures the degree of interest rate smoothing and j represents the number of lags. Several theoretical justifications are advanced in the literature for the inclusion of interest rate smoothing in the Taylor rule such as the fear of disruption in financial markets (Goodfriend, 1991) and uncertainty about the effects of interest rate changes (Sack, 1998).

Orphanides (2003) considers a simple form of a monetary policy rule nesting different variants of Taylor rules as special cases. Smoothing and forward-looking behaviours are allowed in this equation:

$$i_t = \gamma_0 + \gamma_i i_{t-1} + \gamma_\pi \pi_{t+3}^a + \gamma_y (y_{t-1} - y_{t-1}^*) + \gamma_{\Delta y} (\Delta y_{t+3} - \Delta y_{t-1}^*) \qquad (2.1.4)$$

where π_{t+3}^a is the 'year-ahead' inflation forecast starting at $t-1$ (assuming, $t-1$ is the quarter of last available actual data). The term $\Delta y_{t+3} - \Delta y_{t-1}^*$ denotes the year-ahead forecast of output growth relative to its potential. The difference between y_{t-1} and y_{t-1}^* is the output gap in period $t-1$. The variables dated t and later reflect real-time forecasts formed during period t. With γ_i greater than zero, it takes inertial behaviour in setting the interest rate into account. To nest the other various alternatives this specification is somewhat more general than the one estimated by Clarida et al. (i.e., Eq. 2.1.3) in that it includes a growth rate term (Δy_t). Orphanides

(2003) indicates that the interest rate responses to both the output level and output growth are significantly positive in the US.

Orphanides (2007) later re-wrote Eq. (2.1.4) by introducing expectations of the output level y_t:

$$i_t = (1-\gamma_i)(\bar{r}+\pi^*) + \gamma_i i_{t-1} + \gamma_\pi (E(\pi)-\pi^*) + \gamma_y E(y-y^*) + \gamma_{\Delta y} E(\Delta y - \Delta y^*) \quad (2.1.5)$$

where, Eq. (2.1.5) is called the generalised Taylor rule. According to Orphanides (2007), if the natural real rate of interest ($\bar{r}$) is unknown and real-time estimates are subject to significant mis-measurement then this variant of the Taylor rule is shown to be considerably more robust than the initial one. It is worth noting that Eq. (2.1.2) is a special case of Eq. (2.1.5). The generalised Taylor rule allows for smoothing in setting the interest rate ($\gamma_i > 0$). In addition, economic activity takes two forms: (i) the level of the expected output gap ($E(y-y^*)$) and/or (ii) its difference ($E(\Delta y - \Delta y^*)$). Letting $\gamma_i = 1$ and $\gamma_y = 0$ yields the simplification of the generalised Taylor rule which is referenced by a number of subsequent researchers such as Ireland (2004b, 2007[1]):

$$\Delta i_t = \gamma_\pi (\pi - \pi^*) + \gamma_{\Delta y}(\Delta y - \Delta y^*) \quad (2.1.6)$$

Among all the above studies including Clarida et al. (1998, 2000), Ireland (2004b, 2007) and Orphanides (2003, 2007), the Taylor rule and its extended versions have proven valuable for monetary policy analysis. Given that Eq. (2.1.5) is a more generalised rule nesting other versions as special cases, this study therefore utilises it in the econometric estimation in this chapter.

3.2.2 The New Keynesian Model

This section draws on a vast literature such as Rotemberg (1982), Driscoll (2000), Steinsson (2003), Woodford (2003), Ireland (1997, 2004a, 2004b, 2007) and Gali (2015) to present a New Keynesian Model which features firms competition[2] and

[1] The expectation terms vanish in Ireland (2004b, 2007), which will be discuss later in the methodology section.
[2] There are different levels of competition across an economy. As in much of the New Keynesian literature, this study assumes that final goods market is very competitive whereas the intermediate goods market is monopolistic competition.

sticky prices in a market. Since Section 3.2.1 has already introduced the development of the Taylor rule, this section will concentrate on other subdivisions in the model – the household sector and the firm sector. The latter consists of two types of firms, intermediate-goods-producing firms and final-goods-producing firms. This section begins with the baseline New Keynesian Model as discussed in Gali (2015) and then shows, using the results in Driscoll (2000), Steinsson (2003) and Ireland (2007), how it can be augmented to incorporate consumption smoothing, stochastic elasticity, costs of price adjustments, etc. Some equations involved in the simulation estimation will be discussed in more detail in the methodology section. Section 3.2.2.1 explains how to model the households' dynamic optimisation problem. Section 3.2.2.2 concentrates on intermediate-goods-producing firms and final-goods-producing firms.

3.2.2.1 Household

As in Gali (2015), an economy is assumed to be inhabited by a number of identical households where a representative household maximises his utility function as presented in Eq. (2.2.1):

$$\max_{\{C_t\},\{N_t\},\{B_t\},\{M_t\}} : E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t;\ a_t) \tag{2.2.1}$$

$$s.t. \quad P_t C_t + M_t + B_t / R_t \leq M_{t-1} + B_{t-1} + W_t N_t + D_t + TR_t \tag{2.2.2}$$

where C_t is the quantity consumed of a single good available in the economy. The term N_t is interpreted as hours of work or the number of household members who are employed assuming a large household. W_t denotes the nominal wage rate and a_t represents the preference shock. $E\{.\}$ is an expectation operator. The parameter $\beta \in (0,1)$ denotes a discount factor lying between zero and one. It is assumed that the marginal utility of consumption is positive and non-increasing, while the marginal disutility of labour is positive and non-decreasing:

$$U_{c,t} \equiv \frac{\partial U_t}{\partial C_t} > 0, \quad U_{cc,t} \equiv \frac{\partial^2 U_t}{\partial C_t^2} \leq 0 \tag{2.2.3}$$

$$U_{n,t} \equiv \frac{\partial U_t}{\partial N_t} \leq 0, \quad U_{cc,t} \equiv \frac{\partial^2 U_t}{\partial N_t^2} \leq 0 \tag{2.2.4}$$

Eq. (2.2.1) is subject to a sequence of budget constraints given by Eq. (2.2.2) for $t = 0, 1, 2, \dots T$. In other words, a representative household chooses a sequence of consumption C_t and labour supplied N_t in order to maximise utility (as in Eq. 2.2.1) subject to the budget constraint Eq. (2.2.2). Here P_t denotes the price of the consumed good, W_t is the nominal wage, M_t is the household's nominal end-of-period balance in financial assets (excluding bonds) and B_t is the nominal value of the household's end-of-period portfolio of bonds at the price of $1/R_t$. The term D_t represents the dividends accruing to the households who are firms' owners. Although Gali (2015) does not take government transfers (TR_t) into account, both Driscoll (2000) and Ireland (2004b, 2007) include transfers as in Eq. (2.2.2) above. In addition, Driscoll (2000) and Ireland (2004b, 2007) also include real money balance (denoted as M_t/P_t) in the utility function. Therefore, this study can re-write Eq. (2.2.1) as:

$$\max_{\{C_t\},\{N_t\},\{B_t\},\{M_t\}} : E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t, M_t/P_t;\ a_t) \tag{2.2.5}$$

As in Driscoll (2000), in order to derive a conventional IS curve that does not include additive terms like real balances and the unemployment rate, the above additive separability is essential. To justify a standard IS curve without additive terms, Ireland (2004b) develops a theoretical model with microfoundations that allows, but does not require, the real money balance to affect economic activity and inflation. Using the post-1980s US data he shows that the US economy prefers the standard model in which the real balance is absent from the IS relationship. Therefore, this study considers the Driscoll (2000) and Ireland (2004b, 2007) augmentation and includes the term M_t/P_t in the utility function as well.

Another extension to Eq. (2.2.1) is introduced by Fuhrer (2000) who investigates the inclusion of habit formation in the consumer's utility function:

$$\max_{\{C_t\},\{N_t\},\{B_t\},\{M_t\}} : E_0 \sum_{t=0}^{\infty} \beta^t U[(C_t - \gamma C_{t-1}), N_t, M_t/P_t;\ a_t] \tag{2.2.6}$$

where the habit formation parameter γ lies between zero and one, i.e., $1 > \gamma \geq 0$. Fuhrer (2000) shows that this modification significantly improves the short-run dynamic behaviour of a monetary-policy model both qualitatively and statistically. As in Fuhrer (2000), given the link from real spending to inflation in most monetary-policy models, a jump response in real spending (i.e., $\gamma = 0$) is likely to result in a jump response in inflation. Conversely a gradual real spending response to monetary shocks implies a gradual reaction of inflation to policy shocks, which is what is observed in practice. Therefore, a specification with a habit formation parameter tends to accurately gauge the gradual change of inflation. In the methodology section, this study models the objective of a representative household with Eq. (2.2.6) which is subject to the budget constraints as stated by Eq. (2.2.2).

3.2.2.2 Firm

Also following Gali (2015), a large number of identical firms are assumed to operate in an economy. According to Ireland (2007), Leith, Moldovan and Rossi (2009) and Coibion, Gorodnichenko and Wieland (2012), firms operating in the economy can be split into two types, final and intermediate-goods-producing firms.

Prior to Steinsson (2003), the standard final goods production technology is simply a constant elasticity (CES) bundle of intermediate goods. In other words, there are no other factors (i.e., no labour) required to produce finished goods. In literature like Leith et al. (2009), it is standard to assume that final goods are produced by a number of monopolistically competitive firms with $Y_t(i)$ units of intermediate goods $i \in [0,1]$ purchased at the nominal price of $P_t(i)$ to manufacture Y_t units of finished goods. A production function for the finished goods can be written as:

$$Y_t = \left[\int_0^1 Y_t(i)^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)} \tag{2.2.7}$$

It is generally required that the elasticity (i.e., θ) of substitution among different types of intermediate goods is greater than one. As long as $\theta < \infty$, intermediate goods are imperfect substitutes in production, which gives intermediate-goods-producing firms some market power. It is quite straightforward to verify that this production function has constant-returns-to scale (i.e., a given percentage increase in capital and labour input results in an equal percentage increase in output). In other words, doubling all intermediate goods causes output to double. In Steinsson (2003), the elasticity parameter (θ) is assumed to be time-varying rather than a constant. The economic rationale for this assumption is that the variety of goods produced and their substitutability is changing all the time. As a result the elasticity and firms' desired markup over marginal costs (which is measured by $(\theta - 1)/\theta$) changes as well. Therefore, Eq. (2.2.7) is re-written as:

$$Y_t = \left[\int_0^1 Y_t(i)^{(\theta_t - 1)/\theta_t} di\right]^{\theta_t/(\theta_t - 1)} \tag{2.2.8}$$

where θ_t measures the time-varying elasticity of substitution for each intermediate good. The term $\theta_t/(\theta_t - 1)$ can be interpreted as the firm's desired markup over marginal costs. Therefore random fluctuations in θ_t act as shocks to the firm's desired markup. During each period t, the final-goods-producing firm aims to maximise its profits[3]. Combining the idea proposed in Steinsson (2003), Sims (2010) and other literature on New Keynesian models (e.g., Ireland, 2007; Leith et al., 2009; Coibion et al, 2012), the objective of final-goods-producing firms is written in nominal terms as:

$$\underset{\{Y_t(i)\}}{max}: P_t \left[\int_0^1 Y_t(i)^{(\theta_t - 1)/\theta_t} di\right]^{\theta_t/(\theta_t - 1)} - \int_0^1 P_t(i) Y_t(i) di \tag{2.2.9}$$

The necessary condition for a relative maximum is that the first order derivative with respect to each $Y_t(i)$ be equal to zero:

$$P_t \frac{\theta_t}{\theta_t - 1} \left[\int_0^1 Y_t(i)^{\frac{\theta_t - 1}{\theta_t}} di\right]^{\frac{\theta_t}{\theta_t - 1} - 1} \frac{\theta_t - 1}{\theta_t} Y_t(i)^{\frac{\theta_t - 1}{\theta_t} - 1} = P_t(i) \tag{2.2.10}$$

This study simplifies Eq. (2.2.10) with the following steps and solves the demand curve for goods of each intermediate sector i as in Eq. (2.2.14):

[3] Sims (2010) acknowledges that it is more usual that a firm's objective is to maximise its present value (i.e., discounted value) of future profits. However, as in Sims (2010, p. 3), firms buy intermediate goods in each period and a New Keynesian model considers that firms' objective of maximising value is equivalent to maximising their profits in each period.

$$P_t\left[\int_0^1 Y_t(i)^{\frac{\theta_t-1}{\theta_t}}di\right]^{\frac{1}{\theta_t-1}} Y_t(i)^{-\frac{1}{\theta_t}} = P_t(i) \tag{2.2.11}$$

$$Y_t(i)^{-\frac{1}{\theta_t}} = \frac{P_t(i)}{P_t}\left[\int_0^1 Y_t(i)^{\frac{\theta_t-1}{\theta_t}}di\right]^{-\frac{1}{\theta_t-1}} \tag{2.2.12}$$

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t}\left[\int_0^1 Y_t(i)^{\frac{\theta_t-1}{\theta_t}}di\right]^{\frac{\theta_t}{\theta_t-1}} \tag{2.2.13}$$

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} Y_t \tag{2.2.14}$$

According to Eq. (2.2.14), the demand for each intermediate good is negatively related to its relative price and positively related to total production. Given the assumption that final-goods firms operate in a highly competitive environment, their profits approach zero. This yields:

$$Y_tP_t = \int_0^1 P_t(i)Y_t(i)di \tag{2.2.15}$$

Substituting Eq. (2.2.14) into Eq. (2.2.15), this study can solve for the price of final goods:

$$Y_tP_t = \int_0^1 P_t(i)\left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} Y_t di \tag{2.2.16}$$

Simplifying Eq. (2.2.16) with the following steps, this study can derive an expression for the aggregate price level as in Eq. (2.2.19):

$$Y_tP_t = Y_tP_t^{\theta_t}\int_0^1 P_t(i)^{1-\theta_t}\,di \tag{2.2.17}$$

$$P_t^{1-\theta_t} = \int_0^1 P_t(i)^{1-\theta_t}\,di \tag{2.2.18}$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_t}\, di\right]^{\frac{1}{1-\theta_t}} \tag{2.2.19}$$

As already mentioned, the other type of firms are intermediate-goods-producing firms. Gali (2015) assumes a continuum of firms indexed by $i \in [0, 1]$. Each intermediate-goods-producing firm produces a differentiated good but all use an identical technology base represented by the production function:

$$Y_t = Z_t N_t(i) \tag{2.2.20}$$

where Z_t denotes the technology level that is common across firms and evolves over time. The term $N_t(i)$ is the amount of labour supplied to each intermediate-goods-producing firm indexed by $i \in [0, 1]$:

$$N_t = \int_0^1 N_t(i) di \tag{2.2.21}$$

In theoretical literature such as Ireland (2004a, 2004b, 2007) and Coibion et al. (2012), intermediate-goods producing firms act as price setters because their production substitutes imperfectly for one another in the final-goods-producing firm's technology. However, the intermediate-goods-producing firms must choose prices and satisfy representative final-goods-producers' demands at their chosen price $P_t(i)$ during each period $t = 1, 2, \dots T$.

Prices are assumed to be sticky. Rotemberg (1982) argues that firms, fearing upset clients, attribute a cost to changing prices. Rotemberg (1982) presents a theory to justify the proposed price stickiness. Using US data, his tests reject the hypothesis that the price level is not sticky in the US. Hall, Walsh and Yates (2000), using a survey of 654 UK companies, conclude that prices are indeed sticky in the UK. Drawing on a vast literature, Rotemberg (1982) identifies two types of cost that result in price stickiness: firstly, the physical costs of changing current prices and secondly, the reputational costs of price changes – Stiglitz (1979) states that under imperfect information customers tend to go to firms with relatively stable prices. Therefore, Rotemberg (1982) proposes a new rule suggesting that each price-setting firm faces a quadratic cost (Exp_t) of adjusting its nominal prices:

$$Exp_t = \frac{\phi}{2}\left[\frac{P_t(i)}{\Pi^* P_{t-1}(i)} - 1\right]^2 Y_t \tag{2.2.22}$$

during $t = 1, 2, \ldots T$ where $\Pi_t = P_t/P_{t-1}$ and the term Π^* measures the gross steady-state inflation rate. The term $\phi/2$ $(\phi \geq 0)$ governs the magnitude of the adjustment cost. As in Ireland (1997), Eq. (2.2.22) generalises Rotemberg's (1982) proposal so that costs apply to changes in both price levels and the inflation rate. Rotemberg (1982) emphasises that this specification makes an intermediate-goods-producing firm's problem dynamic. In Eq. (2.2.22), the negative effects of price changes on the customer-firm relationships increase with the magnitude of the price change and with the output supplied.

More recently, Ireland (2007) further augments Eq. (2.2.22) by considering the results in Fuhrer and Moore (1995). In their paper, one-time shocks to the inflation rate have persistent impacts on inflation that last well beyond the lifetime of the initial shock. Consequently Ireland (2007) extends Eq. (2.2.22) as:

$$Exp_t = \frac{\phi}{2}\left[\frac{P_t(i)}{\Pi_{t-1}^{\alpha}(\Pi^*)^{1-\alpha} P_{t-1}(i)} - 1\right]^2 Y_t \tag{2.2.23}$$

where Π^* is considered as the inflation target and Π_{t-1} is the inflation rate between t and $t-1$. The parameter α provides information on inflation persistence. When α is equal to zero firms' price setting behaviour is purely forward-looking, which means that it is costless for firms to raise their prices in line with the central bank's inflation target so that inflation is not persistent. On the other hand, when α is equal to one, the price setting behaviour becomes purely backward-looking in the sense that it is costless for firms to increase their prices in line with the previous period's actual inflation, hence inflation is persistent.

Let $D_t(i)$ denote the nominal profits in each intermediate-goods-producing firm. The following process illustrates the derivation of the real profits $D_t(i)/P_t$:

$$\frac{D_t(i)}{P_t} = \frac{P_t(i)Y_t(i)}{P_t} - \frac{W_t N_t(i) Y_t(i)}{P_t} - Exp_t \tag{2.2.24}$$

Recall that W_t is the nominal wage rate. For each unit of intermediate-good $Y_t(i)$, the amount $W_t N_t(i)$ is paid for the labour supplied. Substituting Eq. (2.2.14), Eq. (2.2.20) and Eq. (2.2.23) into Eq. (2.2.24) to yield:

$$\frac{D_t(i)}{P_t} = Y_t(i)\frac{P_t(i)}{P_t} - N_t(i)Y_t(i)\frac{W_t}{P_t} - Exp_t \tag{2.2.25}$$

$$= \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t}\left[\frac{P_t(i)}{P_t}\right]Y_t - \left(\frac{Y_t}{Z_t}\right)\left[\frac{P_t(i)}{P_t}\right]^{-\theta_t}\left(\frac{W_t}{P_t}\right)$$

$$-\frac{\phi}{2}\left[\frac{P_t(i)}{\Pi_{t-1}^{\alpha}(\Pi^*)^{1-\alpha}P_{t-1}(i)} - 1\right]^2 Y_t$$

$$= \left[\frac{P_t(i)}{P_t}\right]^{1-\theta_t} Y_t - \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t}\left(\frac{Y_t}{Z_t}\right)\left(\frac{W_t}{P_t}\right)$$

$$- -\frac{\phi}{2}\left[\frac{P_t(i)}{\Pi_{t-1}^{\alpha}(\Pi^*)^{1-\alpha}P_{t-1}(i)} - 1\right]^2 Y_t$$

The objective of an intermediate-goods-producing firm is to maximise its real market value, Eq. (2.2.26), where the term $\beta^t \Lambda$ denotes the marginal utility to the household of an additional unit of real profits delivered in the form of dividends during period t:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[\frac{D_t(i)}{P_t}\right] \tag{2.2.26}$$

3.2.3 The Estimates of Inflation Goals

Recall the classification of the Taylor-rule studies in Beechey and Osterholm (2012): the first strand of research into monetary policy preference assumes a constant inflation objective and parameter stability within a given sample. The second strand (e.g., Ireland, 2007) seeks to find the policy goal and allows time variation in the inflation objectives. This section focuses on the second strand of studies and provides a comprehensive review of studies regarding estimating inflation targets. The preferences for inflation and output stabilisation are still assumed to be time-invariant in the literature discussed below.

Leigh (2008) relaxes the assumption that the Fed’s inflation target is constant for the duration of the analysis (1979Q3-2004Q1) and allows the Fed’s inflation target to follow a random walk. Treating the natural rate of interest and the inflation target as two variables rather than as constant parameters distinguishes the Leigh (2008) work from much empirical work on policy rules. In applying a time-varying parameter model with the Kalman filter algorithm to the Volcker-Greenspan period, Leigh (2008) finds significant variations in the Fed's implicit target suggesting that the assumption of a constant inflation target seems unnecessarily restrictive.

Erceg and Levin (2003), Cogley and Sbordone (2005), Gavin, Keen and Pakko (2005), Roberts (2006), Salemi (2006) and Smets and Wouters (2007) have developed various macroeconomic models allowing for continual movements in the implicit target for the US. However as concluded in Ireland (2007), each of those studies concentrates on a different set of issues (see, Table 1). None of them focuses on estimating a continually changing inflation target and none of them attempts to specifically model the changes in the inflation target as a deliberate policy response to other shocks which hit the economy. For the purpose of filling this gap in the literature, Ireland (2007) constructs a structural model drawing on contemporary macroeconomic theories to provide identifying restrictions needed to shed light on the patterns, causes and consequences of the changes in the US Fed's inflation target. The macroeconomic theory is from a New Keynesian model, the characteristics of which have already been illustrated in the previous sections. Although Ireland (2007) draws on Clarida et al. (1999) in the development of his model, he extends their approach by using a reduced and simplified generalised Taylor rule (as in Eq. 2.1.6). Ireland (2007) uses this to describe the Fed's monetary policy rule. As discussed in Section 3.2.2, a standard New Keynesian model consists of a household, an intermediate goods-producing firm, a final-goods producing firm and a central bank. It provides an accurate description of not only the central bank but also of the optimising behaviour of households and firms in an economy. Ireland (2007) also highlights that his New Keynesian structural model developed and estimated is consistent with the Lucas (1976) argument by separating out the parameters in the central bank reaction function (i.e., those parameters that change given a change in monetary policy) from additional parameters that describe private tastes and technologies (i.e., those parameters that ought to remain invariant to shifts in the policy rules).

As mentioned in Section 3.2.2, Ireland (2007) incorporates several extensions to the basic New Keynesian model by including the degree of firms' backward-looking behaviour, price stickiness, etc. Since Ireland (2007) also models the Fed's reaction to cost-push shocks and technology shocks, the corresponding response of the inflation target to these two shocks and the coefficients on inflation and the output gap are also taken into account. The estimated structural model in Ireland (2007) describes historical movement in inflation, changes in the output gap and the evolution of the interest rate. According to Ireland (2007), the structural model also allows for the consideration of counterfactual scenarios including what the behaviour of inflation, the output gap and the interest rate would have looked like if the central bank had maintained a constant inflation target over the sample.

Ireland (2007) estimates the simplified generalised Taylor rule (as in Eq. 2.1.6) using a state-space model in which the inflation target is allowed to vary over time. The Fed is allowed to systematically adjust its inflation target in response to either or both of two supply-side shocks, cost-push shocks and technology shocks.[4] The empirical evidence favours the generalised Taylor rule with smoothing behaviour. In addition, the implicit inflation target of the Fed is found to be time-varying between 1959 and 2004. As in Ireland (2007), the target rises from 1.25% in 1959 to over 8% in the mid to late 1970s before falling back below 2.5% in 2004, which rejects the hypothesis of a fixed inflation target as in Clarida et al. (1998, 2000). Ireland (2007) also shows that the Fed's inflation target would fall by 41 basis points (bps) and 6.4 bps following a favourable one-standard-deviation cost-push shock and technology shock respectively. The model's linearity implies that symmetrically the inflation target would rise by the same amount following a similarly sized adverse disturbance.

This study finds at least three shortcomings with Ireland's (2007) study. Firstly, he sets the interest rate in response to current inflation and output gaps while failing to consider central bankers' forward-looking behaviour in making policy decisions. Secondly, he employs a simplified generalised Taylor rule (as in Eq. 2.1.6) to model the Fed's policy interest rate. However, Eq. (2.1.5) implies that the simplified form (Eq. 2.1.6) is only used when central banks do not react to the level of the real output gap (i.e., $\gamma_y = 0$) and the coefficient on interest rate smoothing is equal to one (i.e., $\gamma_i = 1$). Thirdly, he does not specify the possible response of the inflation target to

[4] Ireland (2007) mentions that he once uses a more generalised model, considering the response of inflation target to the demand-side shocks in his preliminary analysis. However, he does not give a full description of the original model (used in his preliminary analysis) in his publication in 2007.

demand-side shocks, i.e., preference shocks. In addition, although the Ireland (2007) specification has been proven to be much more robust for inflation target analysis (than models failing to consider the response of the inflation target to various shocks), it has not been applied to other countries as yet, which may be partially attributed to its complexity. This also motivates the analysis of monetary policy in the UK using the methodology in this study.[5]

Table 1: Macroeconomic Models Allowing for a Time-varying Inflation Target

Author	Issues focused on
Erceg and Levin (2003)	Private agents' inability to disentangle transitory from persistent movements in the inflation target of the Fed.
Cogley and Sbordone (2005)	The stability of the estimated parameters of a Phillips curve in the face of changes elsewhere in the US economy.
Gavin et al. (2005)	The ability of their model to account for the persistence of inflation in the US.
Roberts (2006)	The ability of his model to capture the changing relationship between unemployment and inflation in the US.
Salemi (2006)	The weights placed by the Fed on the stabilisation objectives of output, inflation and interest rate.
Smets and Wouters (2007)	The ability of their (New Keynesian) model to track the post-war US data using an expanded number of variables.
Ireland (2007)	Obtaining estimates of the time-varying (implicit) inflation target and a generalised Taylor rule.

3.2.4 The Estimates of the Price Rigidity

Implementing the structural model that is proposed by Ireland (2007) and augmented in this study requires a preliminary study of the degree of price rigidity. The most significant work in the area of price rigidity estimation is conducted by Taylor (1980) and Calvo (1983). Taylor (1980) assumes an environment of monopolistically competitive firms which face price adjustment constraints. The price adjustment is assumed to be time dependent, e.g., every period the fraction $1/X$ of firms set their prices for X periods. However, Gali and Gertler (1999) argue that in this scenario it is necessary to keep track of price histories of firms which makes the aggregation cumbersome. Calvo (1983) develops a staggered price model which is along the lines

[5] Ireland (2007) mentions that he once uses a more generalised model considering both of the interest reaction to the level of output gap and the response of the inflation target to demand-side shocks in his preliminary analysis. However, he does not give a full description of the original model (used in his preliminary analysis) in his publication in 2007. In addition, even though Ireland (2007) uses a generalised structural model in the preliminary analysis, he still fails to consider the forward-looking behaviour of a central bank which is a major deficit in his study. For the sake of caution, this study uses an augmented (for forward-looking elements, interest rate reaction to the output level and inflation target's response to preference shocks) structural model in the econometric estimation. Details on the augmented structural model are given latter in the methodology section.

of Phelps (1978) and Taylor (1980) but which is analytically tractable. It assumes that in any given period each firm has a constant probability $(1-\xi)$ that it may adjust its prices during that period. This probability is independent of the time elapsed since the last price revision. Hence, the average time over which the price is fixed is given by $(1-\xi)\sum_{k=0}^{\infty} k\xi^{k-1} = 1/(1-\xi)$. Since the adjustment probabilities are unrelated to the firms' price history, the aggregation problem is simplified in Calvo's proposal (see, Gali and Gertler, 1999).

In the Calvo (1983) basic model, all firms are forward-looking, i.e., they change prices optimally using all available information available to forecast future marginal costs. Gali and Gertler (1999) extend this initial formula to allow a subset of firms to use a backward-looking rule to decide prices. They assume that backward-looking firms obey a rule of thumb which has two characteristics: (i) no persistent deviation between the rule and optimal behaviour, i.e., on average this rule is consistent with the optimal behaviour and (ii) the price in period t depends only on information dated $t-1$ or earlier. Forward-looking firms are still assumed to behave exactly as in the Calvo (1983) baseline model. Hence, the proposal of Gali and Gertler (1999) nests the Calvo (1983) model as a special case.

Finally, the specification developed in Gali and Gertler (1999) is a hybrid version of a Phillips curve that lets the inflation rate depend on a combination of expected future inflation, lagged inflation rates and the current state of the real economy. An important note is required for the hybrid Phillips curve: traditional empirical work on the Phillips curve places emphasis on using output gap measures, instead of marginal cost measures, as indicators of real economic activity. Gali and Gertler (1999) maintain that the use of the output gap could raise considerable measurement errors in potential output which will then cause some biases in the estimation results. In light of the difficulties with using the output gap, they instead use measures of real marginal cost in the econometric estimation. Real marginal cost is given by the ratio of the wage rate to the marginal product of labour. Appendix 1 presents the derivation of the Gali and Gertler (1999) hybrid Phillips curve.

By applying Generalised Method of Moments (GMM) to the extended Calvo model, Gali and Gertler (1999) discover that 1/4 of price setters are backward-looking in the US which leads to a rejection of the purely forward-looking model. Moreover, the prices in the US are found to be fixed for round five quarters. Given the evidence from Gali and Gertler (1999), this study uses the hybrid staggered price model for the UK. The results obtained will be used later for simulating the augmented structural model.

3.2.5 The Discussion of the Monetary Policy in the UK

The Bank of England Act came into effect on 1st June 1998. The Act makes the BOE independent in setting the interest rate. It states that in relation to monetary policy, the objective of the BOE shall be: (i) to maintain price stability and (ii) subject to that, to support the economic policy of the government. In March 2013, the MPC reiterated that its current inflation target should be defined as 2% as measured by the 12-month increase in the CPI. However, a further look at the history of the BOE's monetary policy indicates that the BOE's policy towards inflation targeting has been modified over time.

Although the inflation reducing policy was announced in 1976 in the UK, a specific inflation target was only introduced after the sterling crisis in 1992. Following the earlier lead of New Zealand and Canada, the BOE introduced an inflation target in October 1992. The objective was to achieve price stability in the long run which is defined as a RPI inflation range of 1%-4% a year (see, King, 1997; Benati, 2003). In June 1995, the inflation objective was modified to achieve an inflation rate of RPI of 2.5%. In November 2003, the target was re-defined as 2% for the CPI inflation (see, Benati, 2003).

The most noteworthy point is that the inflation target announced reflects the government's commitment to medium-term price stability. The MPC (March 2013) acknowledges that the inflation rate may occasionally depart from its target as a result of shocks and disturbances. According to the MPC (March 2013), attempts to keep the inflation rate at the announced target in these circumstances may result in undesirable volatility in output. This reflects the short-term trade-offs which must be made between inflation and output variability in making monetary policy decisions (see, MPC, March 2013, p.1). It is reasonable to hypothesise that the BOE may have an implicit inflation target that differs from the reference figure for short-term goals. Therefore, for the UK case even with an explicit inflation target the econometric estimation that allows for a time-varying inflation target is also necessary. Based on the above arguments, the rationale for investigating a time-varying implicit inflation target includes (but may not be limited to) (i) the announced inflation target of the BOE has evolved over the sample period of 1993-2013; (ii) the official inflation rate has been calculated based on the CPI data since 1996, however the inflation target was still based on RPI between 1996 and October 2003; (iii) there is no explicit statement in the BOE regarding the short-term inflation target, which leads to the possibility that the implicit short-term target may not be the same as the official inflation target.

Apart from that, Ireland (2007) considers persistence in the inflation rate in the US as the primary motivation for studying the Fed's implicit inflation objective. Referring to Friedman's (1968, p.39) words that inflation is always and everywhere a monetary phenomenon, Ireland (2007) stresses that large and persistent movements in the inflation rate cannot happen without ongoing shifts in the Fed's inflation target. In the UK case, persistence in inflation is also detected in the literature. As in Meenagh, Minford, Nowell, Sofat and Srinivasan (2009), persistence defines the extent to which the effect of shocks persists both in terms of size and length of time. Meenagh et al. (2009) discover a certain degree of persistence in the inflation rate throughout 1992-2003 in the UK. Thus, as stressed in Ireland (2007) an economic model for generating information about the BOE's implicit inflation target becomes quite important. Given the fact that the BOE has its medium to long-run inflation target, the estimation of the augmented structural model should focus on (i) examining the existence of its short-term inflation objective and (ii) distinguishing between the movements in the inflation rate attributable to changes in the inflation target and those driven by other shocks.

3.3 Data

This study sources data from the BOE and the Office for National Statistics (ONS). The data used is monthly as this is the frequency at which the MPC meets to make monetary policy decisions. Another crucial reason for using monthly data is that it better allows for identifying the timing and speed of changes in the implicit short-term inflation target. The sample period is from October 1992 to June 2013.

Figure 1 presents graphs illustrating the movement of major variables over the sample period. This study considers several different measures of the output gap and the interest rate. However, in the estimation it only chooses the ones that have been followed most closely by the BOE.

Figure 1.1: Interest rate, (monthly) %:

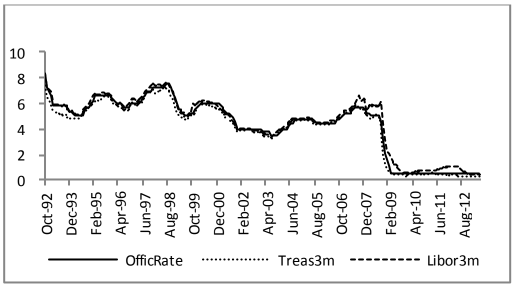


Figure 1.2: The inflation rate, (monthly) %:

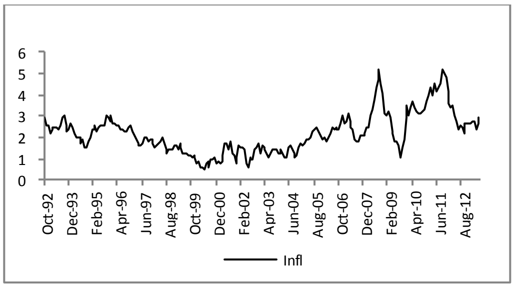


Figure 1.3: Industrial production Index (monthly):

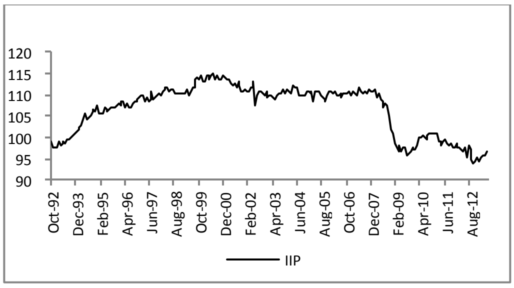


Figure 1.4 Labour income share (quarterly) %:

Figure 1: The Main Variables for Studying the BOE's Policy Goals, 1992-2013

The alternative interest rate measures considered include the official central bank interest rate (OfficRate), the three-month inter-bank sterling lending rate (Libor3m) and the discount rate of three-month Treasury bills (Treas3m). This data is obtained from statistics of the BOE. Nelson (2000) argues that actual interest rate instruments used by the BOE have changed over time. These rates include the bank rate, the minimum lending rate, the two-week repo rate, etc. To deal with this, Nelson (2000), Martin and Milas (2004) and Castro (2011) argue that the Treas3m has a close relationship with all the interest rate instruments used in the BOE's history. Consequently this study uses the Treas3m as the nominal interest rate (i_t) for the sample period analysed.

Following the BOE, the inflation rate (Infl) in Figure 1.2 is calculated as the annual rate of change in the CPI. However, the official CPI statistics in the UK only started in 1996. The historical estimate of inflation back to 1988 is calculated by the ONS based on the RPI. Following the existing literature in this field such as Martin and Milas (2014), this study calculates the inflation rate with the RPI for the period of 1993-1996. This data is available monthly.

The decision to choose monthly data has consequences for the output measure that is used. Data on GDP which is one of the most common output measures is only available on a quarterly basis. Hence, the literature studying monetary policy (e.g., Clarida et al., 1998; Gerlach and Lewis, 2010; Castro, 2011) chooses the index of industrial production (IIP, CPI deflated) to measure real economic activity on a monthly basis. This study follows Clarida et al. (1998), Gerlach and Lewis (2010) and Castro (2011) and uses the IIP to measure the level of output for the UK. Figure 1.3 plots the evolution of the IIP over the sample period investigated.

This study transforms the price index, real IIP and Trea3m as follows:

$$\pi_t = ln\ (Price_t) - ln\ (Price_{t-4}) = ln\left(\frac{Price_t}{Price_{t-4}}\right) = ln\left[\frac{infl_t}{100} + 1\right] \tag{3.1}$$

$$gy_t = ln\ (Output_t) - ln\ (Output_{t-1}) \tag{3.2}$$

$$i_t = \frac{Treas3m_t}{100} \tag{3.3}$$

where $infl_t$ and $Treas3m_t$ are both expressed in percentage terms and t subscripts denote time.

Table 2: Unit Root and Stationary Tests

	ADF	PP	KPSS
π_t	-2.572897*	-2.617447*	0.740526
gy_t	-19.55972*	-19.12465*	0.537088#
i_t	-0.799916	-1.236052	1.418298

*Note: *: Unit root is rejected at a significance level of 10%; #: The stationarity is not rejected at a significance level of 1%; All the test regressions here contain a constant term.*

Table 2 reports the results of unit root and stationarity tests for the variables used in this study. Due to the low power and poor performance of unit root tests in small samples, this study follows the methodology used in Castro (2011). It reports the results of two unit root tests, i.e., augmented Dickey and Fuller (1979) test (ADF) and Phillips and Perron (1988) test (PP) to investigate whether test power is an issue. It also reports the Kwiatkowski, Phillips, Schmidt and Shin (1992) stationarity test (KPSS) results for robustness purposes.

The test results displayed in Table 2 indicate that the power of unit root tests seems to be an issue for the UK. The KPSS test is unable to provide the evidence of stationarity for π_t and i_t. However, the ADF and the PP tests are able to reject the unit root in π_t. Although the evidence fails to support the stationarity hypothesis for i_t for this sample period, this study would expect that if a longer time period is considered it would find evidence of stationarity for i_t.

Table 3 presents the estimates of the Gali and Gertler (1999) Hybrid Phillips Curve (Eq. 5 in the appendix):

$$\pi_t = S_0 mc_t + S_1 E_t\{\pi_{t+1}\} + S_2 \pi_{t-1}$$

where mc_t denotes the percentage deviation of the real marginal cost (MC_t) from its steady-state level. The reader is referred to Appendix 1 for a detailed illustration. This study employs the GMM estimator to estimate the above equation, since the regression is performed on variables some of which (like $E_t\{\pi_{t+1}\}$) are unavailable to market participants at the decision-making moments. Also following Gali and Gertler (1999), it calculates the real marginal cost MC_t as the labour income share (S_t):

$$S_t \equiv W_t N_t / P_t Y_t \tag{3.4}$$

where W_t represents nominal wage per worker, N_t denotes labour, P_t is the price level and Y_t is output. Letting lower case letters represent percentage deviation from the steady-state levels (i.e., the mean) yields:

$$mc_t = s_t \tag{3.5}$$

Figure 1.4 plots the movement of S_t over the sample period 1993:I-2013:II. The data for S_t is obtained from DataStream but is only available quarterly. Hence the GMM estimates here are on a quarterly basis. This study uses the natural logarithm of S_t for estimating s_t . Both contemporaneous and lagged inflation rate measures are percentage changes in the CPI. The MPC's projection of inflation one-quarter ahead (which is only available on a quarterly basis) is taken for $E_t\{\pi_{t+1}\}$. Finally the instruments used in this study include a constant, 1-5 lagged inflation rates, 1-4 lags of labour income share and 1-4 lags of the output gap which are similar to those used in Gali and Gertler (1999). A 12-lag Newey-West estimate of the covariance matrix is used in the GMM estimation.

Table 3: Estimates of the New Hybrid Phillips Curve

	ξ	d	f
Estimate	0.66919	0.27829	0.73210
Std. error	0.03926	0.07535	0.07293
t-Statistic	17.0468	3.69331	10.0389
Prob.	0.00000	0.00040	0.00000

Test of over identifying restrictions for this specification J=10.77.

The parameter ξ is estimated to be 0.67 with a standard error 0.04, which implies that prices are fixed for roughly three to four quarters on average in the UK. That period length is similar to the findings in Ireland (2007) for the US case.

Turning now to the estimated fraction of backward-looking price setters. The parameter d is estimated to be 0.28 with a standard error of 0.08 indicating that roughly a quarter of price setters are backward-looking. Thus Calvo's basic model (1983) is rejected by the data in the case of the UK. However, the quantitative importance of backward-looking behaviour for inflation dynamics is not very large. The implied estimates for the reduced form parameters on lagged versus expected future inflation are 0.31 (for S_2) versus 0.55 (for S_1). The subjective discount factor (f) is estimated to be 0.73.

3.4 Methodology

Using a generalised Taylor rule, this section proceeds to model the implicit inflation target in an inflation-targeting central bank like the BOE. It employs the maximum likelihood method to generate the estimates focusing on three issues: (i) whether the short-term inflation target was allowed to change, (ii) if so, which factors have dominated changes in the short-term inflation target throughout the sample period and (iii) how would the economy have behaved, if the central bank had maintained a fixed inflation target in the sample period.

This section re-presents the structural model initiated by Ireland (2007) and also places emphasis on the extension to the Ireland (2007) specification. Ireland (2007) extends the baseline New Keynesian model of Clarida et al. (1999) to include a generalised Taylor rule (Eq. 2.1.6). With this rule, Ireland (2007) allows a central bank to adjust its inflation target in response to two different shocks, technology shocks and cost-push shocks that hit the domestic economy. The Ireland (2007) model is extended in this chapter to incorporate more forward-looking behaviour and aggregate demand shocks. As already mentioned, this study refers to this as the augmented structural model. In Ireland (2007), a simplified generalised Taylor rule (Eq. 2.1.6) is used where the interest rate reacts to contemporaneous inflation and the output gap and the inflation target responds to supply-side shocks alone. Although Ireland (2007) states that he does a preliminary exercise in which the inflation target responds to both supply and demand-side shocks, he still fails to consider central banks' forward-looking behaviour – this is one of the primary limitations in his structural model. Therefore, instead of employing a simplified interest rate reaction function this study considers the specification used in Ireland's preliminary estimation and then expands the Ireland (2007) structural model to consider more forward-looking characteristics. More specifically, the augmented structural model developed below adjusts the short-term interest rate to stabilise inflation expectations, the level of economic output and growth of output. The implicit inflation objectives not only react to technology shocks and cost-push shocks but also to preference shocks.

As mentioned in the literature section, the (augmented) structural model has primarily New Keynesian features. It shares its basic features with the model in Clarida et al. (1999) and the work of Woodford (2003) and Gali (2015). However, it resembles most closely the model in Ireland (2004a, 2007). The augmented structural model consists of a household, two types of firms (final goods-producing firms and continuum of intermediate goods-producing firms indexed by $i \in [0, 1]$) and a central bank. In each period ($t = 0, 1, 2$ ...), an intermediate goods-producing firm produces a distinct intermediate good that cannot be substituted. Therefore, the intermediate goods can also be indexed by $i \in [0, 1]$ where firm i produces good i. Referring to the previous discussion on the interest rate reaction function and the New Keynesian model in Section 3.2.1 and 3.2.2, this study describes the behaviour of these four agents and the implications for the evolution of equilibrium prices and quantities.

3.4.1 The Household

Section 3.2.2.1 sets out the household decision problem. Considering the discussion in Gali (2015) and the developments in Driscoll (2000) and Fuhrer (2000), this study writes this as:

$$\max_{\{C_t\},\{N_t\},\{B_t\},\{M_t\}} : E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[ln(C_t - \gamma C_{t-1}) + ln\left(\frac{M_t}{P_t}\right) - N_t \right] \tag{4.1}$$

$$0 < \beta < 1 \; and \; 0 \leq \gamma < 1$$

$$s.t. \quad P_t C_t + M_t + B_t/R_t \leq M_{t-1} + B_{t-1} + W_t N_t + D_t + TR_t \tag{4.2}$$

where Eq. (4.2) suggests that the household enters each period of $t = 0, 1, 2 \; ... T$ with money M_{t-1} and bonds B_{t-1}. At the beginning of the period, the household receives TR_t as a lump-sum of nominal government transfer. Next, his/her bond matures providing the household with B_{t-1} additional units of money. The household then uses some of this to purchase B_t/R_t new bonds where R_t represents the nominal interest rate between t and $t + 1$. W_t denotes the nominal wage rate and N_t denotes the hours of work. As in Section 3.2.2.1, Eq. (4.1) is based on Driscoll (2000) and Ireland (2004b, 2007) who advocate separating the utility function into three terms, consumption (C_t), labour supply (N_t) and real money balance (M_t/P_t). It also takes into account Fuhrer's (2000) findings about the habit of consumption that is controlled by the parameter γ.

According to Clarida et al. (1999), Ireland (2004a, 2004b, 2007) and Gali (2015), preference shocks a_t (also considered as demand-side shocks) in Eq. (4.1) follows the stationary autoregressive process:

$$ln(a_t) = \rho_a \; ln(a_{t-1}) + \sigma_a \varepsilon_{at} \tag{4.3}$$

$$0 \leq \rho_a < 1 \; and \; \sigma_a \geq 0$$

where serially uncorrelated innovation ε_{at} has a standard normal distribution.

In summary, the representative household chooses C_t, N_t, B_t and M_t to maximise the expected utility function Eq. (4.1) subject to the budget constraints Eq. (4.2) for period $t = 0, 1, 2, \ldots T$. The Lagrange multiplier method is taken to find out the local maxima of Eq. (4.1) subject to its constraint (as in Eq. 4.2). The first-order condition for the problem can be written as:

$$\Lambda_t = \frac{a_t}{C_t - \gamma C_{t-1}} - \beta\gamma E_t\left(\frac{a_{t+1}}{C_{t+1} - \gamma C_t}\right) \tag{4.4}$$

$$a_t = \Lambda_t\left(\frac{W_t}{P_t}\right) \tag{4.5}$$

$$\Lambda_t = \beta R_t E_t\left(\frac{\Lambda_{t+1}}{\Pi_{t+1}}\right) \tag{4.6}$$

$$\frac{M_t}{P_t} = \left(\frac{a_t}{\Lambda_t}\right)\left(\frac{R_t}{R_{t-1} - 1}\right) \tag{4.7}$$

and Eq. (4.2) for all $t = 0, 1, 2 \ldots T$, where Λ_t denotes the non-negative Lagrange multiplier on the budget constraint and Π_t represents the inflation rate. The t subscripts denote time.

Eq. (4.4) shows the Lagrange multiplier Λ_t with the marginal utility of consumption during period t adjusted to account for the effects of the backward habits which carry over into period $t + 1$. As Eq. (4.1) implies that the expected utility is linear in the labour supply, Eq. (4.5) links the marginal substitution rate between consumption and leisure to the real wage rate. Eq. (4.6) takes the form of the Euler equation and links the intertemporal marginal rate of substitution to the real interest rate. Eq. (4.7) shows the money demand relationship suggesting that the real balances go up as a result of a rise in consumption and a fall in the nominal interest rate.

3.4.2 The Final-goods-producing Firm

This study adopts the idea of time-varying elasticity in Steinsson (2003) and writes the production function for finished goods as:

$$Y_t = \left[\int_0^1 Y_t(i)^{(\theta_t - 1)/\theta_t} di\right]^{\theta_t/(\theta_t - 1)} \tag{4.8}$$

where θ_t measures the time-varying elasticity of substitution for each intermediate good. As mentioned in Section 3.2.2.2, random fluctuations in θ_t act as shocks to the firm's desired markup over marginal costs $\theta_t/(\theta_t - 1)$. Ireland (2004a) stresses that random shocks to θ_t tend to translate into shocks to intermediate goods-producing firm's desired markups of price over marginal cost. In equilibrium, the θ_t shocks behave like cost-push shocks which are introduced by Clarida et al. (1999) into the New Keynesian model. The cost-push shocks capture everything (except for the output gap) that could affect marginal costs. As in Clarida et al. (1999) and Ireland (2004a, 2004b, 2007), cost-push shocks follow a stationary autoregressive process:

$$ln(\theta_t) = (1 - \rho_\theta)\, ln(\theta) + \rho_\theta\, ln(\theta_{t-1}) + \sigma_\theta \varepsilon_{\theta t} \tag{4.9}$$

$$0 \leq \rho_\theta < 1, \theta > 1 \text{ } and \text{ } \sigma_\theta \geq 0$$

where the serially uncorrelated innovation $\varepsilon_{\theta t}$ follows a standard normal distribution.

Taking into account the ideas from Steinsson (2003), Sims (2010) and other literature on New Keynesian models (e.g., Ireland, 2007; Leith et al., 2009; Coibion et al., 2012), the objective of a final-goods-producing firm is given in Eq. (2.2.9). This study re-writes it as:

$$\underset{\{Y_t(i)\}}{max}: P_t \left[\int_0^1 Y_t(i)^{(\theta_t - 1)/\theta_t} di\right]^{\theta_t/(\theta_t - 1)} - \int_0^1 P_t(i) Y_t(i) di \tag{4.10}$$

The derivation of the first order conditions for Eq. (4.10) is presented in Eq. (2.2.10) – Eq. (2.2.14). It is re-stated as:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} Y_t \tag{4.11}$$

for all $i \in [0, 1]$ and $t = 0, 1, 2, \ldots T$. Given the derivation from Eq. (2.2.15) to Eq. (2.2.19), competition drives finished goods-producing firms' profits to zero in equilibrium determining the aggregate price level P_t as:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_t} di\right]^{\frac{1}{1-\theta_t}} \tag{4.12}$$

3.4.3 The Intermediate-goods-producing Firm

The New Keynesian model assumes a continuum of intermediate-goods-producing firms indexed by $i \in [0,1]$. Each firm produces a differentiated good but uses an identical technology. They hire $N_t(i)$ units of labour from the households to manufacture $Y_t(i)$ units of intermediate goods i:

$$N_t = \int_0^1 N_t(i)di \tag{4.13}$$

According to the constant-returns-to-scale technology, the production function could be described as:

$$Y_t = Z_t N_t(i) \tag{4.14}$$

The aggregate technology shock Z_t follows a random walk with drift, distinguishing its effects from those of cost-push shocks. As supply-side disturbances, both shocks tend to move output and inflation in opposite directions in the short run, however only technology shocks have a permanent influence on the level of output (see, Ireland, 2007):

$$ln(Z_t) = ln(z) + ln(\, Z_{t-1}) + \sigma_z \varepsilon_{zt} \tag{4.15}$$

$$z > 1 \; and \; \sigma_z \geq 0$$

where the serially uncorrelated innovation ε_{zt} follows a standard normal distribution.

As discussed in Section 3.2.2.2, Rotemberg (1982) discovers the price stickiness and Fuhrer and Moore (1995) discovers the existence of inertia in the inflation target. This study adopts the findings in both Rotemberg (1982) and Fuhrer and Moore (1995). It follows Ireland (1997, 2007) and re-states the cost of adjusting nominal prices between periods faced by a representative intermediate-goods-producing firm as:

$$Exp_t = \frac{\phi}{2}\left[\frac{P_t(i)}{\Pi_{t-1}^{\alpha}(\Pi_t^*)^{1-\alpha}P_{t-1}(i)} - 1\right]^2 Y_t \tag{4.16}$$

Again the parameter ϕ denotes the magnitude of the price adjustment costs. The term Π_t^* is the monetary authority's short-term inflation target for period t. The extent to which price setting is forward or backward-looking depends on whether α is closer to zero or one. The value of this parameter provides the information on inflation persistence. When α equals one, the price setting behaviour is purely backward-looking.

The discussion in Section 3.2.2.2 also mentions that the goal of an intermediate-goods-producing firm is to maximise its real market value (in Eq. 2.2.26) by choosing $P_t(i)$ where $\beta^t \Lambda$ denotes the marginal utility to households of an additional unit of real profits that is delivered in the form of dividends during period t:

$$\max_{\{P_t(i)\}}: E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[\frac{D_t(i)}{P_t}\right] \tag{4.17}$$

The derivation of $D_t(i)/P_t$ (i.e., the firm's real profits during the same period in light of the requirement that it sells its production at price $P_t(i)$) is presented in Eq. (2.2.25). It is re-stated here as:

$$\begin{aligned} \frac{D_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t}\right]^{1-\theta_t} Y_t - \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} \left(\frac{W_t}{P_t}\right)\left(\frac{Y_t}{Z_t}\right) \\ - \frac{\phi}{2}\left[\frac{P_t(i)}{\Pi_{t-1}^{\alpha}(\Pi_t^*)^{1-\alpha}P_{t-1}(i)} - 1\right]^2 Y_t \end{aligned} \tag{4.18}$$

The first-order conditions for this problem are:

$$\begin{aligned} 0 = (1-\theta_t)\left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} + \theta_t \left[\frac{P_t(i)}{P_t}\right]^{-1-\theta_t} \left(\frac{W_t}{P_t}\right)\left(\frac{1}{Z_t}\right) \\ - \phi\left[\frac{P_t(i)}{\Pi_{t-1}^{\alpha}(\Pi_t^*)^{1-\alpha}P_{t-1}(i)} - 1\right]\left[\frac{P_t(i)}{\Pi_{t-1}^{\alpha}(\Pi_t^*)^{1-\alpha}P_{t-1}(i)}\right] \\ + \beta\phi E_t\left\{\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\left[\frac{P_t(i)}{\Pi_{t-1}^{\alpha}(\Pi_t^*)^{1-\alpha}P_{t-1}(i)} - 1\right]\right. \\ \left.\times\left[\frac{P_t(i)}{\Pi_{t-1}^{\alpha}(\Pi_t^*)^{1-\alpha}P_{t-1}(i)}\right]\left[\frac{P_t}{P_t(i)}\right]\left(\frac{Y_{t+1}}{Y_t}\right)\right\} \end{aligned} \tag{4.19}$$

and Eq. (4.14) for all $t = 0, 1, 2 \ \dots \ T$.

In Eq. (4.19), the coefficient ϕ denotes the parameter of price adjustment costs. These optimal conditions imply that in the absence of price adjustment costs (i.e., $\phi = 0$)

firms sets their prices $P_t(i)$ as a markup $\theta_t/(\theta_t-1)$ over the marginal cost W_t/Z_t. Hence, the term $\theta_t/(\theta_t-1)$ is considered as the firm's desired markup and random fluctuations in θ_t could be considered as shocks to its markup. When ϕ is greater than zero, actual markups deviate from (but tend to move toward) the desired level because firms respond optimally to shocks hitting the economy.

3.4.4 The Symmetric Equilibrium

In a New Keynesian model, it is quite common to assume that all intermediate-goods-producing firms make identical decisions in a symmetric equilibrium. This suggests that $Y_t(i)=Y_t$, $h_t(i)=h_t$, $D_t(i)=D_t$ and $P_t(i)=P_t$ for all $i\in[0,1]$. Ireland (2007) also mentions the market-clearing condition for money and bonds that $M_t=M_{t-1}+T_t$ and $B_t=B_{t-1}=0$ must hold for all $t=0,1,2\ \dots T$ in the equilibrium.

After imposing these conditions, this study is able to simplify the household's budget constraint, i.e., Eq. (4.2). In equilibrium, Eq. (4.2) is reduced to:

$$C_t=\frac{W_tN_t}{P_t}+\frac{D_t}{P_t} \tag{4.20}$$

Substituting Eq. (4.18) into Eq. (4.20) yields:

$$C_t=\frac{W_tN_t}{P_t}+Y_t-\left(\frac{W_t}{P_t}\right)\left(\frac{Y_t}{Z_t}\right)-\frac{\phi}{2}\left[\frac{P_t(i)}{\Pi_{t-1}^{\alpha}(\Pi_t^{*})^{1-\alpha}P_{t-1}(i)}-1\right]^2Y_t \tag{4.21}$$

Simplifying Eq. (2.21) with Eq. (4.14) gives:

$$\begin{aligned}C_t&=\frac{W_tN_t}{P_t}+Y_t-\left(\frac{W_t}{P_t}\right)(N_t)-\frac{\phi}{2}\left[\frac{P_t(i)}{\Pi_{t-1}^{\alpha}(\Pi_t^{*})^{1-\alpha}P_{t-1}(i)}-1\right]^2Y_t\\&=Y_t-\frac{\phi}{2}\left[\frac{P_t(i)}{\Pi_{t-1}^{\alpha}(\Pi_t^{*})^{1-\alpha}P_{t-1}(i)}-1\right]^2Y_t\\&=Y_t-\frac{\phi}{2}\left[\frac{\Pi_t}{\Pi_{t-1}^{\alpha}(\Pi_t^{*})^{1-\alpha}}-1\right]^2Y_t\end{aligned} \tag{4.22}$$

Hence, Eq. (4.22) represents an economy's aggregate resource constraint. To simplify the intermediate-goods-producing firm's optimal price adjustment rule, i.e., Eq. (4.19), this study substitutes Eq. (4.5) into Eq. (4.19) to obtain:

$$\theta_t - 1 = \theta_t \left(\frac{a_t}{\Lambda_t Z_t}\right) - \phi \left[\frac{\Pi_t}{\Pi_{t-1}^{\alpha}(\Pi_t^*)^{1-\alpha}} - 1\right]\left[\frac{\Pi_t}{\Pi_{t-1}^{\alpha}(\Pi_t^*)^{1-\alpha}}\right]$$
$$+ \beta\phi E_t \left\{\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\left[\frac{\Pi_{t+1}}{\Pi_t^{\alpha}(\Pi_{t+1}^*)^{1-\alpha}} - 1\right]\left[\frac{\Pi_{t+1}}{\Pi_t^{\alpha}(\Pi_{t+1}^*)^{1-\alpha}}\right]\right\} \tag{4.23}$$

for all $t = 0, 1, 2\ \dots T$. Eq. (4.22) and Eq. (4.23) together with Eq. (4.3), Eq. (4.4), Eq. (4.6), Eq. (4.9) and Eq. (4.15) form a reduced system model consisting of equations Eq. (4.1) – Eq. (4.19).

Following Ireland (2007), this study defines output growth (g_t^y), inflation (g_t^π) and the nominal interest rate (g_t^r) as:

$$g_t^y = \frac{Y_t}{Y_{t-1}} \tag{4.24}$$

$$g_t^\pi = \frac{\Pi_t}{\Pi_{t-1}} \tag{4.25}$$

$$g_t^r = \frac{R_t}{R_{t-1}} \tag{4.26}$$

for all $t = 0, 1, 2\ \dots T$. The ratio of the nominal interest rate to the inflation rate ($r_t^{r\pi}$) is defined as:

$$r_t^{r\pi} = \frac{R_t}{\Pi_t} \tag{4.27}$$

for all $t = 0, 1, 2\ \dots T$.

3.4.5 The Efficient Level of Output

This section derives the efficient level of output that will be used in the central bank's reaction function presented later. As defined in Ireland (2004a, 2007), a social planner is someone who overcomes the frictions that cause real money balances to show up in the household's utility function and frictions that give rise to explicit costs of nominal price adjustment facing an intermediate-goods producing firm. The social planner for the economy chooses $n_t(i)$ units of household's time to produce $Q_t(i)$ units of each intermediate goods for all $i \in [0, 1]$. Then according to the same constant-returns-to-scale technologies which are described by Eq. (2.2.8) and Eq. (2.2.20) above, those different intermediate goods are used to produce Q_t units of finished goods. In other words, a social planner chooses $n_t(i)$ and Q_t to maximise household's welfare, now measured by:

$$\max_{\{Q_t, n_t(i)\}}: E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[ln(Q_t - \gamma Q_{t-1}) - \int_0^1 n_t(i) di \right] \tag{4.28}$$

$$s.t. \quad Z_t \left[\int_0^1 n_t(i)^{(\theta_t - 1)/\theta_t} di \right]^{\theta_t/(\theta_t - 1)} = Q_t \tag{4.29}$$

subject to the feasibility constraints stated in Eq. (4.29) for all $t = 0, 1, 2, \ldots T$. The first-order conditions for this problem can be written as:

$$\Xi_t = \frac{a_t}{Q_t - \Upsilon Q_{t-1}} - \beta\gamma E_t \left(\frac{a_{t+1}}{Q_{t+1} - \Upsilon Q_t} \right) \tag{4.30}$$

$$a_t = \Xi_t Z_t (Q_t/Z_t)^{1/\theta_t} n_t(i)^{-1/\theta_t} \tag{4.31}$$

and Eq. (4.29) for $t = 0, 1, 2, \ldots T$ where Ξ_t denotes the nonnegative multiplier on the aggregate feasibility constraints for period t. As in Ireland (2007), Eq. (4.31) suggests that it is optimal for the social planner to select $n_t(i) = n_t$ for all $i \in [0, 1]$. Then re-organising (4.31) with the following two steps, this study gets:

$$n_t^{1/\theta_t} = (\Xi_t Z_t / a_t)(Q_t/Z_t)^{1/\theta_t} \tag{4.32}$$

$$n_t = (\Xi_t / a_t)^{\theta_t} Z_t^{\theta_t} (Q_t/Z_t) \tag{4.33}$$

Given the production function for the finished goods in Eq. (4.8), this study writes the relationship between n_t and $n_t(i)$ as:

$$n_t = \left[\int_0^1 n_t(i)^{(\theta_t - 1)/\theta_t} di \right]^{\theta_t/(\theta_t - 1)} \tag{4.34}$$

Substituting Eq. (4.34) into the aggregate feasibility constraint (Eq. 4.29) gives:

$$Z_t n_t = Q_t \tag{4.35}$$

Eq. (4.35) is another form of the aggregate feasibility constraint for the social planner. Then substituting Eq. (4.33) into Eq. (4.35) yields:

$$\Xi_t = a_t / Z_t \tag{4.36}$$

Finally, this study substitutes Eq. (4.36) into the first optimality condition (Eq. 4.30) and obtains the following relationship as presented in Eq. (4.37).

$$\frac{1}{Z_t} = \frac{1}{Q_t - \Upsilon Q_{t-1}} - \beta\gamma E_t\left[\left(\frac{a_{t+1}}{a_t}\right)\left(\frac{1}{Q_{t+1} - \Upsilon Q_t}\right)\right] \tag{4.37}$$

for $t = 0, 1, 2 \ldots T$. As in Ireland (2007), this relationship indicates that similar to the equilibrium level of output (Y_t) Q_t, the efficient level of output also contains a unit root in the process Eq. (4.15) in the case of technology. The term Q_t could also be considered as the potential level of economic output. Therefore, the output gap (x_t) can be stated as:

$$x_t = \frac{Y_t}{Q_t} \tag{4.38}$$

3.4.6 The Central Bank

Recall that Section 3.2.1 describes the development of monetary policy rules. Some recent studies (e.g., Orphanides, 2003, 2007) propose to extend the initial Taylor rule as:

$$\begin{aligned} i_t = (1 - \gamma_i)(\bar{r} + \pi^*) + \gamma_i i_{t-1} + \gamma_\pi(E(\pi) - \pi^*) + \gamma_y E(y - y^*) \\ + \gamma_{\Delta y} E(\Delta y - \Delta y^*) \end{aligned} \tag{4.39}$$

Orphanides (2003) shows that this equation is found to be more robust than the initial one. It is worth emphasising that in a generalised Taylor rule the interest rate response to economic development takes two forms: response to the level of the expected output gap and its difference. Later, Chapter 4 demonstrates that the smoothing parameter approaches 1.0 in the UK. This result motivates the use of a simplified Eq. (4.39) as in Ireland (2007) to describe the BOE's reaction function, i.e., setting $\gamma_i = 1.0$:

$$ln(R_t) - ln(R_{t-1}) = \rho_\pi \, ln\left(\frac{\Pi_t}{\Pi_t^*}\right) + \rho_{gy} \, ln\left(\frac{g_t^y}{g^y}\right) + ln(v_t) \tag{4.40}$$

where v_t denotes the transitory monetary policy. The response parameters ρ_π and ρ_{gy} are chosen by the monetary authority. The smoothing parameter with the value of one suggests that the central bank raises its short-term nominal interest rate (R_t) when the inflation rate (Π_t) is above its target (Π_t^*) or/and the growth rate of output (g_t^y) rises above its steady-level (g^y). Eq. (4.40) is the equation used in Ireland (2007) to derive the implicit inflation target within the Federal Reserve. It also assumes the parameter of γ_y in Eq. (4.39) is zero. However, in the case of the UK there is no evidence to indicate that the BOE does not react to the level of output. Therefore, this study decides to use a more generalised rule to model the BOE's reaction function:

$$\begin{aligned} &ln(R_t) - ln(R_{t-1}) \\ &\qquad = \rho_\pi \, ln\left(\frac{E_t\Pi_{t+1}}{\Pi_{t+1}^*}\right) + \rho_{gy} \, ln\left(\frac{g_t^y}{g^y}\right) + \rho_x ln\left(\frac{x_t}{x}\right) + ln(v_t) \end{aligned} \tag{4.41}$$

for all $t = 0, 1, 2 \; ... T$. Another crucial change in the above equation is to let the interest rate react to the expected inflation rate ($E_t\Pi_{t+1}$) instead of the contemporaneous rate (Π_t). Castro (2011) investigates the horizons of inflation and output forecasts within the BOE. His result points to the fact that it is more appropriate to let the BOE's policy rate respond to inflation expectations and contemporaneous economic activity. Eliminating the unobservable forecast variables (as in Ireland 2007) yields:

$$ln(R_t) - ln(R_{t-1}) = \rho_\pi \, ln\left(\frac{\Pi_{t+1}}{\Pi_{t+1}^*}\right) + \rho_{gy} \, ln\left(\frac{g_t^y}{g^y}\right) + \rho_x ln\left(\frac{x_t}{x}\right) + ln(v_t) \tag{4.42}$$

In Eq. (4.42), the response parameters $\rho_\pi > 0$, $\rho_{gy} \geq 0$ and $\rho_x \geq 0$ are chosen by the central bank. If $\rho_x = 0$, Eq. (4.42) is reduced to Eq. (4.40) as used in Ireland (2007). Under this rule, the central bank tends to raise its nominal short-term interest rate (R_t), whenever the inflation rate (Π_t) is above its target (Π_t^*), whenever the growth rate of output (g_t^y) increases above its steady-level (g^y) and whenever the output gap (x_t) rises above its steady-state level (x). Two types of shocks enter into the reaction function Eq. (4.42). As in Ireland (2007), let the inflation target (Π_t^*) follow a random walk process:

$$ln(\Pi_t^*) = ln(\Pi_{t-1}^*) + \delta_a \varepsilon_{at} - \delta_\theta \varepsilon_{\theta t} - \delta_z \varepsilon_{zt} + \sigma_\pi \varepsilon_{\pi t} \tag{4.43}$$

$$\delta_a \geq 0, \delta_\theta \geq 0, \delta_z \geq 0 \; and \; \sigma_\pi \geq 0$$

for all $t = 0, 1, 2 \ldots T$ where the serially uncorrelated innovation $\varepsilon_{\pi t}$ follows a standard normal distribution. The central bank chooses the three parameters, δ_a, δ_θ and δ_z, to adjust its inflation target in response to demand-side and supply-side shocks, i.e., preference shocks (a_t), cost-push shocks (θ_t) and technology shocks (Z_t). Adverse supply-side shocks (negative realisations of $\varepsilon_{\theta t}$ and ε_{zt}) work to increase goods' prices while favourable supply-side shocks (positive realisations of $\varepsilon_{\theta t}$ and ε_{zt}) will reduce goods' prices (Ireland, 2007). The central bank adjusts its inflation target upwards in response to adverse cost-push shocks and technology shocks and *vice versa*. Adverse demand-side shocks (negative realisations of ε_{at}) work to decrease goods' price while central banks adjust the inflation target downwards in response to adverse preference shock.

It is worth noting that the random walk specification in Eq. (4.43) corresponds to the arguments in Ireland (2007) that large and persistent movements in the inflation rate cannot happen without ongoing shifts in the inflation target. Since Meenagh et al. (2009) have already discovered evidence of persistence in the UK, their result justifies the use of Eq. (4.43) for the BOE. Apart from that, it is also helpful to distinguish effects of the time-varying (short-term) inflation target from those generated by the model's preference shocks, cost-push shocks, technology shocks and monetary policy shocks (i.e., the term of v_t in Eq. 4.42). As in Ireland (2007), the transitory monetary policy shock is set to follow a stationary autoregressive process:

$$ln(v_t) = \rho_v \, ln(v_{t-1}) + \sigma_v \varepsilon_{vt} \tag{4.44}$$

$$0 \leq \rho_v < 1 \; and \; \sigma_v \geq 0$$

where the serially uncorrelated innovation ε_{vt} follows a standard normal distribution.

3.4.7 Constructing a Stationary System

At this stage, this study has built a system with 16 equations: Eq. (4.3), Eq. (4.4), Eq. (4.6), Eq. (4.9), Eq. (4.15), Eq. (4.22) – Eq. (4.27), Eq. (4.37), Eq. (4.38), Eq. (4.42) – Eq. (4.44). However, a problem is that among the 16 variables in this system some of them are stationary while the others inherit unit roots either from the process seen in Eq. (4.15) or the process seen in Eq. (4.43). Ireland (2007) proposes to transform all variables so as to make sure that the system model is stationary.[6] Therefore, this study follows Ireland (2007) and adjusts the 13 variables (Y_t, C_t, Π_t, R_t, Q_t, x_t, g_t^y, g_t^π, g_t^r, $r_t^{r\pi}$, Λ_t, Z_t and Π_t^*) in the system as follows (i.e., Eq. 4.45 – Eq. 4.57). In addition to these 13 variables, the structural model also involves three terms, i.e., a_t, θ_t and v_t, which are stationary according to Eq. (4.3), Eq. (4.9) and Eq. (4.44).

$$y_t = \frac{Y_t}{Z_t} \tag{4.45}$$

$$c_t = \frac{C_t}{Z_t} \tag{4.46}$$

$$\pi_t = \frac{\Pi_t}{\Pi_t^*} \tag{4.47}$$

$$r_t = \frac{R_t}{\Pi_t^*} \tag{4.48}$$

$$q_t = \frac{Q_t}{Z_t} \tag{4.49}$$

$$x_t = \frac{Y_t}{Q_t} \tag{4.50}$$

$$g_t^y = \frac{Y_t}{Y_{t-1}} \tag{4.51}$$

$$g_t^\pi = \frac{\Pi_t}{\Pi_{t-1}} \tag{4.52}$$

[6] Although some statistics studies have proposed a few techniques to deal with non-stationarity, much of the state space literature still prefers to use stationary variables. For example, Jorgensen and Song (2007) argue that non-stationarity may lead to very small estimates of the variance of the latent process.

$$g_t^r = \frac{R_t}{R_{t-1}} \tag{4.53}$$

$$r_t^{r\pi} = \frac{R_t}{\Pi_t} \tag{4.54}$$

$$\lambda_t = Z_t \Lambda_t \tag{4.55}$$

$$z_t = \frac{Z_t}{Z_{t-1}} \tag{4.56}$$

$$\pi_t^* = \frac{\Pi_t^*}{\Pi_{t-1}^*} \tag{4.57}$$

In terms of these stationary variables, a stationary system model can be re-written as:

$$y_t = c_t + (\frac{\phi}{2}) \left[\pi_t \left(\frac{\pi_t^*}{\pi_{t-1}} \right)^{\alpha} - 1 \right]^2 y_t \tag{4.58}$$

$$ln(a_t) = \rho_a \; ln(a_{t-1}) + \sigma_a \varepsilon_{at} \tag{4.59}$$

$$\lambda_t = \frac{a_t z_t}{z_t c_t - \gamma c_{t-1}} - \beta \gamma E_t \left(\frac{a_{t+1}}{z_{t+1} c_{t+1} - \gamma c_t} \right) \tag{4.60}$$

$$\lambda_t = \beta r_t E_t \left[\left(\frac{1}{z_{t+1}} \right) \left(\frac{1}{\pi_{t+1}^*} \right) \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right) \right] \tag{4.61}$$

$$ln(\theta_t) = (1 - \rho_\theta) \, ln(\theta) + \rho_\theta \, ln(\theta_{t-1}) + \sigma_\theta \varepsilon_{\theta t} \tag{4.62}$$

$$ln(z_t) = ln(z) + \sigma_z \varepsilon_{zt} \tag{4.63}$$

$$\begin{aligned} \theta_t - 1 = \theta_t \left(\frac{a_t}{\lambda_t} \right) - \phi \left[\pi_t \left(\frac{\pi_t^*}{\pi_{t-1}} \right)^{\alpha} - 1 \right] \left[\pi_t \left(\frac{\pi_t^*}{\pi_{t-1}} \right)^{\alpha} \right] \\ + \beta \phi E_t \left\{ \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left[\pi_{t+1} \left(\frac{\pi_{t+1}^*}{\pi_t} \right)^{\alpha} \right. \right. \\ \left. \left. - 1 \right] \left[\pi_{t+1} \left(\frac{\pi_{t+1}^*}{\pi_t} \right)^{\alpha} \right] \left(\frac{y_{t+1}}{y_t} \right) \right\} \end{aligned} \tag{4.64}$$

$$g_t^y = \left(\frac{y_t}{y_{t-1}} \right) z_t \tag{4.65}$$

$$g_t^{\pi} = \left(\frac{\pi_t}{\pi_{t-1}}\right)\pi_t^* \tag{4.66}$$

$$g_t^{r} = \left(\frac{r_t}{r_{t-1}}\right)\pi_t^* \tag{4.67}$$

$$r_t^{r\pi} = \frac{r_t}{\pi_t} \tag{4.68}$$

$$1 = \frac{z_t}{z_t q_t - \gamma q_{t-1}} - \beta\gamma E_t\left[\left(\frac{a_{t+1}}{a_t}\right)\left(\frac{1}{z_{t+1}q_{t+1} - \gamma q_t}\right)\right] \tag{4.69}$$

$$x_t = \frac{y_t}{q_t} \tag{4.70}$$

$$\begin{aligned} ln(r_t) - ln(r_{t-1}) \\ &= \rho_\pi\, ln(\pi_{t+1}) - ln(\pi_t^*) + \rho_{gy}\, ln\left(\frac{g_t^y}{g^y}\right) + \rho_x\, ln\left(\frac{x_t}{x}\right) \\ &+ ln(v_t) \end{aligned} \tag{4.71}$$

$$ln(\pi_t^*) = \delta_a\varepsilon_{at} - \delta_\theta\varepsilon_{\theta t} - \delta_z\varepsilon_{zt} + \sigma_\pi\varepsilon_{\pi t} \tag{4.72}$$

$$ln(v_t) = \rho_v\, ln(v_{t-1}) + \sigma_v\varepsilon_{vt} \tag{4.73}$$

for all $t = 0, 1, 2\ \ldots T$. As a_t, θ_t and v_t are all stationary, Eq. (4.59), Eq. (4.62) and Eq. (4.73) are the re-statement of Eq. (4.3), Eq. (4.9) and Eq. (4.44) respectively.

3.4.8 The Steady State in the Absence of Shocks

When there are no shocks hitting the economy, it tends to converge to a steady-state growth path along which all of the stationary variables described in Section 3.4.7 are constant. This yields: $y_t = y$, $c_t = c$, $\pi_t = \pi$, $r_t = r$, $q_t = q$, $x_t = x$, $g_t^y = g^y$, $g_t^\pi = g^\pi$, $g_t^r = g^r$, $r_t^{r\pi} = r^{r\pi}$, $\lambda_t = \lambda$, $a_t = a$, $\theta_t = \theta$, $z_t = z$, $v_t = v$ and $\pi_t^* = \pi^*$. The assumption of no shocks also suggests that the steady-state values of a, θ, z and v all equal 1.0. Eq. (4.43) indicates $\Pi_t^* = \Pi_{t-1}^*$ (i.e., $\pi^* = \pi_t^* = 1$) in that condition. The adjustment cost stated in Eq. (4.16) equals zero in the equilibrium state. Thus with Eq. (4.58) this study obtains: $c = y$ and $g^y = z$. Similarly, Eq. (4.66) and Eq. (4.67) suggest that $g^\pi = 1$ and $g^r = 1$ respectively. Eq. (4.71) implies that: $\Pi_t = \Pi_t^*$, i.e., $\pi = \pi_t = 1$.

Given the above results including $\theta_t = \theta$, $\lambda_t = \lambda$, $a_t = a = 1$ and $\pi_t = \pi_{t-1} = \pi_t^* = 1$ in the steady state, this study re-writes Eq. (4.64) and yields:

$$\lambda = \frac{\theta}{\theta - 1} \tag{4.74}$$

With $\lambda_t = \lambda$, $z_t = z$, $c_t = c = y$, and $a_t = a = 1$, this study re-writes Eq. (4.60) as:

$$\lambda = \left(\frac{1}{y}\right)\left(\frac{z}{z-\gamma} - \frac{\beta\gamma}{z-\gamma}\right) \tag{4.75}$$

Then substituting Eq. (4.74) into Eq. (4.75) and re-organising the result produces:

$$y = \left(\frac{\theta - 1}{\theta}\right)\left(\frac{z - \beta\gamma}{z-\gamma}\right) \tag{4.76}$$

Similarly re-writing Eq. (4.69) based on the results that $z_t = z$, $q_t = q$ and $a_t = a = 1$ to obtain:

$$q = \frac{z - \beta\gamma}{z - \gamma} \tag{4.77}$$

So that combing Eq. (4.70) together with Eq. (4.76) and Eq. (4.77) gives:

$$x = \frac{\theta - 1}{\theta} \tag{4.78}$$

Finally with $z_t = z$, $r_t = r$, $\pi_t = \pi_t^* = 1$ and $\lambda_t = \lambda_{t+1} = \lambda$ this study re-writes Eq. (4.61) and obtains:

$$r = \frac{z}{\beta} \tag{4.79}$$

So that combing Eq. (4.79) together with Eq. (4.68) gives:

$$r^{r\pi} = \frac{z}{\beta} \tag{4.80}$$

3.4.9 Constructing a Linearised System Model

In order to describe how the economy responds to various shocks, this study follows the methods used in Ireland (2007) and log-linearises the stationary system (described in Eq. 4.58 – Eq. 4.73) around the steady state level (derived in Section 3.4.8). Let:

$$\hat{y}_t = ln\left(\frac{y_t}{y}\right) \tag{4.81}$$

$$\hat{c}_t = ln\left(\frac{c_t}{c}\right) \tag{4.82}$$

$$\hat{\pi}_t = ln(\pi_t) \tag{4.83}$$

$$\hat{r}_t = ln\left(\frac{r_t}{r}\right) \tag{4.84}$$

$$q_t = ln\left(\frac{q_t}{q}\right) \tag{4.85}$$

$$\hat{x}_t = ln\left(\frac{x_t}{x}\right) \tag{4.86}$$

$$\hat{g}_t^y = ln\left(\frac{g_t^y}{g^y}\right) \tag{4.87}$$

$$\hat{g}_t^\pi = ln(g_t^\pi) \tag{4.88}$$

$$\hat{g}_t^r = ln(g_t^r) \tag{4.89}$$

$$\hat{r}_t^{r\pi} = ln\left(\frac{r_t^{r\pi}}{r^\pi}\right) \tag{4.90}$$

$$\hat{\lambda}_t = ln\left(\frac{\lambda_t}{\lambda}\right) \tag{4.91}$$

$$\hat{a}_t = ln(a_t) \tag{4.92}$$

$$\hat{\theta}_t = ln\left(\frac{\theta_t}{\theta}\right) \tag{4.93}$$

$$\hat{z}_t = ln\left(\frac{z_t}{z}\right) \tag{4.94}$$

$$\hat{v}_t = ln(v_t) \tag{4.95}$$

and

$$\hat{\pi}_t^* = ln(\pi_t^*) \tag{4.96}$$

denote the percentage deviation of each stationary variable from its steady-state value. The first-order Taylor approximation to the aggregate resource constraint (as expressed in Eq. 4.58) implies that $\hat{c}_t = \hat{y}_t$. This allows the term $\hat{c}_t$ to be eliminated from the system. Then the first-order approximations to the remaining 15 equations imply:

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \sigma_a \varepsilon_{at} \tag{4.97}$$

$$\begin{aligned}(z-\gamma)(z-\beta\gamma)\hat{\lambda}_t & \\ &= \gamma z \hat{y}_{t-1} - (z^2 + \beta\gamma^2)\hat{y}_t + \beta\gamma z E_t \hat{y}_{t+1} \\ &+ (z-\gamma)(z-\beta\gamma\rho_a)\hat{a}_t - \gamma z \hat{z}_t\end{aligned} \tag{4.98}$$

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + \hat{r}_t - E_t \hat{\pi}_{t+1} \tag{4.99}$$

$$\hat{e}_t = \rho_e \hat{e}_{t-1} + \sigma_e \varepsilon_{et} \tag{4.100}$$

$$\hat{z}_t = \sigma_z \varepsilon_{zt} \tag{4.101}$$

$$(1+\beta\alpha)\hat{\pi}_t = \alpha\hat{\pi}_{t-1} + \beta E_t \hat{\pi}_{t+1} - \psi\hat{\lambda}_t + \psi\hat{a}_t - \hat{e}_t - \alpha\hat{\pi}_t^* \tag{4.102}$$

$$\hat{g}_t^y = \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \tag{4.103}$$

$$\hat{g}_t^\pi = \hat{\pi}_t - \hat{\pi}_{t-1} + \hat{\pi}_t^* \tag{4.104}$$

$$\hat{g}_t^r = \hat{r}_t - \hat{r}_{t-1} + \hat{\pi}_t^* \tag{4.105}$$

$$\hat{r}_t^{r\pi} = \hat{r}_t - \hat{\pi}_t \tag{4.106}$$

$$0 = \gamma z \hat{q}_{t-1} - (z^2 + \beta\gamma^2)\hat{q}_t + \beta\gamma z E_t \hat{q}_{t+1} + \beta\gamma(z-\gamma)(1-\rho_a)\hat{a}_t - \gamma z \hat{z}_t \tag{4.107}$$

$$\hat{x}_t = \hat{y}_t - \hat{q}_t \tag{4.108}$$

$$\hat{r}_t - \hat{r}_{t-1} = \rho_\pi \hat{\pi}_{t+1} - \hat{\pi}_t^* + \rho_{gy} \hat{g}_t^y + \rho_x \hat{x}_t + \hat{v}_t \tag{4.109}$$

$$\hat{\pi}_t^* = \delta_a \varepsilon_{at} - \delta_e \varepsilon_{et} - \delta_z \varepsilon_{zt} + \sigma_\pi \varepsilon_{\pi t} \tag{4.110}$$

$$\hat{v}_t = \rho_v \hat{v}_{t-1} + \sigma_v \varepsilon_{vt} \tag{4.111}$$

for all $t = 0, 1, 2\ ...\ T$ where in Eq. (4.100), Eq. (4.102) and Eq. (4.110) cost-push shocks $\hat{\theta}_t$ have been re-normalised as $\hat{e}_t = (1/\phi)\hat{\theta}_t$. The new parameters ρ_e, σ_e, ψ and δ_e have been defined as $\rho_e = \rho_\theta$, $\sigma_e = \sigma_\theta/\phi$, $\psi = (\theta - 1)/\phi$ and $\delta_e = \delta_\theta$ so that like $\varepsilon_{\theta t}$, ε_{et} they have standard normal distributions.

Eq. (4.97), Eq. (4.100), Eq. (4.101) and Eq. (4.110) respectively describe the process for exogenous preference shocks, cost-push shocks, technology shocks and monetary policy shocks. The model's economic content is concentrated in Eq. (4.98), Eq. (4.99), Eq. (4.102), Eq. (4.109) and Eq. (4.110). Eq. (4.99) is a New Keynesian (forward-looking) IS curve and links the marginal utility of consumption to its own expected future value and to the value of the *ex ante* real interest rate. Eq. (4.98) assesses the marginal utility of consumption. It includes both forward-looking and backward-looking terms based on preference. Eq. (4.102) describes a hybrid forward-looking and backward-looking New Keynesian Phillips curve with the product of ψ and the real marginal cost term ($\hat{a}_t - \hat{\lambda}_t$), the cost-push shock ($\hat{e}_t$) and a term α from the price adjustment cost formulation (in Eq. 2.2.23 and Eq. 4.16) showing the degree of backward-looking behaviour. Eq. (4.109) and Eq. (4.110) forms a forward-looking generalised Taylor rule by taking the endogenous evolution of the central bank's inflation target into account.

As a first step in solving the log-linearised system above, it is necessary to re-write Eq. (4.109) by substituting Eq. (4.103) and Eq. (4.108) into the central bank's reaction rule (Eq. 4.109):

$$\hat{r}_t - \hat{r}_{t-1} = \rho_\pi \hat{\pi}_{t+1} - \hat{\pi}_t^* + \left(\rho_x + \rho_{gy}\right)\hat{y}_t - \rho_x \hat{q}_t - \rho_{gy}(\hat{y}_{t-1} - \hat{z}_t) + \hat{v}_t \tag{4.112}$$

As a second step, note that Eq. (4.103) – Eq. (4.106) and Eq. (4.108) can be used to solve for $\hat{g}_t^y$, $\hat{g}_t^\pi$, $\hat{g}_t^r$, $\hat{r}_t^{r\pi}$ and $\hat{x}_t$ in terms of $\hat{y}_t$, $\hat{\pi}_t$, $\hat{r}_t$, $\hat{q}_t$, $\hat{\lambda}_t$, $\hat{a}_t$, $\hat{e}_t$, $\hat{z}_t$, $\hat{v}_t$ and $\hat{\pi}_t^*$. It suggests using the ten equations, Eq. (4.97) – Eq. (4.102), Eq. (4.107) and Eq. (4.110) – Eq. (4.112) to solve for the ten variables $\hat{y}_t$, $\hat{\pi}_t$, $\hat{r}_t$, $\hat{q}_t$, $\hat{\lambda}_t$, $\hat{a}_t$, $\hat{e}_t$, $\hat{z}_t$, $\hat{v}_t$ and $\hat{\pi}_t^*$ before returning to the five equations Eq. (4.103) – Eq. (4.106) and Eq. (4.108) to solve for the remaining five variables $\hat{g}_t^y$, $\hat{g}_t^\pi$, $\hat{g}_t^r$, $\hat{r}_t^{r\pi}$ and $\hat{x}_t$.

3.4.10 Constructing a State Space Model for the System

Since this study extends the Ireland (2007) structural model by considering more factors such as the central bank's forward-looking behaviour, it is necessary to re-create matrices carefully for the augmented structural model. An important contribution in this study is to develop matrices in this section for estimating the augmented structural model.

Re-present the system consisting of Eq. (4.97) – Eq. (4.111) as:

$$AE_t s_{t+1}^0 = Bs_t^0 + C\xi_t \tag{4.113}$$

$$\xi_t = P\xi_{t-1} + X\varepsilon_t \tag{4.114}$$

where:

$$A = \begin{bmatrix} z^2+\beta\gamma^2 & 0 & 0 & 0 & 0 & -\beta\gamma z & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1+\beta\alpha & 0 & 0 & 0 & 0 & -\beta & 0 \\ 0 & 0 & 0 & z^2+\beta\gamma^2 & 0 & 0 & 0 & -\beta\gamma z \\ 0 & 0 & 1 & 0 & 0 & 0 & -\rho_\pi & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.115}$$

$$s_t^0 = [\hat{y}_{t-1}, \hat{\pi}_{t-1}, \hat{r}_{t-1}, \hat{q}_{t-1}, \hat{\lambda}_t, \hat{y}_t, \hat{\pi}_t, \hat{q}_t]' \tag{4.116}$$

$$B = \begin{bmatrix} \gamma z & 0 & 0 & 0 & -(z-\gamma)(z-\beta\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & -\psi & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma z & 0 & 0 & 0 & 0 \\ -\rho_{gy} & 0 & 1 & 0 & 0 & \rho_x+\rho_{gy} & 0 & -\rho_x \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.117}$$

$$C = \begin{bmatrix} (z-\gamma)(z-\beta\gamma\rho_a) & 0 & -\gamma z & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \psi & -1 & 0 & 0 & -\alpha \\ \beta\gamma(z-\gamma)(1-\rho_a) & 0 & -\gamma z & 0 & 0 \\ 0 & 0 & \rho_{gy} & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.118}$$

$$\xi_t = [\hat{a}_t, \hat{e}_t, \hat{z}_t, \hat{v}_t, \hat{\pi}_t^*]' \tag{4.119}$$

$$P = \begin{bmatrix} \rho_a & 0 & 0 & 0 & 0 \\ 0 & \rho_e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_v & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.120}$$

$$X = \begin{bmatrix} \sigma_a & 0 & 0 & 0 & 0 \\ 0 & \sigma_e & 0 & 0 & 0 \\ 0 & 0 & \sigma_z & 0 & 0 \\ 0 & 0 & 0 & \sigma_v & 0 \\ \delta_a & -\delta_e & -\delta_z & 0 & \sigma_\pi \end{bmatrix} \tag{4.121}$$

$$\varepsilon_t = [\varepsilon_{at}, \varepsilon_{et}, \varepsilon_{zt}, \varepsilon_{vt}, \varepsilon_{\pi t}]' \tag{4.122}$$

Eq. (4.113) and Eq. (4.114) together describe the final version of the augmented structural model. The former takes the form of a system of linear difference equations which is driven by exogenous shocks in the latter. To solve this system, Ireland (2007) suggests separating unstable and stable components and then solving the unstable components. This study follows Ireland (2007) to use the algorithm outlined in Klein (2000). Klein provides solutions that take the form of a state-space specification linking the behaviour of the stationary model's three observable variables (i.e., $\hat{g}_t^y$, $\hat{g}_t^\pi$ and $\hat{r}_t^{r\pi}$) to unobservable ones. Therefore the model parameters are estimated by the Kalman filter algorithms described in Hamilton (1994, ch.13) to obtain the maximum likelihood estimates of the model's structural parameters and to optimally explore the information contained in the observable data to derive the behaviour of unobservable variables including the central bank's implicit time-varying short-term inflation target, $\hat{\pi}_t^*$.

3.5 Empirical Evidence

Preliminary analysis reveals that using the MPC's inflation forecast data will lead to more equations to estimate, which results in difficulties in solving the augmented structural model. Therefore, this study uses the actual future inflation rate to represent the BOE's inflation forecasts (see, Eq. 4.41-4.42). It uses the seasonally adjusted CPI to measure the nominal price level P_t. The Trea3m is used as the measure of the nominal interest rate R_t in the structural model. Prior to the use of IIP, P_t and R_t in the econometric estimation, these three series of raw data are passed through the stationary-inducing transformations outlined in Section 3.4.7 for solving the structural model. It is worth noting that every time the inflation target (for the UK) is mentioned in this section, it refers to the hypothesised short-term implicit inflation target of the BOE.

3.5.1 Maximum Likelihood Estimates of Parameters

The augmented structural model assumes that the five innovations ε_{at}, ε_{et}, ε_{zt}, ε_{vt} and $\varepsilon_{\pi t}$ are independent and serially uncorrelated. Since the econometric estimation in this study considers the possible response of the policy rate (i.e., Trea3m) to the level of the output gap and also allows the inflation target to respond to demand-side shocks, it has two more parameters to estimate in comparison to Ireland (2007). The augmented model has 19 parameters: z, β, ψ, γ, α, ρ_π, ρ_x ρ_{gy}, ρ_a, ρ_e, ρ_v, σ_a, σ_e, σ_z, σ_v, σ_π, δ_a, δ_e and δ_z to estimate. The 19 parameters are obtained from the system of Eq. (4.97) – Eq. (4.111).

The preliminary attempts in the data section (in Table 3) to estimate the Gali and Gertler (1999) hybrid Phillips curve indicate that prices are fixed for roughly 3-4 quarters on average in the UK. This result is also similar to the findings of Ireland (2007) based on the US data suggesting that individual goods prices remain fixed on average for 3.7 quarters or just under one year. Hence, the empirical estimation here uses a value of $\psi = 0.1$ as Ireland (2007) suggested for the coefficient on the real marginal cost term in a pure forward-looking New Keynesian Phillips curve.

Estimation of the (augmented) structural model requires this study to fix the value of z and β at $z = g^y$ and $\beta = z/r^{r\pi}$. This is to ensure that the steady-state rate of real output growth ($g^y = z$) and the steady-state ratio of the nominal interest rate to inflation rate ($r^{r\pi} = z/\beta$, as in Eq. 4.80) match the mean of these two variables as measured in the data. It suggests that along the steady-state growth path, the growth rate of expected inflation approaches zero (expected inflation is unchanging). Indeed in the UK data, the sample average of inflation growth rate is quite small. This is consistent with the findings of Ireland (2007) based on the US data. In the UK, the inflation rate at the end of the 20-year sample period is at approximately the same point as at the beginning of the sample period (in Figure 1.2). The average quarterly rate of changes in expected inflation over the last twenty years is very close to zero. Thus, according to Ireland (2007) all variables are de-meaned prior to estimation in a manner consistent with the implication of the theoretical (augmented) structural model. This study fixes $z = 0.9999$ and $\beta = 0.9793$ in advance.

Table 4 presents the maximum likelihood estimates together with standard errors of the remaining 16 parameters of the augmented structural model. Standard errors are obtained using a parametric bootstrapping procedure which simulates the estimated model. Firstly, it draws a random sample from the real output growth rate, the growth rate of inflation and the ratio of the nominal interest rate to the inflation rate. It draws a sample size of T (i.e., the same number of observations as the original sample of real UK data). In a second step, it re-estimates the augmented structural model with the simulated data drawn from the first step. This constitutes the first run of the bootstrap procedure. This procedure is repeated 1,000 times which gives a distribution of each parameter. The standard errors displayed in Table 4 correspond to the standard deviation of individual parameter estimates based on the 1,000 replications.

For the sake of comparison, Table 4 includes four sets of estimates for the augmented structural model: (i) the estimation of an unconstrained version with an endogenous inflation target in which all 16 parameters are estimated freely, (ii) the estimation of an augmented structural model with exogenous inflation target, (iii) the estimation of an augmented structural model with purely backward-looking consumption behaviour and (iv) the estimation of an augmented structural model with purely forward-looking consumption behaviour. The models in the estimation (ii-iv) are three versions of the constrained models. The exogenous inflation target model assumes $\delta_a = \delta_e = \delta_z = 0$, which means the central bank does not modify its short-term (implicit) inflation target in response to supply-side or demand-side shocks. The backward-looking structural model fixes α at 1.0 and estimates the other 15 parameters freely. In the forward-looking structural model, the parameter α is fixed at zero and the other 15 parameters are estimated freely. For all four versions of the structural model, the estimates of γ shift from 0.15 to 0.18 indicating a low degree of backward-looking behaviour of household's consumption which is measured by the habit-formation parameter γ.

The maximised value of the log-likelihood function (L^*) for each augmented structural model is shown at the bottom of Table 4. As already mentioned, the 'exogenous target' model is a constrained version of the 'endogenous target' structural model. The results show that L^* falls considerably when the constraints $\delta_a = \delta_e = \delta_z = 0$ are imposed. Furthermore, when either the constrained $\alpha = 1$ or $\alpha = 0$ is imposed in estimating the structural model with pure backward-looking or forward-looking price setting, the maximised value of the log-likelihood function L^* drops as well. Hence, the likelihood ratio tests firmly reject the null hypotheses that one (or more) of the three constraints is true at 5% level of significance. This motivates the concentration on the results of the unconstrained model with the endogenous inflation target in the subsequent analysis.

With an α of 0.3606 lying between zero and one, the endogenous model produces a hybrid forward-looking and backward-looking Phillips curve. For the unconstrained augmented model, the estimates show that both the expected inflation rate and current real economic activity (including the output level and its growth rate) enter significantly into the generalised Taylor rule. With ρ_π, ρ_x and ρ_{gy} equalling 0.8173, 0.7270 and 0.0259 respectively, the BOE places different weights on inflation, output and its growth ratio. A rise of one percent in expected inflation induces the BOE to increase the nominal rate by roughly 0.8173 percent, while a one-percent-increase in the real output level causes it to raise the rate by 0.7270 percent. The significant estimate of ρ_x provides the evidence for using a generalised Taylor rule instead of its simplification in the New Keynesian structural model.[7] The BOE also reacts to a one-percent-rise in the real output growth by raising the nominal interest rate by 0.026

[7] Recall that in the literature section 3.2.1 (Eq. 2.1.6), a simplified generalised Taylor rule assumes that the interest rate does not respond to the level of economic activity. Hence, the parameter on the level of output is assumed to be zero in Eq. (2.1.6). However, the econometric estimation in Table 4 indicates that this assumption is too restrictive using the UK data because the estimated ρ_x is significantly greater than zero.

percent. The policy reaction to expected inflation, however, appears higher than the associated response to economic activity. This is similar to the findings in Ireland (2007) for the US case. Here it is worth noting that the response parameters on inflation and output indeed differ substantially from those obtained using a GMM (e.g., Clarida, et al., 1998; Castro, 2011). This is likely to result from the possible time-variation in the BOE's short-term inflation target. Moreover, comparing the point estimates of δ_a, δ_e and δ_z together with their standard errors implies that the response of the short-term inflation target to technology shocks is much more important. In contrast Ireland (2007) finds that the cost-push shock is the dominant factor in the US.

Table 4: Parameter Estimation with Maximum Likelihood Estimates

Parameter	Augmented structural model with endogenous target		Augmented structural model with exogenous target		Augmented structural model with backward-looking		Augmented structural model with forward-looking	
	Estimate	SEE	Estimate	SEE	Estimate	SEE	Estimate	SEE
γ	0.17806	0.03893	0.17381	0.04621	0.15108	0.04349	0.15872	0.03463
α	0.36063	0.12423	0.35477	0.06030	1.00000	---	0.00000	---
ρ_π	0.81734	0.66079	3.60100	1.47302	0.59388	0.17211	0.30035	0.54665
ρ_x	0.72703	0.22640	0.29348	0.13454	0.31759	0.08501	0.95439	0.21140
ρ_{gy}	0.02594	0.02326	0.08931	0.03884	0.03760	0.02133	0.00256	0.01479
ρ_a	0.99502	0.00333	0.99632	0.00209	0.99315	0.00430	0.98522	0.00651
ρ_e	0.00003	0.16961	0.99998	0.00000	0.00001	0.05481	0.45349	0.20304
ρ_v	0.75557	0.05584	0.75472	0.04930	0.67879	0.07452	0.82511	0.05129
σ_a	0.57797	0.19228	0.74541	0.19553	0.46427	0.18938	0.20268	0.08159
σ_e	0.00099	0.00052	0.00121	0.00017	0.00256	0.00019	0.00000	0.00026
σ_z	0.01114	0.00092	0.00012	0.00313	0.00890	0.00094	0.01079	0.00076
σ_v	0.00578	0.00160	0.01064	0.00401	0.00389	0.00068	0.00481	0.00121
σ_π	0.00079	0.00042	0.00080	0.00045	0.00085	0.00048	0.00017	0.00032
δ_a	0.00000	0.00017	0.00000	---	0.00001	0.00023	0.00000	0.00015
δ_e	0.00002	0.00026	0.00000	---	0.00000	0.00028	0.00065	0.00031
δ_z	0.00072	0.00025	0.00000	---	0.00053	0.00033	0.00066	0.00020
L^*	3150.3753		3145.0341		3140.3423		3143.8896	

3.5.2 The Time-varying Inflation Target and the Contribution from Each Shock

As the maximum likelihood estimation favours the unconstrained endogenous model, this section focuses on the implication of the unconstrained (augmented) structural model with the endogenous inflation target.

The results indicate that δ_a is close to zero (in Table 4) which implies that the inflation target does not react significantly to demand-side shocks (i.e., preference shocks). The remaining three parameter estimates σ_π (the inflation target's response to exogenous shocks), δ_e (the inflation target's response to cost-push shocks) and δ_z (the inflation target's response to technology shocks) are all relatively small, which is consistent with the estimates of Ireland (2007) for the US economy. However, a key difference between the US and the UK is that σ_π differs significantly from zero in the UK. This suggests that σ_π together with the parameters δ_e and δ_z attribute all movements in the BOE's inflation target to this central bank's reaction to supply-side shocks and exogenous shocks. In the US, Ireland (2007) discovers that exogenous shocks to the inflation target play no role in explaining the movement of inflation and only supply-side shocks contribute to the shifts in the Fed's implicit goals. A clear interpretation of the estimates in Table 4 emerges from Figure 2. It plots the impulse responses obtained from the unconstrained endogenous model.

In particular, the figure shows that under the estimated policy rule the inflation rate falls immediately by 9 basis points following a favourable one-standard-deviation cost-push shock (i.e., cost decline, $\varepsilon_{et} > 0$). Inflation will be kept below the steady-state level for roughly one quarter. This is consistent with the implication of the augmented structural model: given the estimated parameters of ρ_e, σ_e, δ_e and δ_z, the model (Eq. 4.100, Eq. 4.102 and Eq. 4.110) expects an inflation rate decline caused by a positive cost-push shock. The model's (assumed) linearity then indicates that symmetrically inflation rises by the same amount following a similar sized adverse cost-push disturbance. Figure 2 also confirms that cost-push shocks and technology shocks act as supply-side disturbances moving output and inflation in an opposite direction. As noted, a favourable cost-push shock tends to lead output to rise by 1.4 basis points before returning to its steady-state level. In addition, a larger and more persistent increase in output is observed following a positive technology shock. The large estimated value for δ_z implies a 7.2-basis-point adjustment of inflation after a one-standard-deviation technology shock. This impact is expected to last over 18 months.

Consistent with prior expectations, a one-standard-deviation innovation in monetary policy shock works to raise the real interest rate by roughly 21 basis points and keeps the real rate above its steady-state level for around a period of one year. Monetary tightening generates a decline in the output level by roughly 52 basis points and a decrease in inflation by about 22 basis points – the movement in output and inflation tends to persist for around one year. Meanwhile, a one-standard-deviation shock to the inflation target tends to cause a temporary response in output and the real rate of interest but permanent changes in inflation and the nominal interest rate.

The last column of Figure 2 indicates how the welfare-theoretic measure of the output gap (i.e., x_t defined in Eq. 4.38) as a percentage deviation of output in equilibrium from its efficient level (Q_t defined in Eq. 4.37) reacts to each of the five shocks. As shown by Eq. (4.37), cost-push shocks, monetary policy shocks and inflation target shocks do not enter the Q_t equation, i.e., they do not influence the efficient level of the output (Q_t). Therefore, the movement of the output gap (x_t) is consistent with output (y_t) after suffering each of these shocks. However, preference shocks and technology shocks enter significantly into Eq. (4.37) generating impulse responses in the output gap that differ from those in the output level as plotted in Figure 2.

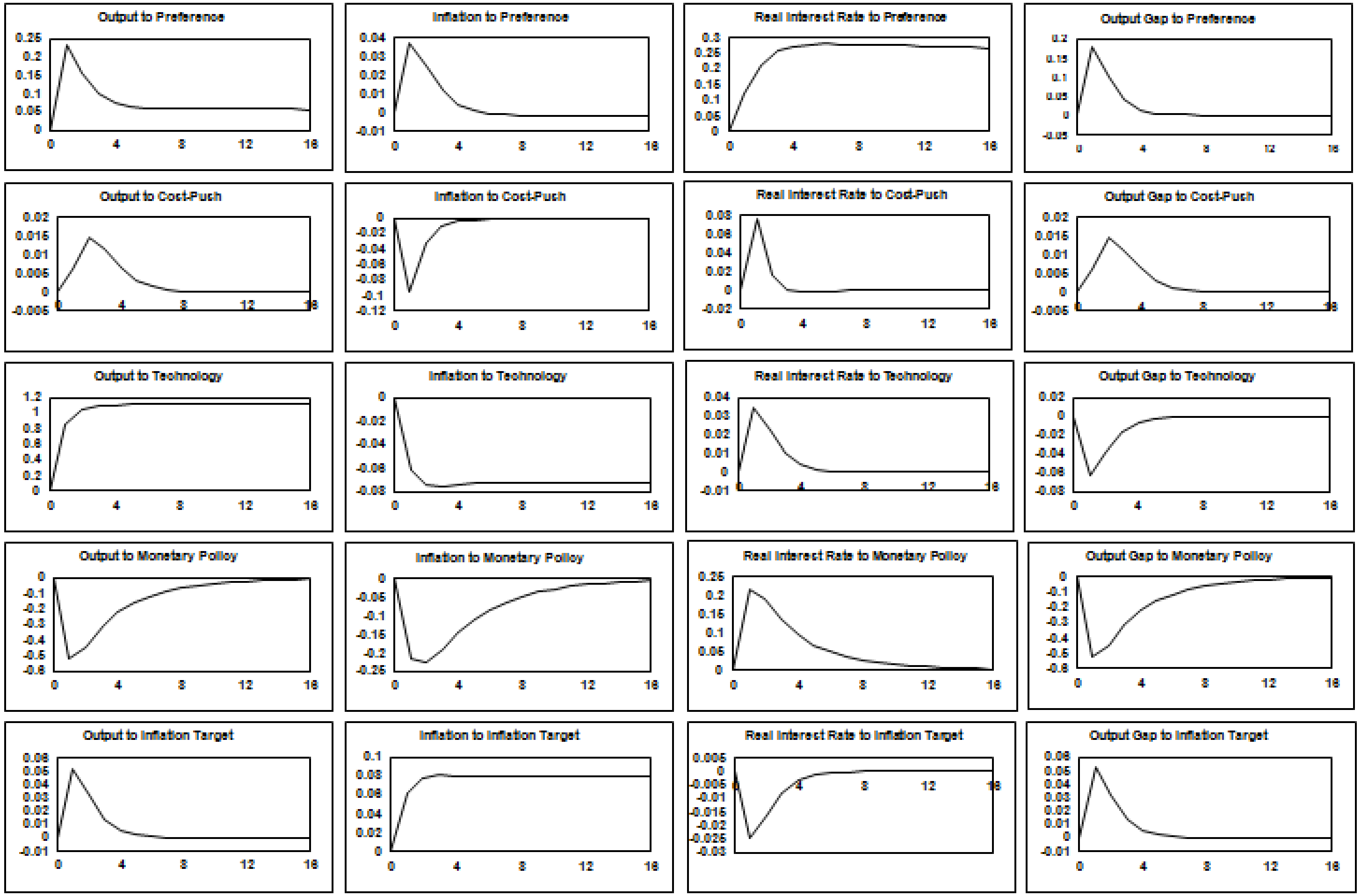


Figure 2: Impulse Responses from the Structural Model with an Endogenous Inflation Target

Figure 3 shows the movement of the monthly inflation rate and estimated inflation target over the sample period. The estimates reflect information contained in the full sample using the methodology referenced in Ireland (2007) i.e., using the smoothing algorithms described by Hamilton (1994, p.394-397) and generalised by Kohn and Ansley (1983) to handle cases like the one that arises here where the state covariance matrix turns out to be singular. The figure indicates that the inflation target falls from 2.63% in January 1993 to a low of 1.05% in August 2000. The low short-term inflation target during 2000-2003 explains the lower inflation rate in that period (compared to the target of 2.5%)[8]. The estimated inflation target then appears to rise considerably and hit a peak of 3.60% in September 2011. Since then it has been kept in a range of 3.00%-3.55%.

By way of explaining these estimated high short-term inflation targets, this study refers to an open letter written by the governor of the BOE to the Chancellor[9] in June 2008. Given the ongoing rise in inflation and an above 3% rate of inflation in May 2008, the letter clarifies that the MPC expects to return the inflation rate to the long-run target (2%) in roughly 24 months – as the committee believes that if the bank rate was set to bring inflation back to 2% within one year, the result would be unnecessary volatility in output and employment. Therefore, it seems reasonable to infer that in at least a two-year horizon from June 2008 the MPC might have had several temporary (short-term) targets for inflation and allowed the rate to remain above the BOE's long-run objective of 2%. This is what is reflected in Figure 3.

In the subsequent open letter dated August 2010, the governor of the BOE acknowledges that high inflation may remain for one-year longer than previously expected. The MPC announces that inflation will remain high until the end of 2011, which partially explains why the estimates of the short-term target keep rising 24 months after the issue of the June 2008 open letter. The decline in the implicit short-term target of inflation in early 2012 is consistent with the MPC's expectation. As in the open letter in November 2011, the governor maintains that although the inflation rate is still above the 2% target it will fall sharply in 6 months. This expectation leads the BOE to modify its implicit inflation target downward.

[8] Recall: between 1995 and 2003, the inflation target was defined as an inflation rate of the RPI of 2.5%.
[9] If inflation moves away from the target by more than one percentage point in either direction, the governor of the BOE is required to provide the Chancellor with an explanation in an open letter (see, MPC, March 2013).

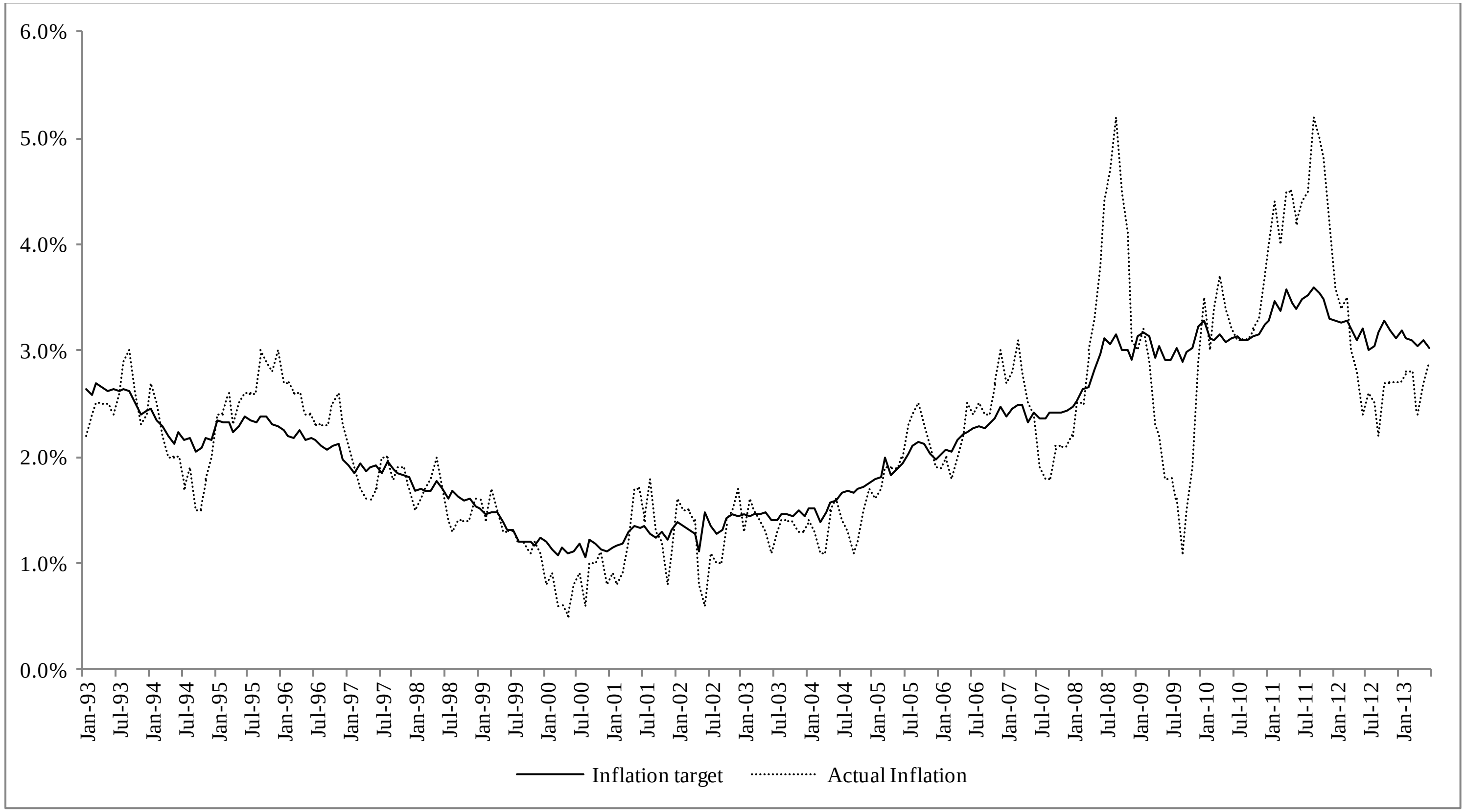


Figure 3: The Actual Inflation Rate and the Inflation Target Implied by the Augmented Structural Model with the Endogenous Target

3.5.3 Estimates of the Counterfactual Inference

As stated at the beginning of the methodology section, the estimated model provides a detailed answer to questions such as how would the domestic economy have behaved if the BOE had kept a time-invariant inflation target over the sample period. Figure 4 compares the actual paths for inflation, the short-term nominal rate and the real output gap obtained from the historical data to the counterfactual paths that according to the augmented structural model would have been realised under a fixed inflation target (i.e., assuming the four parameters σ_{π}, δ_a, δ_e and δ_z all equal zero).

Estimates from the augmented structural model indicate that the inflation rate would have become much less volatile and more stable under the counterfactual scenario. In particular, the variance of inflation falls by roughly 2/3 under the constant inflation target. Without allowing the implicit short-term inflation target to shift downward, the counterfactual estimate is higher than the actual rate prior to 2008. In the most recent crisis, the implicit target is estimated to increase, which explains why actual inflation appears above the inflation rate under the counterfactual scenario at some dates between 2008 and 2013. Through the Fisher effect, the nominal interest rate follows the inflation rate by being less volatile under the constant inflation target. However, it is surprising that the counterfactual nominal interest rate always remains above the actual rate, even when the counterfactual inflation rate is below the actual one after 2008. This is because although inflation declines under the counterfactual case, its deviation from the assumed-target is still greater than the deviation of actual inflation from the implicit short-term objectives. In other words, the counterfactual inflation rate is lower than the actual inflation rate in 2008-2013, but it is still much higher than the target – if the BOE didn't change the implicit target. Because real economic activity (measured by the output growth rate) looks much the same under the counterfactual path as it does historically, the greater deviation of inflation (from the inflation target) leads to the higher nominal interest rate under the counterfactual scenario for the period from mid-2008 to 2013.

In addition, it is particularly interesting to discover that although the movement of output growth looks much similar under the counterfactual scenario, output growth has a larger variance (increase roughly by 8.2%) under the counterfactual scenario.

This seems consistent with the MPC's (March 2013) judgement that any attempts to fix inflation is likely to result in great volatility in output.

Figure 4.1: Actual versus counterfactual inflation rate (%):

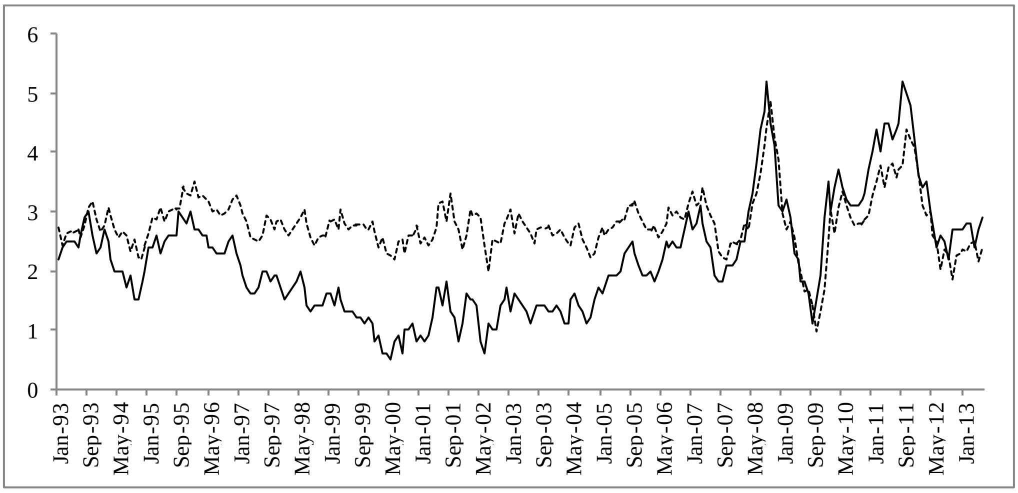


Figure 4.2: Actual versus counterfactual interest rate (%):

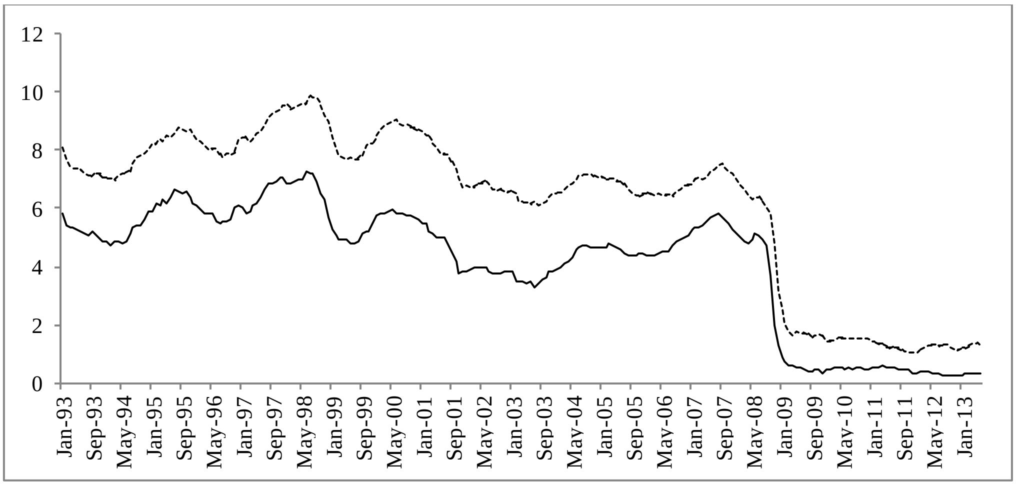


Figure 4.3: Actual versus counterfactual output growth rate (%):

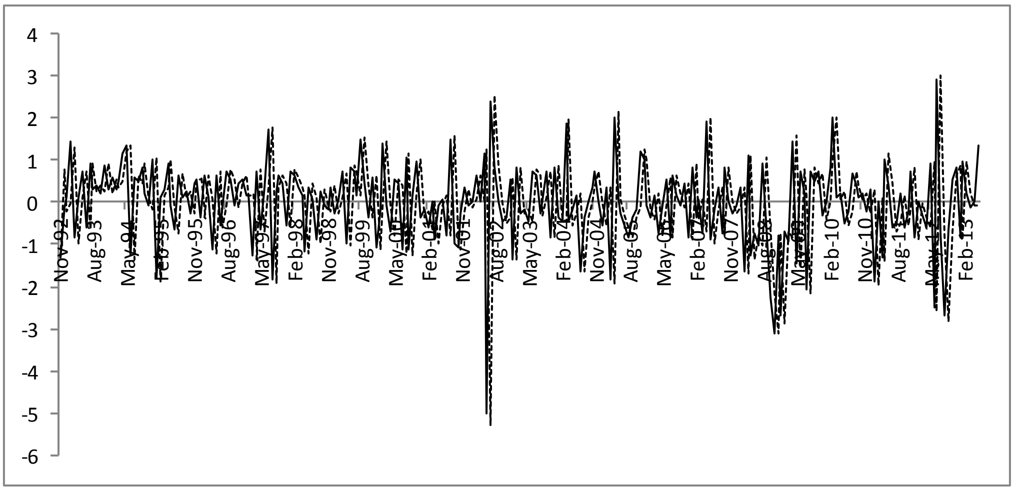


Figure 4: Actual UK Data (heavy lines) and Counterfactual Paths (thin lines) Obtained Under a Constant Inflation Target Using the Structural Model

3.6 Conclusions

This chapter focuses on inflation targeting. It hypothesises that for an inflation targeting country, in this case the UK, the central bank implicitly targets a short-term inflation objective which is different from the official inflation target based the MPC's long-term consideration. This means that the central bank adjusts inflation gradually to achieve the long-run inflation objective. The hypothesis is motivated by several factors. Firstly, the MPC (March 2013) acknowledges in the BOE's remit that the official inflation target reflects its long-run objectives. The committee brings the inflation rate to its official target gradually, because attempts to adhere to the official target may result in extreme volatility in output. Therefore there are short-term trade-offs between inflation and output stability. With the above statement, it is reasonable to infer that the BOE may have some short-run objectives regarding movement in the inflation rate. Secondly, empirical literature in the UK such as Martin and Milas (2004) and Castro (2011) have estimated the BOE's inflation objectives. However, their estimates seem quite different from the announced target. This observation suggests that the MPC may have an implicit goal to be distinguished from the official one. In addition, the following three points imply that the implicit inflation target, if it exists, may be time-varying: (i) the BOE has changed its official inflation objectives twice after October 1992; (ii) the inflation rate is measured using the CPI after 1996, however its target was still on the basis of the RPI between 1997 and 2003. Thus, the BOE may have to frequently adjust its implicit inflation target due to the difference between the measures of the CPI and the RPI; (iii) at an empirical level, Martin and Milas (2004) and Castro (2011) both imply that the BOE's inflation objective may change over time. Therefore, the hypothesis is re-stated as the BOE has an implicit time-varying inflation target for its short-term purpose. However, this hypothesis does not assume that the BOE is working on short-term goals instead of promoting long-run price stability. The concentration of this study is to examine whether the BOE has an implicit objective when it moves inflation to its long-term target rate.

In order to capture the timing and speed of switch of any shifts in the implicit goal, this study uses data on a monthly basis as this is the frequency with which the MPC meets to make monetary policy decisions. The sample starts in October 1992 when the MPC began to target inflation in the UK. It ends June 2013 so as to include as long a time series as possible. The methodology used in this study is based on the previous work by Ireland (2007). However, several changes are made here. Firstly, this study considers a more generalised structural model to examine the above hypothesis. Instead of setting the inflation target to supply-side shocks alone, it considers the preliminary estimation of Ireland (2007) by allowing the implicit goals of the BOE to respond to preference, cost-push and technology shocks. It also uses the generalised Taylor rule in Orphanides (2003, 2007) to let the short-term interest rate react to the inflation rate, the output level and its growth rate. The most important adjustment in this study is to include the central bank's inflation expectations – the central bank sets monetary policy on the basis of its inflation expectations. It is generally agreed in the literature and justified by the recent study in Chapter 4 that the BOE targets expected inflation rather than the contemporaneous inflation rate. For comparative purpose, this study produces the estimation of four versions of the (augmented) structural model to decide the most appropriate model for examining the earlier hypothesis. The maximum likelihood estimates show that the structural model with the endogenous target best suits the UK data.

The estimates of parameters and the implicit short-run inflation target shown in Table 4 and Figure 3 together with the counterfactual histories traced out in Figure 4 bring some important findings. It is obvious that the short-term inflation target that enters into the BOE's reaction function changes a lot throughout the sample period. It falls from 2.63% in January 1993 to the bottom of 1.05% in August 2000. Then it rises markedly and hits a peak of 3.60% in September 2011. Since then the target has been kept in a narrow range of 3.00%-3.55%. The counterfactual tests indicate that keeping the inflation target fixed between October 1992 and June 2013 would have led to less-volatile inflation. However, the short-term interest rate would have been much higher but less volatile via the Fisher effect. Output growth would have been more volatile if the implicit inflation target is assumed to be constant. All the above results indicate that the BOE has an implicit inflation target which is time-varying over the sample period. This implicit inflation objective differs itself from the official inflation target that is based on the BOE's medium to long-run consideration. The other crucial finding that arises is that among the three shocks hitting the UK technology shocks dominate the shifts in the inflation target. The impact of technology shocks on both output and inflation is estimated to persist for over 1.5 years.

Appendix:

Appendix 1: Gali and Gertler's Hybrid Phillips Curve (1999):

Assume, as in Calvo's model (1983) that each firm is able to adjust its price in any given period with a fixed probability $(1-\xi)$. Therefore changing price is independent of time. As in Gali and Gertler's extension (1999), two types of firms are assumed to co-exist. A fraction $1-d$ of the firms behave like the firms in Calvo's model: they set prices optimally given the constraints on the timing of adjustments and using all the information in order to forecast future marginal costs (i.e., forward-looking). The remaining firms instead use a simple rule of thumb that is based on the recent history of aggregate price behaviour (i.e., backward-looking).

The aggregate price level evolves according to:

$$p_t = \xi p_{t-1} + (1-\xi)\bar{p}_t^*, \quad 0 \le \xi \le 1 \tag{1}$$

where $\bar{p}_t^*$ is an index for the prices newly set in period t. It could be expressed as:

$$\bar{p}_t^* = (1-d)p_t^f + dp_t^b, \quad 0 \le d \le 1 \tag{2}$$

where p_t^f denotes the prices set by forward-looking firms and p_t^b the price set by backward-looking firms. As mentioned earlier, the forward-looking firms in Gali and Gertler's (1999) model behave exactly as in the baseline Calvo's (1983) specification. Accordingly, p_t^f is expressed as:

$$p_t^f = (1-f\xi)\sum_{k=0}^{\infty}(f\xi)^k E_t\{mc_{t+k}^n\}, \quad 0 \le f \le 1 \tag{3}$$

where the parameter f denotes the subjective discount factor and mc_t is the percent deviations of real marginal cost from its steady-state level. The real marginal cost is measured as the ratio of the wage rate to the marginal product of labour.

Given the two features of backward-looking firms described in Section 3.2.4, Gali and Gertler's (1999) formulate the pricing behaviour of these firms as follows:

$$p_t^b = \bar{p}_{t-1}^* + \pi_{t-1} \tag{4}$$

This equation implies that backward-looking firms set their price (p_t^b) based on the average prices set in the most recent price adjustments ($\bar{p}_{t-1}^*$) with a correction for the inflation. Combining Eq. (1) – Eq. (4), this study obtains the hybrid Phillips curve:

$$\pi_t = S_0 mc_t + S_1 E_t\{\pi_{t+1}\} + S_2 \pi_{t-1} \tag{5}$$

where:

$$S_0 = (1-d)(1-\xi)(1-f\xi)S^{-1}, \quad S_1 = f\xi S^{-1}, \quad S_2 = dS^{-1} \tag{6}$$

with $S = \xi + d[1 - \xi(1-f)]$.

Conclusively, as emphasised in Gali and Gertler (1999) the hybrid Phillips curve (Eq. 5) differs from the other specifications in two principal ways. Firstly, it employs the real marginal cost instead of the output gap as the forcing variable. Secondly, all the coefficients in this hybrid model are explicit functions of three model parameters: (i) ξ that measures the degree of price stickiness, (ii) d the degree of backwardness in price setting and (iii) f the discount factor.

CHAPTER 4

LINEAR SHORT-TERM INTEREST RATE MODELLING IN THE UNITED KINGDOM

4.1 Introduction

"Theory of interest rate determinants has been a weak spot in economics for a long time, and explanation and determination of the interest rate still give rise to more disagreement among economists than any other branch of economic theories." (Haberler, 1946, p. 195)

The purpose of this chapter is to model the short-term interest rate in the United Kingdom (UK). It uses quarterly data. The sample period begins in 1993:I when the Monetary Policy Committee (MPC) started to target inflation and ends in 2013:II.

The primary objective of the Bank of England (BOE) defined in Bank of England Act 1998 is to meet the inflation target which is measured as the twelve-month increase in the Consumer Price Index (CPI). Subject to that the BOE is expected to support economic policy of the UK government including its objective for growth and employment. As disclosed in another release on monetary policy trade-offs (MPC, August 2013), the MPC acknowledges that it considers a wide range of indicators to set the official short-term interest rate including the current inflation rate, the MPC's own inflation projection, the output gap, real GDP growth, the unemployment rate, etc. This results in the process of making interest rate decisions quite complicated and unobservable. This study aims to simplify the process and to describe the BOE's monetary policy.

Due to the inherent complexity in setting monetary policy, researchers have kept an ongoing interest in describing an interest rate reaction function with a simple reduced formula. In 1993, Taylor found that a very simple rule of only inflation and the output gap was able to describe the behaviour of the United States (US) Federal Reserve (Fed) between 1987-1992 very well. Following Taylor (1993), this linear equation has been widely used to explain monetary policies in many countries. However, there is still a continuous debate on how to empirically model monetary policy.

This chapter uses the longest time series to date of expected inflation data to evaluate the BOE's forward-looking behaviour over a 20-year period. As Barnett, Groen and Mumtaz (2010) conclude, the work which has been done on this topic generally focuses on the US economy. For example, Kim, Osborn and Sensier (2005) and Melosi (2012) employ surveys of inflation expectations from surveys of professional forecasts; Leduc, Sill and Stark (2007) and Malik and Banerjee (2013) use surveys such as the Livingston Survey for the US. For other countries, Hock and Zimmerman (2005) use a survey on a group of professional economists to get a measure of inflation expectations for Switzerland and Cristadoro and Veronese (2011) employ a survey of professional forecasters to obtain data on expected inflation for India. In the UK, Barnett et al. (2010) use the MPC's projected inflation. However, the sample period of Barnett et al. (2010) lasts from 1993 to 2007 which may not be sufficiently long enough to contain enough variation in the inflation rate to identify slope parameters.

Furthermore, this chapter introduces an optimal measure of the output gap. The Taylor (1993) rule suggests that the short-term interest rate can be modelled well by targeting both inflation and output biases (deviation of inflation and output from their target/long-run values). Although the BOE explicitly defines the inflation rate as twelve-month increases in the CPI, it does not have an explicit measure for the economic output gap. In the 'Monetary Policy Trade-offs and Forward Guidance' (henceforth, Guidance) of the MPC (August 2013), the committee judges that it would be better to use real activity measures (such as real GDP) rather than nominal indicators (such as nominal GDP) to assess economic activity. For the MPC, three principal real activity indicators are available (deviation of real GDP from its long-run trend, growth of real GDP and the unemployment rate). However, there is no single economic activity indicator that is able to provide all information regarding economic development. This is the motivation behind this study to create a composite total output index (TOI) as a better measure of the output gap for the UK.

To investigate the BOE's reaction to variability in the state of the UK financial system, this chapter uses the financial conditions index (FCI) estimated in Chapter 2. As mentioned in that chapter, this index is created by combining a dynamic model averaging method and a time-varying parameter factor-augmented vector autoregressive with stochastic volatility (DMA-TVP-FAVAR) model. The FCI to be used in this study optimally summarises the financial information in the UK.

The remainder of this chapter is organised as follows: Section 4.2 reviews the literature on monetary policy, Section 4.3 discusses data issues, Section 4.4 estimates an optimal TOI, Section 4.5 examines the BOE's interest rate reaction function and Section 4.6 concludes.

4.2 Literature Review

"*The interest on money is regarded as the premium on the current cash over deferred cash*." (Roos and Szeliski, 1942, p.501). In general, current theories of interest rate determinants take two forms, the 'pure' theory and the 'monetary' theory of interest.

The pure theory regards the interest rate as the price of capital (see, Keynes, 1937). In this framework, the interest rate is determined by demand and supply for loanable funds. The initial pure theory requires that there should be a state of definite and fixed expectation and that there should be a state of full employment. Keynes (1937) argues that this assumption is too restrictive in reality. Hence he modifies the classical pure theory by introducing expectations so as to make it more realistic. This 'realistic' pure theory takes into account factors that may affect expectations.

The monetary theory is well expounded by Wicksell (1898), Taylor (1993), etc. In 1898, Wicksell proposed to distinguish between the actual market rate (as affected by monetary policy) and the natural rate of interest. He defines the latter as the rate at which the demand for loan capital equals the supply of savings. According to Wicksell (1898), central banks try to move the market rate to the natural rate of interest to achieve price stability. In 1993, Taylor established a linear algebraic rule for the interest rate which not only links the actual short-term interest rate to the natural rate but also to inflation and output biases. The role of expectations is also mentioned in the monetary theory. Clarida, Gali and Gertler (1998, 2000) modify the initial Taylor rule and link the interest rate to expected inflation and output stabilisation.

This study emphasises the monetary theory of interest. In the literature review section, it discusses the estimation of an interest rate reaction function in detail. Section 4.2.1 presents a short discussion on the 'rules versus discretion' debate in monetary policy. Section 4.2.2 reviews the development of monetary policy rules which is a starting point for the latter econometric analysis. Section 4.2.3 considers data and measurement issues. Section 4.2.4 discusses linear estimates of a Taylor rule in the literature. Section 4.2.5 includes the literature on an augmented Taylor rule (for an FCI).

4.2.1 The Discretion versus Policy Rules Debate

Simons (1936) first raises the choice between rules and discretion while setting monetary policy. A discretionary policy means that a central bank's future actions are not restricted. Under discretion a central bank is free to act in accordance with its own judgement. By contrast a policy rule involves exercise of control over a central bank in a way which restricts its actions. As discussed by Gerald and Dwyer (1993), policy rules usually limit discretion but do not eliminate it.

Simons (1936, p.3) emphasises the value of a law instead of reliance on an authority's discretion, because "*definite, stable, legislative rules of the game as to money are of paramount importance to the survival of a system based on freedom of enterprise*".

Following Simons (1936), macroeconomists have presented a vast discussion on this issue. Some believe that economic growth can be enhanced and price stability can be achieved by implementing a monetary rule. Rules are argued to reduce monetary policy mistakes, to improve transparency of monetary policy and to end political influence on central banks. Others including Laidler (1991) and Friedman and Kuttner (1996) believe that monetary policy can work fine without rules. Friedman and Kuttner even cite attempts to implement rules in the past that have failed to create a reliable policy.

Kydland and Prescott (1977), Barro and Gordon (1983), Blanchard and Fischer (1989) and Taylor (1993) criticise discretionary policy because they are made on a period-by-period basis without considering any connection between the policy choices made over time. As McCallum (1989, ch. 12) maintains, central banks should consider the cumulative consequences of their policies which may be destabilising for an economy, in his mind, a discretionary policy does not foster such prudence. Hence, Lear (2000) concludes that a crucial advantage of decision making based on rules is that it views policy not just as a sequence of unrelated decisions but as a way to achieve optimal outcomes by following a consistent regime over a long time period.

The insight of Kydland and Prescott (1977) is that discretion entails costs of economic instability and excessive inflation as a result of activist monetary policy. In addition, Barro and Gordon (1983) stress that a rule that commits a central bank to a non-activist and non-discretionary policy will make economic activity more stable by creating a more credible and certain policy aimed at price stability. Summarising the views in Kydland and Prescott (1977), Barro and Gordon (1983) and Blanchard and Fischer (1989), Taylor (1993) concludes that rules have major advantages over discretion in improving economic performance. Given the above conclusions, monetary studies have derived several simple rules to guide monetary policy, to avoid inflation bias and political pressures and to achieve the stabilisation of inflation and economic growth. The most famous rules include, but are not limited to, the K-percent rule and the Taylor rule.

4.2.2 The Development of Monetary Policy Rules

Over the last couple of decades, economic researchers have made a great number of proposals for monetary policy rules. A large number of studies (for instance, Simon, 1936; Cooper and Fischer, 1972; McCallum, 1988, 1993) have investigated relative advantages and disadvantages of alternatives. Simons (1936) argues that some proposals are not operational as they fail to distinguish the objective of monetary policy with its instrument. In other words, the concepts involved are not under the control of central banks. According to Simons (1936), an operational policy rule should be simple and transparent to communicate and implement which requires an appropriate choice of policy instruments (also see, Orphanides, 2007) like the short-term interest rate that is under the control of central banks. This section discusses different monetary rules with various instruments and objectives. It begins with the K-percent rule (also called the K-rule) and then moves on to the development of the Taylor rule.

4.2.2.1 The K-percent Rule

In 1983, Friedman proposed using money supply as the policy instrument. His K-percent rule is derived from the quantity theory of money. The underlying consideration in the rule is that central banks choose a constant growth rate of money supply (k) to correspond to the sum of the inflation target (π^*) and an economy's potential growth rate (Δy^*) and adjust for any trend in the velocity of money (Δv^*):

$$\Delta m = \pi^* + \Delta y^* - \Delta v^* \tag{2.1}$$

where m denotes money stock in transaction. The velocity of money is expressed as the average number of times per year each unit of money is used to buy goods and services.

McCallum (1988, 1993) modifies Eq. (2.1) by allowing for some automatic response of money growth to economic growth:

$$\Delta m = g_{ny}^* - \Delta v^* - \theta_{g_{ny}} (g_{ny} - g_{ny}^*) \tag{2.2}$$

where $g_{ny} = \pi + \Delta y$ and $g_{ny}^* = \pi^* + \Delta y^*$. The terms g_{ny} and g_{ny}^* denote economic growth of nominal income and potential nominal income respectively.

An important advantage of using the K-percent rule (or its extensions) is that monetary authorities are able to implement it without much information. If the money velocity has a cyclical characteristic, the only required element for calibrating this rule is the potential growth rate of output. The techniques of estimating potential output will be discussed later. However, Orphanides (2007) argues that potential instability in money demand caused by either a temporary disturbance or persistent changes resulting from financial innovation complicates the use of the money supply as a policy instrument. After examining the existing models, Taylor (1993) also concludes that there is a consensus that policy rules that focus on money supply do not deliver as good a performance as policies focusing on price levels and real output. The above statement partially explains why the K-percent rule is rarely used in practice and also why some monetary authorities (such as the Fed, the BOE and the European Central Bank) prefer to use interest rate instruments when making monetary policy decisions.

4.2.2.2 The Wicksell Rule

Prior to Friedman's K-percent rule, Wicksell (1898) had proposed a rule formulated using an interest rate instrument. He argues that central banks are expected to maintain price stability, which can be achieved by pegging the short-term interest rate at an economy's natural rate of interest i^* (also called equilibrium interest rate). The term i^* is stated as:

$$i^* = \bar{r} + \pi^* \tag{2.3}$$

where $\bar{r}$ denotes real equilibrium interest rate (ERR). This rule does not require central banks to identify their ERRs. As in Wicksell (1898), a simple law that responds to prices is sufficient to achieve satisfactory stability:

"If prices rise, the rate of interest is to be raised; and if prices fall, the rate of interest is to be lowered; the interest rate is henceforth to be maintained at its new level until a further movement of prices calls for a further change in one direction or the other." (Wicksell, 1898, p. 189)

In algebraic terms, the Wicksell (1898) rule is expressed as:

$$i = \delta\pi \tag{2.4}$$

where i represents an economy's nominal short-term interest rate. However, this rule failed to draw much attention in monetary policy discussions. According to Taylor (1993), Bryant, Hooper and Mann (1993) and Orphanides (2007), this may be attributed to the fact that this rule concentrates too much on price stability while lacking explicit reference to real economic activity.

4.2.2.3 The Taylor Rule and Its Extensions

As is now well known, the landmark paper by Taylor (1993) specifies how the Fed in the US adjusts its nominal interest rate (i) to its current inflation (π, as measured over the last four quarters) and the output gap (q):

$$i = \pi + 0.5q + 0.5(\pi - 2) + 2 \tag{2.5}$$

where the output gap (q, also called output bias) is measured by the percentage deviation of real output (y) from a target (known as the potential output, y^*). This rule has the feature that the nominal federal interest rate (i) rises if the inflation rate (π) rises above the target of 2% and/or real output (y) is above its potential level (y^*). If both inflation and real output remain at their target levels, the real federal rate would equal 2%. According to Taylor (1993), the parameters specified in this rule have well explained the Fed's behaviour between the late 1980s and early 1990s. The Taylor (1993) finding is consistent with the earlier result in Bryant et al. (1993) who examine policy rules that set deviations of the short-term nominal interest rate (i) from its baseline path (i^*) in proportion to the deviation of target variables (z) from their targets (z^*):

$$i - i^* = \gamma(z - z^*) \tag{2.6}$$

This equation nests the Wicksell (1898) rule and the Taylor (1993) rule as two special cases. Across the alternatives[1], the rule that targets both inflation and real output biases yields the most satisfactory economic performance (Bryant et al, 1993):

$$i - i^* = \gamma_\pi(\pi - \pi^*) + \gamma_y(y - y^*) \tag{2.7}$$

Hence, by linking interest rate adjustment directly to the inflation rate and the output gap, the Taylor rule provides a convenient and effective tool for studying monetary policy. Following this lead, the Taylor (1993) rule has been augmented by many others (see, for instance, Clarida et al., 1998, 2000; Castro, 2011) by allowing for both forward-looking behaviour and interest rate smoothing.

Since central banks (like the BOE) acknowledge that they attempt to anchor their inflation expectations (rather than current inflation) to their inflation targets, Clarida et al. (1998, 2000) propose using a forward-looking version of the Taylor rule. The forward-looking rule allows a central bank to consider a broad array of information (not just contemporaneous or past values of inflation and the output gap) to form beliefs about the future condition of an economy. According to Clarida et al. (2000), this feature is highly realistic:

[1] Taylor (1993, p.200): *all the policy rules evaluated in the Bryant et al.'s (1993) comparison are interest rate rules. The monetary authority is assumed to adjust its interest rate in response either to (i) deviation of the money supply from the target, (ii) deviations of the exchange rate from the target, or (iii) weighted deviations of the inflation rate and real output from the target.*

$$i - i^* = \gamma_\pi (E(\pi) - \pi^*) + \gamma_y E(y - y^*) \quad (2.8)$$

where E(.) denotes expectations. Subsequent studies such as Fourcans and Vranceanu (2004), Sauer and Sturm (2007) and Castro (2011) also highlight the importance of using a forward-looking Taylor rule in monetary policy analysis. Castro (2011) compares the performance of a simple Taylor rule (i.e., an interest rate reaction function of only past inflation and economic activity) with a forward-looking specification. His results indicate that the simple Taylor rule using past data cannot capture the reaction of the European Central Bank (ECB) to the inflation rate. In the UK, the BOE behaves in a forward-looking manner.

In addition to adding expectations, Clarida et al. (1998, 2000) further adjust Eq. (2.8) by considering interest rate smoothing. They argue that Eq. (2.8) assumes an immediate adjustment of the interest rate to its desired level which is still too restrictive to explain changes in the interest rate. Interest rate smoothing is added to control for the observed autocorrelation in the interest rate:

$$i_t = \left(1 - \sum_{j=1}^{n} \rho_j\right)\left[i^* + \gamma_\pi (E(\pi) - \pi^*) + \gamma_y E(y - y^*)\right] + \sum_{j=1}^{n} \rho_j \, i_{t-j} \quad (2.9)$$

where the sum of ρ_j captures the degree of interest rate smoothing and j represents the number of lags. Several theoretical justifications are advanced in the literature for the inclusion of interest rate smoothing in the Taylor rule such as the fear of disruption in financial markets (Goodfriend, 1991) and uncertainty about the effects of interest rate changes (Sack, 1998).

4.2.3 Data and Measurement Issues

As discussed in Section 4.2.2, the original Taylor rule (in Eq. 2.5) assumes that central banks set interest rates based on current or historical inflation and the output gap. However, Clarida et al. (1998, 2000) argue that they are more likely to consider all information and make monetary policy decisions on the basis of expectations. This statement leads to a discussion regarding how to appropriately measure inflation expectations and the output gap before estimating an interest rate reaction function.

4.2.3.1 The Inflation Expectation Measures

One common method in the monetary policy literature to calibrate expected inflation is to substitute actual values for unobserved ones, i.e., a 'substitution' approach:

$$E_t(\pi_{t+k}) = \pi_{t+k} = (p_{t+j}/p_{t+j-4}) \tag{2.10}$$

This is done in Orphanides (2001, 2003), Clarida et al. (1998, 2000), Castro (2011), etc. The k subscripts denote the number of periods ahead. This substitution method does make sense when central banks predict future information rationally. However, in practice actual inflation does not always reflect the prior expectations, especially when some unexpected disturbance occurs which is not known by central banks at the time of monetary policy setting. An important contribution of this study is to explicitly use measures of inflation expectations in estimating an interest rate reaction function for the UK. It uses the longest time series data of expected inflation that is published by the MPC to evaluate the BOE forward-looking behaviour over a 20-year period. The inflation projection data is obtained under the assumption that the policy interest rate is maintained unchanged within the forecasting horizon.

In the Monetary Policy Trade-offs and Forward Guidance (MPC, August 2013), the MPC also compares the appropriateness of various potential price indicators:

An indicator based on current inflation performs well from a data availability view, as CPI data is published monthly and CPI inflation is widely reported in the media and official press. However, it has two significant drawbacks: (i) CPI inflation can be affected by cost shocks, (ii) monetary policy takes time to impact the UK economy thus in determining policy what matters is not the current or historical inflation rate but the outlook for inflation around two years ahead – the MPC estimates that it may take roughly two years for official interest rate decisions have their fullest effect on inflation. In 2011, Cogley, Paoli, Matthes, Nikolov and Yates provide a Bayesian analysis for the optimal monetary policy in the UK. They (Cogley et al., 2011) discover that Bayesian policy has characteristics suitable for inflation and output stabilisation in forward-looking models.

The MPC's inflation forecasting report has been published quarterly since February 1993 in the form of charts showing central projections. Using the committee's own projection for inflation has clear advantages over current inflation, because it represents the central bank's best collective judgement of expected inflation and it would be straightforward for the BOE to look through factors temporarily buffeting inflation. Although other measures of inflation expectations have indicated a viewpoint on future prospects for inflation as well, the market-based indicators of inflation expectations typically reference the Retail Price Index (RPI) measure of inflation rather than CPI measure of inflation. Surveys of professional forecasters are based on a relatively small sample size, which may not give a good indication of inflation expectations across a wider economy.

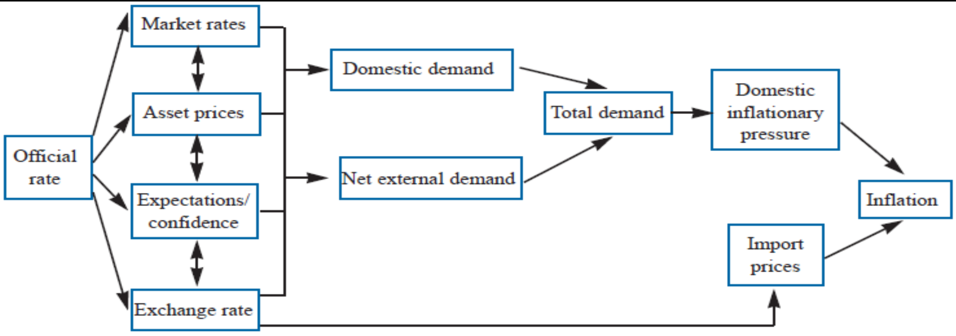
Therefore, this study chooses the committee's central projection for inflation as the expected inflation rate in the econometric estimation. To measure the current inflation rate, this study follows the official statistics and uses the CPI inflation for the period from 1996 to 2013. Since the official CPI only started in 1996 and historical estimates back to 1988 were calculated by the Office for National Statistics (ONS) based on archived RPI data, this study uses the RPI inflation rate for 1993-1996 (this has now become the norm for the UK studies; see, for instance, Castro, 2011).

4.2.3.2 The Output Gap Measures

Unlike the inflation rate in the UK which has a clear definition as being the 12-month increase in the CPI, output measures are not explicitly defined in the Monetary Policy Trade-offs and Forward Guidance of the MPC (August 2013). What is clarified in its press is that the real activity indicators are favoured by the committee: "*it would be better to employ separate indicators for price stability and real activity, rather than combining the two by using either nominal GDP growth or the nominal GDP shortfall.*" (MPC, August 2013, p. 33)

For real activity, the MPC judges that three important indicators are available: (i) the difference between real GDP and its potential level, (ii) the growth rate of real GDP and (iii) the unemployment rate (see, MPC, August 2013). Although the committee believes that among these three variables the unemployment rate is the most suitable indicator, the rate of unemployment is more likely to be considered as a threshold. As acknowledged by the MPC in August 2013, if the 6.5% threshold condition were met it will not result in an immediate change in the BOE's policy interest rate. Instead it will re-assess whether or not to raise its policy rate in light of its assessment of economic growth, which indicates that additional macroeconomic indicators are required. Given the difficulty in selecting output measures for estimating the Taylor rule, this study constructs a composite index to summarise all information across different output measures. The Total Output Index (TOI) helps to simplify the process of setting monetary policy and make a central bank's decisions more transparent and communicable.

Figure 1: The Transmission Mechanism of Monetary Policy



Source: The Monetary Policy Committee (June 2012, p.3)

To understand the criteria of an optimal output measure, this study draws on the MPC's view of the transmission mechanism of monetary policy that is displayed in Figure 1. As the first stage, official interest rate decisions affect market interest rates including the mortgage rate and the bank deposit rate to varying degrees. Meanwhile, policy actions also influence expectations about the future condition of the UK economy, confidence of consumers and investors, the exchange rate and prices of assets. In the second round, these changes affect spending, saving and investment behaviour of individuals and firms. As explained by the MPC (June 2012), individuals are affected by changes in monetary policy in three ways. Firstly, they face new interest rates on their savings and debt, so their disposable incomes alter. Secondly, their financial wealth changes due to the changes in asset prices. Thirdly, changes in the foreign exchange rate alter the relative prices of services and goods priced in the domestic currency. In addition to individuals, firms suffer a shift in the market interest rate, the exchange rate and asset prices as well. For example, an increase in the official interest rate has a direct impact on all firms that rely on loans linked to the short-term money-market interest rate, which in turn reduces the profits of such firms.

A fall in asset prices will reduce the net worth of a firm, which makes it harder to borrow. Appreciation of sterling in the foreign exchange market is expected to worsen the competitive position of UK based firms, which generates lower sales and profit margins. In summary, changes in the financial positions of individuals and firms are expected to result in changes in their spending, savings and investment plans. This may result in a change in aggregate demand.

The third-round effect is the most important part in this study. As Figure 1 illustrates, the demand relative to the domestic supply capacity – in the labour market and elsewhere – is a key influence on domestic inflationary pressure. When economic output is at its potential, production levels do not place upward or downward pressures on the prices in goods markets and employment levels will not exert impacts on unit labour costs. However, when a positive output gap appears firms are working above their normal-capacity levels. This is likely to create domestic inflationary pressures. For some firms, their unit labour costs rise as a result of the demand for labour. Therefore, it is generally agreed within the MPC (June 2012, p. 9) that "*booms in the economy which take output above its potential level are usually followed by a pick-up of inflation*". When there is a negative output gap, the reverse is true.

This description is consistent with the underlying idea of a Phillips curve initiated by Phillips (1958). He proposes a hypothesis that when the demand for a commodity or service is high relatively to its supply, prices are expected to rise. Conversely, when the demand is lower than the supply, prices are expected to fall. His statistical evidence supports this hypothesis. Although the original Phillips curve focuses on the inverse relationship between wage changes and unemployment, Samuelson and Solow (1960) further modify it to explain the negative relationship between inflation and the unemployment rate. In the inflation literature such as King and Watson (1994), Gordon (1997) and Lown and Rich (1997), the Samuelson-Solow Phillips curve has been augmented to include lagged inflation rates to reflect the inflation inertia in reality. Instead of using the unemployment rate as an activity indicator, they use the output gap in their exercise:

$$\pi_t = \sum_{i=1}^{p} (\rho_{t,i}\pi_{t-i}) + \sum_{j=1}^{q} (\rho_{t,j}y_{t-j}^{f}) + \eta_t \tag{2.11}$$

where π_{t-i} is the i-period lags of inflation rates and y_{t-j}^{f} is a real activity indicator. Carlin and Soskice (2006, p. 74) refer to the 'new' Phillips curve and explain that the deviation of output from its medium-run equilibrium (i.e., y_{t-j}^{f}) should be thought of in percentage terms. If output is above its medium-run level, the y_{t-j}^{f} is positive and this will raise inflation (π_t) above its last period's level. If output is below its medium-term level, the y_{t-j}^{f} becomes negative and this will decrease inflation (π_t). Only when $y_{t-j}^{f} = 0$ is inflation constant at last period's rate.

Considering the above statement from both the MPC and the Phillips curve literature, this study establishes a criteria regarding the optimal measure of an output gap: it should contain as much information about the future rate of inflation as possible. In other words, it is the real activity indicator that predicts inflation most accurately.

Finally, the movement of the foreign exchange rate has a significant effect on inflation as well. This is called imported inflation. It arises as the exchange rate alters the sterling prices of imported goods and services, which may be considered as important determinants of many firms' costs and of the retail prices of many goods and services. As illustrated in the MPC's report on its monetary transmission mechanism (June 2012), the effect of exchange rate changes on the domestic inflation is direct in the UK. An appreciation of sterling lowers inflation, while a depreciation raises it. Hence, to fully capture the information contained in output measures this study adds the exchange rate (e_t) to a conventional Phillips curve. This means that the term y_{t-1}^{f} is expected to explain the variation in future inflation (π_t) in addition to the proportion that has been explained by its own lags and imported inflation:

$$\pi_t = \sum_{i=1}^{p} (\rho_{t,i}\pi_{t-i}) + \sum_{j=1}^{q} (\delta_{t,j}y_{t-j}^{f}) + \sum_{l=1}^{h} (\lambda_{t,l}e_{t-l}) + \eta_t \tag{2.12}$$

In Stock and Watson (1999), this augmented equation is called a generalised Phillips curve. Stock and Watson (1999) investigate the forecasts of the US inflation and find that the forecasts can be improved using such a generalised function based on a number of indicators including six exchange rate measures. The main deficit of Stock and Watson (1999) is that their study is purely data-driven. In other words, they do not have sufficient economic rationale for including additional variables such as stock prices, money and credit quantity aggregates and the interest rate. As illustrated by the MPC (June 2012) and also in Chapter 1-2, financial conditions affect real economic output in the first round and then shift inflation through aggregate demand. Therefore, it takes different time lags for financial variables and output to have their impact on inflation. Including financial and real activity indicators contemporaneously to model the inflation rate as in Stock and Watson (1999) and Koop and Korobilis (2014), does not have sufficient economic foundation.

Unlike Stock and Watson (1999), this study only incorporates the real effective exchange rate index to account for the imported inflation that is noted in the MPC's press.

Given the belief that the real output indicator has already incorporated information from the past financial markets, this study excludes stock market variables, the interest rate and quantity indicators in a generalised Phillips curve. It extracts the co-movement from a group of real output measures. The real output measures used to construct a TOI are discussed in the data section 4.3. The estimated common factor is expected to best predict the variation in inflation. Although this is a new topic in the literature, Koop and Korobilis (2014) have already developed a statistical model, the DMA-TVP-FAVAR model, summarising the valuable financial information for forecasting output. Chapter 1-2 applies this technique to the UK financial market and discovers that it outperforms other existing approaches for extracting factors from candidate variables. Therefore, this study opts to use the DMA-TVP-FAVAR model in the econometric exercise to create the best output measure. The reader is referred to Chapter 1-2 for a detailed discussion of this model.

4.2.3.3 The *Ex-post* versus Real-time Data

Another note regarding the data refers to the kind of data used in measuring variables. Most existing studies (e.g., Clarida et al., 2000) on central bank behaviour use *ex-post* revised data (also called, *ex-post* data) i.e., the data published in the latest release to estimate monetary policy rules. An instrumental variable estimation procedure or a generalised method of moment (GMM) is employed, since future economic variables used as regressors are correlated with disturbance terms.

This traditional method to estimate the rules is criticised by Orphanides (2001) who assesses whether the evaluation of the Fed's policy rules with revised data could be inappropriate. The results suggest that in the US the estimated policy rules based on *ex-post* data may provide misleading description of the Fed's monetary policy and obscure the behaviour suggested by information available to the Fed in real time. Orphanides (2001) recommends using real-time data which is the data published at the time when the Fed makes monetary decisions.

However, the use of real-time data as in Orphanides (2001) also has its limitations. As in Kim and Nelson (2006), if the real-time forecasts in Orphanides (2001) were made under the assumption that the nominal Fed rate would remain constant within the time horizon of forecasting there would be no endogeneity in the regression. If not, they would induce an endogeneity problem in monetary policy rule estimation. At an empirical level, Osterholm (2005) investigates the US economy and discovers that real-time data generates only minor difference to *ex-post* data. For the Eurozone, Adema (2004), Sauer and Strum (2007) and Castro (2011) show that the use of real-time data for the Eurozone, instead of *ex-post* data, does not lead to substantially different results. In addition to the endogeneity, another problem arising from using real-time data is that real-time data is always difficult to access as very few records of the real output gap can be found to produce such a time series. Given the reasons in Kim and Nelson (2006) and the results from Adema (2004), Osterholm (2005), Sauer and Sturm (2007) and Castro (2011), this study opts to use *ex-post* data for the current inflation rate, output measures and the interest rate. Since the MPC's inflation forecasts are made under the assumption that the bank rate will remain the same in the forecasting horizon, the endogeneity in the regression is avoided and the real time inflation forecasts are used in the econometric estimation.

4.2.4 The Linear Estimates of a Taylor Rule

Table 1: Summary: Linear Estimation of Taylor rule

Method	Author	Year	Countries	Version
	Clarida et al.	1998	the US	Forward-looking
	Mehra	1999	the US	Forward-looking
GMM	Clarida et al.	2000	the US, the UK, the Eurozone	Forward-looking
	Chadha et al.	2004	the US, the UK, Japan	Forward-looking
	Castro	2011	the US, the UK, the Eurozone	Forward-looking
	Rudebusch	1998	US	Backward-looking
VAR	Neumann and Hagen	2002	Inflation targeting countries	Backward-looking
	Hsing	2004	Japan	Backward-looking

Several empirical studies into Taylor rules estimate a linear function using the policy interest rate to assess whether the inflation-stabilising Taylor principle has been met. This section provides a full review of the literature in this field emphasising some contributions that motivate the analysis presented in this study. The summary of the linear estimation of a Taylor rule is reported in Table 1 in order to have a review of the econometric methods used in the existing studies.

4.2.4.1 The Generalised Method of Moments Estimator

The GMM estimator is widely used in the literature (see, for instance, Clarida et al., 1998, 2000; Mehra, 1999; Chadha et al., 2004; Castro, 2011) for estimating a forward-looking Taylor rule with either *ex-post* or real-time data. Clarida et al. (1998, 2000) highlight the primary reason for using the GMM: it works well when the regression of the policy interest rate is made on variables which are unknown by central banks at decision making moments. This is also relevant to this study. Although this study uses the MPC's inflation forecasts as the expected inflation rate, the expected output gap and the natural rate of interest are still unknown when the BOE sets the interest rate.

To illustrate how the GMM estimator works, re-state Eq. (2.9) as:

$$i_t = \left(1 - \sum_{j=1}^{n} \rho_j\right) \left[i^* + \gamma_\pi (E_t(\pi_{t+k}) - \pi^*) + \gamma_y E_t(y_{t+s} - y^*_{t+s})\right] + \sum_{j=1}^{n} \rho_j \, i_{t-j} \tag{2.13}$$

where π_{t+k} refers to the k-period ahead inflation rate and y_{t+s} is the s-period ahead real output. Simplify Eq. (2.13) as:

$$i_t = \varphi_0 + \varphi_\pi E_t(\pi_{t+k}) + \varphi_y E_t(\bar{y}_{t+s}) + \sum_{j=1}^{n} \rho_j \, i_{t-j} + \mu_t \tag{2.14}$$

where $\bar{y}_{t+s}$ is the s-period ahead output gap and μ_t is an independent and identically distributed (IID) stochastic error term. That is:

$$\varphi_0 = \left(1 - \sum_{j=1}^{n} \rho_j\right)(\bar{r} + \pi^*) - \left(1 - \sum_{j=1}^{n} \rho_j\right)\gamma_\pi \pi^* \tag{2.15}$$

$$\varphi_\pi = \left(1 - \sum_{j=1}^{n} \rho_j\right)\gamma_\pi \tag{2.16}$$

$$\varphi_y = \left(1 - \sum_{j=1}^{n} \rho_j\right)\gamma_y \tag{2.17}$$

Using the 'substitution' method yields:

$$i_t = \varphi_0 + \varphi_\pi \pi_{t+k} + \varphi_y \bar{y}_{t+s} + \sum_{j=1}^{n} \rho_j \, i_{t-j} + \varsigma_t \tag{2.18}$$

where the error term ς_t now becomes a linear combination of forecasting errors of inflation, output and the disturbance μ_t.

To implement the GMM estimator, the following orthogonality condition is imposed:

$$E[\varsigma_t | v_t] = 0 \tag{2.19}$$

where v_t is a vector of variables within a central bank's information set at the time it chooses the interest rate that is orthogonal to ς_t. Clarida et al. (1998, 2000) use a set of lagged variables which helps to predict inflation and the output gap together with other contemporary variables which are not correlated to the current disturbance μ_t.

Another advantage of using GMM is that it is a robust estimator. Unlike the standard ordinary least squares (OLS) and maximum likelihood estimation (MLE), it does not require information about the exact distribution of the disturbance. It only requires a set of instrument variables contained in v_t which the central bank will probably use at the decision making moment. Considering that the dimension of the instrument vector v_t exceeds the number of parameters being estimated, over identifying restrictions must be tested in order to assess the validity of the specification and the instruments used. In that context, Hansen's (1982) over identification test is employed in previous studies such as Clarida et al. (1998, 2000) and Castro (2011). Under the null hypothesis that the instrument set is considered valid, the rejection of orthogonality suggests that the central bank does not adjust its policy to the information contained in the instrument variables.

Castro (2011) investigates the Taylor rule in the US, the UK and the Eurozone and discovers that a basic Taylor rule (backward-looking without interest rate smoothing) produces quite good results for the US but not for the UK. As in Castro (2011), the BOE's policy rate is better modelled using a forward-looking Taylor rule with smoothing behaviour. In the Eurozone the ECB's monetary policy cannot be characterised by a simple Taylor rule, but it can be described by a rule that takes future expectations (beside the past and current information) into account.

A drawback in the GMM studies is that they always assume a time-invariant equilibrium real interest rate ($\bar{r}$) that equals to its sample average. Then, the inflation target (π^*) is calculated based on the $\bar{r}$ and the parameters, φ_0 and φ_π in Eq. (2.18). Based on Eq. (2.15) and Eq. (2.16), π^* is given as:

$$\pi^* = \frac{\varphi_0 - \bar{r}\left(1 - \sum_{j=1}^{n} \rho_j\right)}{1 - \sum_{j=1}^{n} \rho_j - \varphi_\pi} \tag{2.20}$$

which is assumed to be constant over the period examined. This means that the GMM estimator does not consider any possible movement in π^*.

4.2.4.2 The Vector Autoregressive Estimator

The vector autoregressive (VAR) model is a system of linear equations, one for each variable. In the structural form, each equation specifies one of the variables as a linear function of its own lags as well as current and past values of the other variables in the system. Hence, it has the advantage of (i) handling all the questions (regarding the response of the interest rate and the impact of monetary policy) at one time and (ii) providing useful information about aggregate relationships between variables.

Rudebusch (1998), Neumann and Hagen (2002) and Hsing (2004) consider VAR models for the backward-looking version of a Taylor rule in the US, inflation targeting (IT) countries and Japan respectively. Hsing (2004) applies a VAR model to examine the monetary policy reaction function for the Bank of Japan and discovers that the overnight call rate reacts positively to a shock to the output gap and inflation gap as expected, while the reaction of the interest rate to the inflation gap goes on longer than that of the interest rate to the output gap. In Hsing (2004), the inflation target is also time-invariant over the sample studied – the announced inflation target is used which assumes that the central bank has strictly followed its commitment of pegging inflation to the target of 2%.

4.2.5 The Augmented Taylor Rule

In all the aforementioned econometric estimations, the nominal interest rate is modelled to react to the stabilisation of inflation and the output gap. An important aspect emerging from the recent literature is the role played by FCIs in the monetary policy reaction function. As asserted in Castro (2011), an FCI may contain valuable information regarding financial health, as well as the information on future economic activity, thus he expects an increase in the interest rate when the FCI goes up and *vice versa*. Other studies such as Clarida et al. (1998), Chadha et al. (2004) and Montagnoli and Napolitano (2005) also provide evidence on central banks' response to financial indicators. Chapter 1 summarises the theoretical rationale for including indicators such as the real exchange rate, asset prices and spreads in the interest rate reaction function.

Chadha et al. (2004) stresses that the exchange rate is both a source of extraneous shocks and a mechanism for adjusting to a fundamental shock in an open economy. Thus they expect central banks to react to the movements of the exchange rate. Corsetti and Pesenti (2005) theoretically investigate the optimal monetary policies in open economies and discover that central banks should target the exchange rate. At an empirical level, studies such as Clarida et al. (1998), Chadha et al. (2004), Fourcans and Vranceanu (2004) and Lubik and Schorfheide (2007) provide findings that the exchange rate enters interest rate rules significantly. Chadha et al. (2004) use the GMM estimator and examine whether the parameter on the exchange rate is statistically significant using data for the US, the UK and Japan for the 1979-2000 period. Their results indicate that central banks may use the exchange rate not only as part of their information set for changing the interest rate, but also to set the interest rate to offset deviations of the exchange rate from the equilibrium level. Lubik and Schorfheide (2007) examine the Gali and Monacelli (2005) model and obtain a similar conclusion for the UK.

Goodhart and Hofmann (2002), Chadha et al. (2004) and Montagnoli and Napolitano (2005) believe it important for central banks to target asset prices. Montagnoli and Napolitano provide three reasons for monetary policy to respond to asset prices. Firstly, as argued by Borio and Lowe (2002), asset price misalignments may endanger the stability of financial markets. Secondly, they play a role in the monetary transmission – asset prices are closely associated with aggregate demand and inflationary pressures. Thirdly, they contain useful information about financial market expectations of macroeconomics. Therefore, as in Kontonikas and Montagnoli (2006), it is theoretically optimal for central banks to target asset prices. At an empirical level, Chadha et al. (2004) show that when committed to keeping inflation and output stabilised, the Fed and the BOE act in response to prices of assets at least on occasions when there is a need to prevent an abrupt correction in markets.

The theoretical evidence for the relationship between interest rates and spreads is given by Curdia and Woodford (2009) and Teranishi (2012). They show that a spread-adjusted Taylor rule can improve upon an unadjusted policy rule. Castro (2011) finds the responses of monetary policy to two type of spreads, (i) credit spreads and (ii) changes in futures interest rate spreads. Using the GMM estimator Castro (2011) confirms the finding of Driffill, Rotondi, Savona and Zazzara (2006) that the Fed reacts to information contained in the futures interest rate spreads and credit spreads. The empirical findings indicate that (i) the Fed works to lower the volatility of the difference between the future interest rate and the current rate and (ii) it reacts to the expected improvement of economic conditions and consequent inflationary pressures. However, Castro (2011) fails to find a significant response from the BOE to the variation in the futures interest rate spread.

The above studies justify the inclusion of an FCI in an augmented Taylor rule as in Castro (2011). In Chapter 1, this study explores the best weighting method for constructing an FCI in the UK. The criterion for choosing that optimal method focuses on the estimated FCIs' forecasting performance. Therefore the best FCI is defined as an index that predicts economic development as well as possible. Finally, Chapter 1 points to the factor model which allows for changes in loadings as the optimal variable weighting method. Following that chapter, Chapter 2 continues the development of the optimal FCI for the UK. With a set of 21 financial variables (which is wider than the coverage of almost all existing FCIs in the UK), Chapter 2 applies a joint model of dynamic model averaging and time-varying parameter factor-augmented VAR. It creates the optimal FCI which is interpreted as a measure of the deviation from the long-run trend of the UK financial system. The implication of incorporating such an index into an augmented Taylor rule is to target the deviation of asset prices from their trends.

4.3 Data

This study sources data from the BOE, the ONS and the OECD. The data used is quarterly. The sample period covers 1993:I-2013:II. During this time the MPC has been operating an inflation targeting approach and reporting its inflation forecasts on a quarterly basis. This study considers several different measures of the output gap, inflation and the interest rate. However, in the estimation it only chooses the ones that have been followed most closely by the BOE. A detailed description of the variables mentioned and their respective sources is presented in Appendices 1-3.

The alternative interest rate measures considered include the official central bank interest rate, the three-month inter-bank sterling lending rate and the discount rate of three-month Treasury bills. Nelson (2000) argues that the actual interest rates that have been used by the BOE have changed over time including the bank rate, the minimum lending rate, the two-week repo rate, etc. In addition to Nelson (2000), Martin and Milas (2004) and Castro (2011) also argue that the three-month treasury bills discount rate (Trea3m) has a closer relationship with all the interest rate instruments used in the Bank's history. Following the literature, this study uses the Trea3m as the nominal interest rate for the sample period analysed.

Following the BOE, the inflation rate (Infl0) is computed as the annual rate of changes in the CPI. However, the CPI statistics only started in 1996. The historical estimate of inflation back to 1988 is calculated by the ONS based on the RPI. Following the existing literature such as Castro (2011), this study calculates the inflation rate as the RPI for the period 1993-1996. To assess the forward-looking behaviour of the BOE, this study employs the MPC's forecasts for inflation (InflK) at different dates where $K = 1, \dots, 6$ denotes the expected inflation rate with K quarters ahead. The inflation projection is published in the form of charts showing the mean projection (i.e., central projection) together with the estimation of uncertainty based on the historical mean absolute error. As discussed in the Literature Section 4.2.3.1, this represents the MPC's best collective judgement of the outcome of inflation.

In order to ensure the stationarity of these variable, this study uses the inflation rate rather than the index:

$$\pi_t = ln\ (CPI_t) - ln\ (CPI_{t-4}) = ln\left(\frac{Infl0_t}{100} + 1\right) \tag{3.1}$$

$$\pi_{t+k}^f = ln\left(\frac{InflK_t}{100} + 1\right), k = 1, 2, \dots, 6 \tag{3.2}$$

One of the econometric estimations in this study focuses on creating an optimal measure of the output bias. Since the MPC (August 2013) judges that using a nominal indicator could be interpreted as changing its price stability objective and it is better to employ separate indicators for price stability and real activity, this study decides to transform all activity indicators used to their real terms. It considers eight variables to extract an output index (i.e., the TOI) summarising all information available.

The first three indicators are sourced from the Monetary Policy Trade-offs and Forward Guidance (MPC, August 2013) in which the committee acknowledges that they use the output gap, the real GDP growth rate and the unemployment rate to measure real economic development. The output gap which refers to the difference between real GDP and its potential level directly relates the interest rate to reducing the margin of slack in the economy. The major disadvantage associated with an output gap indicator lies in the estimation of the potential level of output that is the highest level of real GDP that can be sustained over the long run. Hodrick and Prescott (1997) maintain that potential GDP has a smoothly varying trend and that this trend could be well approximated by passing real GDP through the Hodrick-Prescott (HP) filter with a smoothness parameter (λ):

$$\sum_{t=1}^{T}(y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1}[(s_{t+1} - s_t) - (s_t - s_{t-1})]^2 \tag{3.3}$$

Technically the HP filter is a two-sided linear filter that estimates a smoothed series (s) of y by minimising the variance of y around s subject to a penalty which constrains the second difference of s. In other words, the HP filter chooses s to minimise the result of Eq. (3.3). The penalty parameter controls the smoothness of the series s_t. The larger the parameter (λ) is, the smoother the s_t will be. As λ approaches the infinity, s_t approaches a linear trend.

Alternatively, Clarida et al. (2000) use the potential GDP estimated from a fitted quadratic function of time assuming that the trend in GDP is deterministic as opposed to being stochastic. From a theoretic perspective, both the HP filter and the quadratic deterministic trend are variants of a detrended method that seeks to reduce the variability of a particular trend component. However, in practise the level of output is likely to behave differently in response to different real shocks such as shocks related to the changes in technology, productivity and consumer preferences. As argued in Chadha, Sarno and Valente (2004), the de-trended output may not capture those situations in reality. This explains why the MPC judges that the GDP gap (the percentage deviation of the real GDP from its HP-filter level as in Appendix 1-2) alone is not sufficient for making monetary decisions.

To overcome the disadvantages associated with measuring the potential level of real GDP, the MPC also considers the real GDP growth rate as the output measure. As illustrated by Orphanides and Williams (2002), when the GDP gap is uncertain it may be better to relate monetary policy to changes in the real GDP gap rather than the level. That is because there is less uncertainty about the changes in the GDP gap than its starting level. Furthermore, the real GDP growth rate performs well against the communication criterion, because it is widely reported in the press and the MPC is required to support the UK government's objective for economic growth. It is clearly specified in the Monetary Policy Trade-offs and Forward Guidance (MPC, August 2013) that the real GDP growth, set at a rate above its historical average, may have a particular powerful effect on the public's expectations about the economic outlook. The major difficulty in using real GDP growth is that the data for the growth rate of real GDP is prone to revision. For example, in the period since 1993 there are four occasions when the 4-quarter real GDP growth rate went from below 3% to above 3% in the preliminary release. Among these four occasions, the growth rate was revised twice within two quarters (see, MPC, August 2013, p.27).

The inherent shortcomings in the real GDP gap and its growth rate motivates the MPC to continue exploring additional output indicators. The third one it uses is the unemployment rate. Although the MPC judges that the unemployment rate is the more suitable output measure compared to the real GDP gap and its growth rate, the unemployment rate is usually specified as a threshold. If the threshold condition were met, the MPC has to re-assess its economic outlook with the inclusion of more indicators to decide whether to raise its short-term interest rate or not. This indicates that the MPC uses the unemployment rate jointly with the real GDP gap and/or its growth rate in practice.

However, the empirical work on monetary policy may use other variables including the real industrial production index, real labour productivity, etc. Since the data for GDP is not available on a monthly basis, Clarida et al. (1998) and Castro (2011) employ the index of industrial production as an economic activity measure. A series of Phillips curve studies (e.g., Gali and Gertler, 1999) argue that in light of difficulties in using the real GDP gap, the real margin cost from its steady-state value could be considered as an appropriate proxy for the output gap. The most common measure of the margin cost in the literature is represented by unit labour costs. Hence, this study employs the deflated index of unit labour cost as another real activity indicator. This is also the case in Chadha et al. (2004) whose Taylor rule estimation for the US, the UK and Japan consider the (wages adjusted) margin cost as an alternative output measure. In another study done by Ireland (2007), labour productivity is used as the measure of economic output. Since the variables weighting model to be used is purely data-driven and the DMA model is expected to decide variables which are more important, it is crucial to include as many relevant output indicators as possible. Therefore, this study draws on extensive readings and includes all activity indicators in the monetary literature (such as Clarida et al., 1998; Gali and Gertler, 1999; Chadha et al., 2004; Ireland, 2007; Castro, 2011). Then it uses the DMA-TVP-FAVAR model to weight each possible index and create the optimal TOI. In addition to the six variables mentioned above (real GDP level, real GDP growth, the unemployment rate, unit labour costs, the industrial production index and labour productivity), this study also considers gross value added and real household disposable income. A detailed description of the constituents is available in Appendix 1. This study uses the deviation of each output indicator from its equilibrium level in the estimation.[2]

Table 2 reports the results of unit root and stationary tests for the variables used in this study. Due to the low power and poor performance of unit root tests in small samples, this study follows the methodology used in Castro (2011). It reports the results of two unit root tests, i.e., augmented Dickey and Fuller (1979) test (ADF) and Phillips and Perron (1988) test (PP) to investigate whether test power is an issue. It also reports the Kwiatkowski, Phillips, Schmidt and Shin (1992) stationarity test (KPSS) results for robust purposes.

The test results displayed in Table 2 indicate that the power of unit root tests seems to be an issue for the UK. The ADF and the PP tests are unable to reject the unit root in Trea3m, π_t and Unem Gap. However, the KPSS test is able to provide evidence of stationarity for both π_t and Unem Gap. Although the evidence fails to support the stationarity hypothesis for Trea3m given the sample period, if this study were to consider a longer time period it would expect to find evidence of stationarity for Trea3m.

[2] Appendix 2 defines RGDP Gap, RGDPG Gap, Unem Gap, RIPX Gap, RGVA Gap, RULCX Gap, RHDI Gap and RLP Gap.

Table 2: Unit Root and Stationary Tests

	ADF	PP	KPSS
Trea3m	-1.1871	-0.8100	0.7837
π_t	-1.2727	-1.9433	0.4658#
π_{t+1}^f	-4.6545*	-3.0706*	0.2284#
π_{t+2}^f	-5.0216*	-3.3656*	0.1986#
π_{t+3}^f	-4.7972*	-3.2791*	0.2829#
π_{t+4}^f	-4.3440*	-3.5365*	0.5793#
π_{t+5}^f	-3.4263*	-3.1928*	0.8128
π_{t+6}^f	-3.1670*	-2.8703*	0.8940
RGDP Gap	-6.3198*	-3.8441*	0.0225#
RGDPG Gap	-4.3255*	-2.6538*	0.6028#
Unem Gap	-2.1377	-2.2749	0.3263#
RIPX Gap	-5.8423*	-4.0664*	0.0212#
RGVA Gap	-6.2740*	-3.7185*	0.0228#
RULCX Gap	-5.1488*	-4.5603*	0.0198#
RHDI Gap	-8.0641*	-8.1935*	0.0389#
RLP Gap	-6.1751*	-3.4547*	0.0243#

*Note: *: Unit root is rejected at a significance level of 10%; #: the stationarity is not rejected at a significance level of 1%; all the test regressions here contain a constant. RGDP denotes the real GDP; RGDPG denotes the growth rate of the real GDP; Unem denotes the unemployment rate; RIPX denotes the real industrial production index; RGVA is short for the real gross value added; RULCX denotes the real unit labour cost index; RHDI is the real household disposable income; RLP represents the real labour productivity. The reader is referred to Appendix 1-3 for a detailed illustration of the variables involved in this study.*

4.4 Estimate an Optimal Artificial Total Output Index

Following Koop and Korobilis (2014) and Chapter 1-2, this study re-writes the p-lagged time-varying parameter factor-augmented VAR (TVP-FAVAR) with stochastic volatility (SV) model as follows:

$$x_t = \lambda_t^f y_t^f + u_t, \quad u_t \sim N(0, V_t) \tag{4.1}$$

$$\begin{bmatrix} w_t \\ y_t^f \end{bmatrix} = c_t + B_{t,1} \begin{bmatrix} w_{t-1} \\ y_{t-1}^f \end{bmatrix} + \cdots + B_{t,p} \begin{bmatrix} w_{t-p} \\ y_{t-p}^f \end{bmatrix} + \varepsilon_t, \quad \varepsilon_t \sim N(0, Q_t) \tag{4.2}$$

where x_t is an $n \times 1$ vector of real activity indicators in estimating a TOI. In the empirical work $w_t = (\pi_t, e_t)'$ where π_t denotes the CPI inflation rate and e_t denotes the sterling effective exchange rate index. Both u_t and ε_t are zero-mean Gaussian errors with covariance V_t and Q_t. The term λ_t^f is a loading factor. $B_{t,1}, \ldots B_{t,p}$ are VAR parameters. This models differs from the traditional factor model (and also conventional principal component analysis, PCA) in that it allows for time-variation in both parameters and loadings. Negro and Otrok (2008) and Eickmeier, Lemke and Marcellino (2011) suggest a model where the factor loadings are set as random walks. Primiceri (2005) and Nakajima (2011a) also assume that VAR parameters follow a random walk process. Following these papers, this study sets λ_t^f and $B_{t,1}, \ldots B_{t,p}$ as:

$$\lambda_t^f = \lambda_{t-1}^f + v_t, \quad v_t \sim N(0, W_t) \tag{4.3}$$

$$\beta_t = \beta_{t-1} + \eta_t, \quad \eta_t \sim N(0, R_t) \tag{4.4}$$

where $\beta_t = \left(c_t', vec\left(B_{t,1}\right)', \ldots, vec\left(B_{t,p}\right)'\right)'$. Given the recommendation in Primiceri (2005) regarding heteroskedasticity, this study lets V_t and Q_t be time-variant. As in Primiceri (2005) and Koop and Korobilis (2014), the covariance matrix V_t is diagonal thus ensuring that u_t is a vector of idiosyncratic shocks.

As mentioned earlier, this study considers the DMA method proposed by Raftery, Karny and Ettler (2010) and applied in Koop and Korobilis (2014) and Chapter 2. A joint model of the DMA and TVP-FAVAR is developed by Koop and Korobilis (2014). It is written as:

$$x_t^{(l)} = \lambda_t^f y_t^{f\,(l)} + v_t \tag{4.5}$$

$$\begin{bmatrix} w_t \\ y_t^{f\,(l)} \end{bmatrix} = c_t + B_{t,1} \begin{bmatrix} w_{t-1} \\ y_{t-1}^{f\,(l)} \end{bmatrix} + \cdots + B_{t,p} \begin{bmatrix} w_{t-p} \\ y_{t-p}^{f\,(l)} \end{bmatrix} + \varepsilon_t \tag{4.6}$$

where quantities specific to the DMA-TVP-FAVAR model M_l is denoted by a superscript (l). Hence, $x_t^{(l)}$ is a subset of x_t and $y_t^{f\,(l)}$ is the TOI implied by M_l. With n activity indicators in the information set x_t, this study assesses $L\ (= 2^n - 1)$ models – it will remove from the model set the one with zero activity indicators. The DMA method produces the optimal TOI by averaging over different $y_t^{f\,(l)}$ (for $l = 1, 2, \ldots, L$) and the weight on each individual $y_t^{f\,(l)}$ is estimated with Eq. (4.7):

$$p_{t|t-1,l} = \frac{p_{t-1|t-1,l}^{\alpha}}{\sum_{j=1}^{L} p_{t-1|t-1,j}^{\alpha}} \quad (4.7)$$ [3]

The reader is referred to Chapter 1-2 or Koop and Korobilis (2014) for a full algorithm for calculating the DMA-TVP-FAVAR model.

In Eq. (4.7), the exponent α is a forgetting factor. The associated model updating equation is stated as:

$$p_{t|t,l} = \frac{p_{t|t-1,k} Pr_k(Data_t|Data_{1:t-1})}{\sum_{j=1}^{K} p_{t|t-1,j} \, Pr_j(Data_t|Data_{1:t-1})} \quad (4.8)$$

The predictive likelihood ($Pr_l(Data_t|Data_{1:t-1})$) represents the predictive density for model l evaluated at $Data_t$. It is considered as a measure of forecasting performance. As in Koop and Korobilis (2013), the calculation of $\pi_{t|t-1,l}$ and $\pi_{t|t,l}$ does not require using simulation methods (such as MCMC) and hence is very simple and fast. The factor $0 < \alpha \leq 1$ is a forgetting factor. Most existing DMA literature, including Raftery et al. (2010), Koop and Korobilis (2013, 2014) and Chapter 2, uses a benchmark value of $\alpha = 0.99$. This means that when using data on a quarterly basis the prediction three years ago has 88% as much weight as the forecast in the last quarter. This study follows the Chapter 2 and Koop and Korobilis (2014) and uses a forgetting factor value of 0.99 in the econometric estimation.

Table 3: Lag Length Selection

	Schwarz information criterion	Hannan-Quinn information criterion
VAR 1: (π_t, e_t, RGDP Gap)	2 lags	2 lags
VAR 2: (π_t, e_t, RGDPG Gap)	2 lags	2 lags
VAR 3: (π_t, e_t, Unem Gap)	2 lags	2 lags

Note: this table display the results of lag length selection using the Schwarz information criterion. It is computed as: $SIC = -\frac{2LK}{T} + \frac{n \log T}{T}$ *where LK denotes the log likelihood and n is the number of parameters using T observations. For a robustness purpose, it also includes the results based on the Hannan-Quinn information criterion. The notations in each bracket refer to the variables included in each VAR.*

Before proceeding with the forecasting exercise, it is important to highlight that the aim of this study is to construct an optimal composite index of the total output gap for

[3] Eq. (4.7) requires to set the initial condition for $p_{0|0,l}$. This study lets $p_{0|0,l} = 1/L$ which is also done in the prior DMA studies including Raftery et al. (2010) and Koop and Korobilis (2012, 2013, 2014).

the UK. The econometric estimation is based on the criterion mentioned in the literature review section 4.2.3.2. The optimal TOI is an output indicator that forecasts inflation as well as possible. The TVP-FAVAR model is used to weight variables and summarise information in a group of output variables. The DMA technique is taken to deal with index-constituent-selection issues. The sample period covers from 1993:I to 2013:II. The order of the DMA-TVP-FAVAR model is determined with the Schwarz information criterion (SIC) applied to different constant-parameter VARs. Table 3 presents the results. Since a TOI is not available at the moment of choosing lag length, this study uses the RGDP Gap, RGDPG Gap and Unem Gap to represent the real activity in the UK. As already mentioned, they are all important indicators according to Monetary Policy Trade-offs and Forward Guidance (MPC, August 2013). For the purposes of robustness, this study uses the Hannan-Quinn (HQ) information criterion to examine the lag selection results. Both the SIC and the HQ tests show that it is optimal to include two lags (i.e., $p = 2$ in the system of Eq. 4.5-4.6).

It is important to notice that while using multiple TVP-FAVAR models there is a risk that in a specific period the TOI has no value, i.e., the weight assigned to each activity indicator is zero. In this case, the estimated TOI cannot capture the economic growth in that period. Therefore, this study decides to impose a restriction that real GDP gap (i.e., the RGDP Gap) is always included. This means that the RGDP Gap is not subject to model averaging and this study does the DMA with the remaining seven variables. Figure 2 shows the two TOIs estimated respectively with the DMA-TVP-FAVAR model and the TVP-FAVAR model (i.e., no DMA process). Since all real activity indicators used are expressed as the deviation from their potential levels, a positive TOI indicates that the economic output is above its equilibrium and *vice versa*. In general, the two TOIs exhibit a similar trend but the DMA-TVP-FAVAR based TOI is more smoothing. It is interesting to find that the difference between the two indices is quite significant during the recent financial crisis from 2007-2009. However, at this stage, it is difficult to express any view on whether one TOI is better or worse than the other.

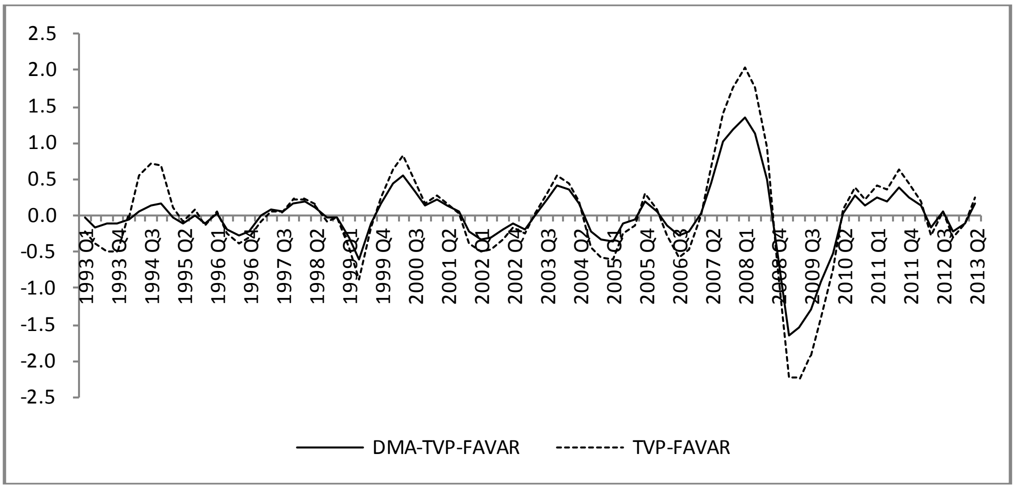


Figure 2: TOIs Estimated with the TVP-FAVAR and DMA-TVP-FAVAR models

To examine the forecasting results improved by the DMA-TVP-FAVAR model, this study investigates the forecasting performance of TOIs for inflation. Given that the estimation period runs from 1993:I to 2013:II the evaluation period covers the period from 1994:I to 2013:II for $h = 1, 2, \ldots, 4$ quarters ahead. The evaluation of forecasting accuracy is based on the mean squared forecast errors (MSFEs) divided by the MSFEs produced by a benchmark which is a time-varying parameter VAR (TVP-VAR) with stochastic volatility using three macroeconomic variables (the inflation rate, the RGDP Gap and an index of the real effective exchange rate).

Table 4 presents the (relative) MSFEs for various indicators of real activities. It starts with the benchmark model. Then it examines the forecasting performance of two further important variables used by the MPC, i.e., (i) the difference between the real GDP growth rate and its steady state and (ii) the deviation of the unemployment rate from its medium-term equilibrium. Finally, Table 4 reports the MSFEs of TOIs produced by the TVP-FAVAR model and the DMA-TVP-FAVAR model respectively. Several results stand out. Firstly, among the three output indicators, the unemployment rate serves as the best measure for predicting the inflation rate. Secondly, the MSFEs of the two TOIs are reported in the last two rows. It is encouraging to find that the MSFEs of two TOIs produced by the TVP-FAVAR model and the DMA-TVP-FAVAR model are much smaller than those produced by RGDP Gap, RGDPG Gap or Unem Gap at $h = 1, \ldots, 3$. Thirdly, the results in Table 4 are also consistent with the findings in Boivin and Ng (2006) that extracting factors with all data available is not always optimal. As in Table 4, the DMA-TVP-FAVAR model gives a TOI with lower MSFEs compared with the TOI produced by the TVP-FAVAR model.

Table 4: Forecasting Performance of Real Activity Indicators for the Inflation Rate

	h=1	h=2	h=3	h=4
TVP-VAR (benchmark: π_t, e_t, RGDP Gap)	0.2656	0.7607	1.2577	1.6550
TVP-VAR (π_t, e_t, RGDPG Gap)	*1.1266	*1.1640	*1.1954	*1.1964
TVP-VAR (π_t, e_t, Unem Gap)	*0.9750	*0.9567	*0.9518	*0.9346
TVP-FAVAR (π_t, e_t, TOI)	*0.9320	*0.9129	*0.9363	*0.9461
DMA-TVP-FAVAR (π_t, e_t, TOI)	*0.8838	*0.8899	*0.9252	*0.9466

*Note: This study employs the Diebold-Mariano (1995) test to examine whether the forecast errors differ significantly from the benchmark's MSFEs. The test is developed by Diebold and Mariano (1995) and comprehensively described in Garratt, Koop, Mise and Vahey (2009). If an MSFE has a *, it means that method forecasts significantly different from the benchmark TVP-VAR.*

Following Koop and Korobilis (2014) and Chapter 2, in Appendix 4 this study reports evidence on which constituent receives the highest weight in the DMA procedure. The number in each panel denotes the probability that the DMA method assigns to models that contain the variable named in the title on the panel. For example, the RIPX Gap is assigned roughly 40% chance of being considered by the MPC. It is worth noting that there is no variable switching. In other words, from a statistic point of view, the output measure which receive significant weights in 1993 are always considered by the MPC when evaluating economic activity. Furthermore, the real GDP growth rate and the unemployment rate enter into the MPC's range of consideration at each point in time, which is consistent with its August 2013 Guidance. However, Appendix 4 indicates that the real unit labour cost and the real household disposable income contain no useful information about the future evolution of inflation. In addition to the three variables (the output gap, the GDP growth rate and the unemployment rate) mentioned in the August 2013 Guidance, Appendix 4 shows that the industrial production index, real gross value added and labour productivity are also helpful for predicting inflation.

Considering the time-varying DMA weights (as in Appendix 4) and the findings that the DMA-TVP-FAVAR based TOI has lower MSFEs (as in Table 4), this study concludes that this TOI is the optimal activity indicator for the UK. It will be used in the subsequent econometric estimation to examine whether the inflation and output stabilisation policy has been met in the UK.

4.5 Estimate a Linear Interest Rate Rule

Using a simple linear Taylor rule, this section proceeds to model the BOE's monetary policy with the GMM estimator. It also examines whether a standard Taylor rule can be augmented with an FCI. The MPC's inflation projection is used to measure the inflation expectation. The estimated 'optimal' TOI (obtained using the DMA-TVP-FAVAR model) is used to measure the deviation of real economic output from its long-run trend.

However, it is worth noting that the TOI is obtained in Section 4.4 under the assumption/criterion that an optimal output measure should include as much information about future inflation as possible. Thus, it is essential to test whether the optimal TOI contains additional output information that is not included in inflation expectations. For robustness purposes, this study also uses the rolling PCA to construct two indices, an output index (RTOI, based on the same output measures used in Section 4.4) and a financial conditions index (RFCI, based on the same financial indicators in Chapter 2). Appendix 5 presents a brief introduction of the rolling PCA.

4.5.1 The Taylor Rule and the GMM Estimator

Recall Eq. (2.9) which describes Clarida et al.'s (1998, 2000) forward-looking Taylor rule with interest rate smoothing:

$$i_t = \left(1 - \sum_{j=1}^{n} \rho_j\right)\left[i^* + \gamma_\pi (E_t(\pi_{t+k}) - \pi^*) + \gamma_y E_t(y_{t+s} - y^*)\right] + \sum_{j=1}^{n} \rho_j \, i_{t-j}$$

where $i^* = \bar{r} + \pi^*$ and y^* is the potential level of output. The sum of parameters lies between zero and one. The term i_t denotes the Trea3m. The inflation target (π^*) is assumed to be constant throughout the sample period. This equation assumes partial adjustment of the actual interest rate i_t to the target rate r_t^*:

$$i_t = \left(1 - \sum_{j=1}^{n} \rho_j\right) r_t^* + \sum_{j=1}^{n} \rho_j \, i_{t-j} \tag{5.1}$$

where the sum of ρ_j gauges the degree of interest rate smoothing. The target rate of interest is expressed as:

$$r_t^* = (\bar{r} + \pi^*) + \gamma_\pi (E_t(\pi_{t+k}) - \pi^*) + \gamma_y E_t(y_{t+s} - y^*) \quad (5.2)$$

Specifically central banks adjust i_t in each period to eliminate a fraction, $1 - \sum_{j=1}^{n} \rho_j$, of the gap between their target rate (r_t^*) and some linear combination of their past values $\sum_{j=1}^{n} \rho_j \, i_{t-j}$. Following the earlier GMM studies, this study assumes the equilibrium real interest rate $\bar{r}$ to be equal to its sample average. The real interest rate at each point of time (t) is obtained by subtracting inflation from the nominal Trea3m. According to the Taylor rules in the existing studies including Clarida et al. (1998) and Castro (2011), in order for monetary policy to be stabilising, the coefficient on the inflation bias γ_π should exceed 1.0 and the coefficient on the output gap γ_y should be positive. A coefficient larger than 1.0 on the inflation bias means that central banks raise their real interest rates to react to higher inflation rates, which will exert stabilising effects on inflation. However, Clarida et al. (1998) discover that the BOE does not increase the interest rate in response to the expected higher inflation rate. In other words, the γ_π is smaller than 1.0 in the UK. To examine whether this finding is plausible or not, this study uses the MPC's central projection for inflation in the GMM estimation. On the other hand, a less-than-one γ_π implies an accommodative behaviour of the interest rate to the expected inflation rate, which may generate self-fulfilling bursts of inflation and output. A positive coefficient on the output gap means that in situations when output is below its potential level a fall in the interest rate would have a stabilising effect on economic activity.

Defining $\gamma_0 = i^* - \gamma_\pi \pi^*$ and $\bar{y}_{t+s} = y_{t+s} - y^*$ yields:

$$i_t = \left(1 - \sum_{j=1}^{n} \rho_j\right) [\gamma_0 + \gamma_\pi E_t(\pi_{t+k}) + \gamma_y E_t(\bar{y}_{t+s})] + \sum_{j=1}^{n} \rho_j \, i_{t-j} + \mu_t \quad (5.3)$$

where μ_t is an IID stochastic error. Eliminating unobserved forecasted variables from this equation, Eq. (5.3) can be re-written in terms of realised variables:

$$i_t = \left(1 - \sum_{j=1}^{n} \rho_j\right)\left[\gamma_0 + \gamma_\pi \pi_{t+k}^f + \gamma_y \bar{y}_{t+s}\right] + \sum_{j=1}^{n} \rho_j \, i_{t-j} + \varsigma_t \tag{5.4}$$

where π_{t+k}^f is the MPC's projection of inflation as described in Eq. (3.2). With π_{t+k}^f in the above equation, the BOE's reaction function is estimated using forecast data on a quarterly basis. The error term ς_t is expressed as a linear combination of the forecast errors of output and the disturbance μ_t.

It is worth reiterating that this study considers various measures of output bias $\bar{y}_t$ such as the optimal TOI produced in Section 4.4, the rolling-PCA-based RTOI, etc. Since the optimal TOI is estimated as the best predictor of inflation, this study also tests the hypothesis that the optimal TOI not only contains information about future inflation but also includes additional information regarding the current status of real economic activity that is not reflected in π_{t+k}^f.

Drawing on extensive readings, this study provides several observations to support the above hypothesis. Firstly, TOI_t projects actual inflation while π_{t+k}^f represents the MPC's central inflation projection which is obtained under the assumption that the current interest rate (at the time of the meeting) is maintained within the forecasting horizon. Secondly, the MPC updates its forecasts (π_{t+k}^f) on a quarterly basis. However, most constituents in the optimal TOI_t (for instance, the unemployment rate, and the industrial production index) are reported on a more frequent basis (e.g., monthly). Thirdly, as in Chapter 3 the prices are sticky and on average are fixed for 3-4 quarters in the UK. Therefore, sticky inflation in the UK may miss some important information in the output indicators. To test this hypothesis, this study will estimate the response coefficient on the output gap (i.e., φ_y in Eq. 5.6) to see whether it is significant when using another output index, the RTOI, as the output measure. The RTOI extracts the co-movement of same output measures (see, Appendix 2) used for estimating the optimal TOI but does not attempt to be the best inflation predictor. Hence, the significance of the response parameter on the RTOI can be considered as evidence that the optimal TOI contains output information which is not included in the BOE's inflation forecasts π_{t+k}^f.

If this hypothesis is valid, the BOE's reaction function could be re-expressed as:

$$i_t = \left(1 - \sum_{j=1}^{n} \rho_j\right)\left[\gamma_0 + \gamma_\pi \pi_{t+k}^{f} + \gamma_y TOI_{t+s}\right] + \sum_{j=1}^{n} \rho_j \, i_{t-j} + \varsigma_t \tag{5.5}$$

where the short-term interest rate is modelled to react to the MPC's expected inflation and the optimal TOI.

In practice, consider the reduced form of Eq. (5.4):

$$i_t = \varphi_0 + \varphi_\pi \pi_{t+k}^{f} + \varphi_y \bar{y}_{t+s} + \sum_{j=1}^{n} \rho_j \, i_{t-j} + \varsigma_t \tag{5.6}$$

The new vector of parameters is related to the former as follows:

$$(\varphi_0, \varphi_\pi, \varphi_y)' = (1 - \sum_{j=1}^{n} \rho_j)(\gamma_0, \gamma_\pi, \gamma_y)' \tag{5.7}$$

Given the estimates of φ_0, φ_π, φ_y and Eq. (5.7), this study could recover the implied values of γ_0, γ_π and γ_y:

$$\gamma_0 = (\bar{r} + \pi^*) - \gamma_\pi \pi^* = \frac{\varphi_0}{1 - \sum_{j=1}^{n} \rho_j} \tag{5.8}$$

$$\gamma_\pi = \frac{\varphi_\pi}{1 - \sum_{j=1}^{n} \rho_j} \tag{5.9}$$

$$\gamma_y = \frac{\varphi_y}{1 - \sum_{j=1}^{n} \rho_j} \tag{5.10}$$

Combining Eq. (5.8) and Eq. (5.9) produces:

$$\pi^* = \frac{\gamma_0 - \bar{r}}{1 - \gamma_\pi} = \frac{\varphi_0 - \bar{r}\left(1 - \sum_{j=1}^{n} \rho_j\right)}{\left(1 - \sum_{j=1}^{n} \rho_j\right) - \varphi_\pi} \tag{5.11}$$

Since the term $\sum_{j=1}^{n} \rho_j$ is used to capture the degree of interest rate smoothing and to eliminate the serial correlation in the error term, the number of lagged interest rates (i.e., the value of j) is determined using the Durbin-Watson (DW, Durbin and Watson, 1951) test. This is also the case in studies like Castro (2011).

Eq. (5.6) is estimated using the GMM estimator. As in Section 4.2.4.1, this technique is suitable for estimating the interest rate reaction function. This is because as in Eq. (5.6), the regression is run on variables, some of which are unavailable to central banks at decision-making moments. As in Clarida et al. (1998, 2000), let v_t be a vector of variables in the central bank's information set at the time it chooses the real rate of interest and that are orthogonal to ς_t. The possible elements of v_t include any lagged variables which are helpful for predicting inputs and any contemporaneous variables uncorrelated with the current interest rate shock μ_t. As π_{t+k}^{f} represents the MPC's mean inflation forecasts that are available for the MPC at decision-making moments, the selection of instrument variables will concentrate on those highly correlated with the output gap and the target interest rate, r_t^*.

In this study, the choice of instruments is motivated by the view that the BOE's output forecasts and the target interest rate depend on the historical values of TOIs, the unemployment rate, the 10-year government benchmark bond yield (Yield10yr), the US 3-month treasury bill rate (US3m) and the sterling effective exchange rate index (ERI). Firstly, the output gap is the most common instrument in the existing GMM literature like Clarida et al. (1998, 2000), Mehra (1999), Chadha et al. (2000) and Castro (2011). Secondly, as most constituents in the optimal TOI relate to real GDP and the unemployment rate receives relatively low weight, this study adds the rate of unemployment in the UK to the instrument set. According to the MPC (August, 2013), the unemployment rate is a crucial indicator of labour market slack. Primiceri (2005) and Malik and Banerjee (2013) both consider it as a real economic measure. Thirdly, Castro (2011) emphasises that the Yield10yr has useful and valuable information about the future evolution of the interest rate making the long-term interest rate more informative as an instrument than the short-term rate. According to Castro (2011), the Yield10yr is proven to be a good instrument for the BOE's forward-looking monetary policy. Finally, given the openness of the UK economy this study follows Clarida et al. (1998) and Chadha et al. (2000) and includes the ERI and foreign interest rates (the US3m) in the information set. Therefore, the instrument set has a constant, 1-6 lagged value of the TOI, Unem Gap, Yield10yr, ERI and US3m.

For robustness purposes, this study will examine whether the estimation results will be improved by adding 1-6 lagged inflation rates as additional instruments. This is motivated by the fact that they may contain useful information about the evolution of future inflation and the real interest rate. The existing GMM literature using lagged rates of inflation as instrument variables include but are not limited to Mehra (1999), Clarida et al. (1998, 2000), Chadha et al. (2004), Castro (2011), etc.

Combining Eq. (5.6) together with Eq. (2.19) yields the following set of orthogonality conditions which is exploited for estimation:

$$E\left\{i - \left[\varphi_0 + \varphi_\pi \pi_{t+k}^{f} + \varphi_y \bar{y}_{t+s}\right] - \sum_{j=1}^{n} \rho_j \, i_{t-j} | v_t \right\} = 0 \tag{5.12}$$

Following Castro (2011), this study uses an optimal weighting matrix, which accounts for various forms of heteroskedasticity and autocorrelation in μ_t in the estimation. The weighting matrix is estimated with the Bartlett kernel method of Newey and West (1987). As noted in Cliff (2003), an optimal weighting matrix requires an estimation of the parameter vector, yet at the same time the estimation of the parameters requires a weighting matrix. To solve this dependency, this study sets the initial parameter using the two-stage least squares (TSLS). Then it calculates the weighting matrix with the last updated parameter estimates.

Considering that the dimension of the instrument vector v_t may exceed the number of parameters being estimated, over identifying restrictions must be tested to assess the validity of the specification and the instrument variables. In this context, the Hansen (1982) over identification test (using J-statistic) is implemented. Under the null hypothesis that the above instruments are valid, rejection of orthogonality implies that a central bank does not adjust its behaviour to information contained in the instrument variables. In that case that some instruments are correlated with v_t , the set of orthogonality conditions will be violated which leads to rejection of the model.

4.5.2 The Estimates of a Taylor Rule

As the first step in estimating the Taylor rule, this study uses the SIC to determine the horizons of inflation and output gap forecasts:

$$SIC = -\frac{2LK}{T} + \frac{n \log T}{T} \tag{5.13}$$

where n in Eq. (5.13) is the number of parameters estimated using T observations and LK is log likelihood which is conducted by looking at the difference between the log likelihood values of both restricted and unrestricted versions of an equation:

$$LK = -\frac{T}{2}[1 + log(2\pi) + log(\hat{\epsilon}'\hat{\epsilon}/T)] \tag{5.14}$$

where $\hat{\epsilon}'\hat{\epsilon}$ denotes sum of squared residuals. Previous studies follow different ways to select forecasting horizons. Clarida et al. (1998) choose the horizons based on their intuition. They select a lead length of 12 months for the inflation rate for all countries investigated including the US, the UK, Japan, Germany, Italy and France. In their study in 2000, Clarida et al. (2000) use a horizon of one quarter instead for both inflation and output forecasts for the US. Chadha et al. (2004) also assume the target horizon of one quarter for both predicted inflation and the output gap. Castro (2011) employs a quantitative method with the SIC. When using the SIC, the values of k and s are determined by choosing the specification with the lowest SIC value. Table 5 presents the results.

Table 5: Lead Length Selection (for the inflation rate and output gap) Based on the SIC, 1993:I-2012:IV

s= \ k=	1	2	3	4	5	6
Panel 1: Inflation Expectation: MPC's forecast; Output: ‘Optimal’ TOI						
s=0	1.11186	1.13444	1.13682	1.13644	1.12548	1.09278
s=1	1.02190	1.04401	1.06660	1.07151	1.07513	1.05452
s=2	1.03393	1.04523	1.05480	1.05765	1.05161	<u>1.01464</u>
s=3	1.08201	1.09002	1.08869	1.09218	1.07368	1.03563
Panel 2: Inflation Expectation: MPC's forecast; Output: rolling-PCA-based TOI						
s=0	1.09711	1.10761	1.10185	1.10356	1.08852	1.05688
s=1	1.10850	1.11181	1.09628	1.09572	1.07670	1.05078
s=2	1.11969	1.11215	1.09416	1.09576	1.06883	<u>1.03260</u>
s=3	1.11222	1.12807	1.12871	1.13877	1.11167	1.07729

Note: The estimated SIC value in this table is based on the GMM estimation. This study adds two lags of interest rates, i.e., $j = 2$ in Eq. (5.6) in order to eliminate any serial correlation in error term.

Panel 1 provides the SIC values when inflation expectation is measured with the MPC’s inflation forecasts and the optimal TOI (estimated in Section 4.4) is used to measure the output gap. For the purpose of robustness, this study estimates the value of the SIC when the output gap is measured with the rolling-PCA-based output index, RTOI. The estimation is presented in Panel 2. It is encouraging to find that the conclusion in relation to the forecasting horizon is insensitive to the choice of various output measures. In both Panel 1 and 2, the SIC value is minimised at $s = 2$ and $k = 6$. It is quite important to emphasise that Table 5 does not suggest that the BOE focuses on the inflation rate and the output gap with only 6 and 2-quarters ahead. The purpose of choosing the forecasting horizons of inflation and output biases (as in Table 5) is to measure the BOE's expectation in a simple way.

With the results in Table 5, this study presents the estimation of the backward-looking, contemporaneous and forward-looking Taylor rules in Table 6. Only one lag of the interest rate (i.e., $j = 1$) is considered in Table 6. Reg. (1) and (2) show the least square (LS) estimates of the backward-looking rules (i.e., $s = k = -1$) which assumes that the BOE reacts to historical information on inflation and the output gap. In Reg. (1), this study employs the optimal TOI (as estimated in Section 4.4) to measure the changes in the output gap. In Reg. (2), it uses the RTOI as the measure of the output gap. However, both Reg. (1) and (2) indicate that a backward-looking rule cannot model the interest rate movement well. In Reg. (1), despite the estimation for φ_y being reasonable this estimate is insignificant and the results provide a negative estimate of φ_π. In Reg. (2), although the estimated φ_y becomes significant the φ_π is negative. Furthermore, the DW test discovers that one lagged interest rate alone is insufficient to eliminate serial correlation in the error term in both regressions. This motives further estimation of the Taylor rule with two interest rate lags in Table 7.

In Reg. (3) and (4) that respectively models a contemporaneous Taylor rule with the optimal TOI and RTOI as the output gap measure, the similar problems are exhibited. The coefficients on the output gap (φ_y) are significantly positive, however φ_π is far from the prior expectation. The DW test also implies that two interest rate lags may be required to capture the BOE's interest rate smoothing behaviour. The unsatisfactory estimation in the backward-looking and contemporaneous rules motivate this study to test another version of the Taylor rule which considers forward-looking elements. The estimated results are displayed in Reg. (5) and (6). The GMM estimator is required in both Reg. (5) and (6).

Although the DW statistical test implies that the serial correlation in the error terms is not eliminated in Reg. (5) which uses the optimal TOI as the output gap measure, the coefficients on the expected inflation and output biases are both significantly positive. The overidentification test suggests that the instrument variables are valid. In Reg. (6), this study estimates the forward-looking Taylor rule but uses the RTOI to measure the output gap. Compared to the estimated coefficients in Reg. (5), the coefficient on the output gap (φ_y) declines dramatically and the estimated φ_π rises. Moreover, both φ_π and φ_y are statistically significant in Reg. (6). This finding provides two important implications. Firstly, this is evidence that the BOE's short-term interest rate could be explained with a forward-looking Taylor rule. Secondly, it indicates that the optimal TOI (which is produced by the DMA-TVP-FAVAR model) includes (i) information about the future inflation and (ii) information which is not reflected in the inflation forecasts of the MPC.

To examine the robustness of the above conclusions and to eliminate the serial correlation in the error terms, this study estimates the forward-looking Taylor rule by considering two lagged interest rates. The GMM estimation results are given in Panel 1 of Table 7.

The first panel of Table 7 focuses on the estimation of Eq. (5.6) while using different output measures. It is organised so that in Reg. (7) the optimal TOI is employed to measure the output gap and in Reg. (8) the rolling-PCA-based output index (the RTOI) is used. Reg. (9) investigates whether the GMM regression can be improved by adding lagged inflation rates to the instrument set. In Reg. (9), the optimal TOI is used. Reg. (10) and (11) are robustness tests which use the same instruments as Reg. (9) but employs different output measures. In Reg. (10) this study employs the RTOI, and in Reg. (11) it uses the RGDPG Gap (the difference between the real GDP growth and its steady-state level) as a measure of the output gap. Reg. (12) presents the results of the subsample analysis in order to examine the changes in the response coefficients when a shorter sample period is used. Several results stand out:

Firstly, according to the DW test the problem of autocorrelation is eliminated by incorporating two interest rate lags in Table 7. In other words, two lags of interest rates are required and are sufficient to account for the smoothing behaviour of the BOE.

Secondly, although the statistical evidence is a little weak in Reg. (7) it still provides evidence that the BOE targets expected inflation and the output gap. As mentioned, the instrument set includes a constant, 1-6 lags of TOI, Yield10yr, ERI, Unem Gap and US3m. This assumes that the MPC's inflation forecasts are available at decision making moments and hence the selection of instrument variables focuses on those highly correlated with the expected TOI and target the interest rate. In Reg. (9), this study discovers that with the additional 1-6 lagged inflation rates in the information set, φ_π turns out to be much larger and more significant. This means that despite the π_{t+k}^f being available to the MPC at the moment of making policy decisions, lagged inflation rates still contains valuable information for the BOE to set the short-term interest rate. A possible explanation is that the BOE makes interest rate decisions every month but the MPC's central projection for the inflation rate is only made on a quarterly basis. Therefore, updated inflation projections may not be available in some months and the lagged inflation rates should provide relevant information about the future inflation for those periods.

Thirdly, comparing the estimated coefficients in Reg. (8) to those in Reg. (7) (and/or comparing the φ_π and φ_y in Reg. 10 to those in Reg. 9), this study obtains similar conclusions from Table 6. When shifting to the RTOI as the measure of the output gap, the estimated φ_y tends to fall substantially and meanwhile the φ_π gets larger. The estimated φ_ys on the RTOI are statistically significant in both Reg. (8) and Reg. (10) based on different instrument variables. As mentioned earlier, this means that the optimal TOI not only provides useful information about future inflation but also gives information about the output gap that is not included in the MPC's inflation forecasts. This finding supports the use of the estimated optimal TOI in modelling the short-term interest rate in the UK.

For robustness purposes, this study completes another GMM regression (in Reg. 11) which uses the RGDPG Gap as an output measure. The empirical results confirm the reaction of the short-term interest rate to output stabilisation in the UK.

Fourthly, as displayed in Reg. (9) which uses the optimal TOI and adds the lagged inflation rate to the instrument set, the sum of smoothing parameters (i.e., rho) is around 0.946. This estimated value is larger than findings in earlier articles including

Clarida et al. (1998) and Castro (2011). This means that the BOE gives a much larger weight to the backward-looking interest rate compared to the weight on the target interest rate. With Eq. (5.11), the GMM estimator obtains a plausible estimate of the long-run inflation target (π^*). Eq. (5.11) provides a value for π^* of 2.32% when the horizon of inflation forecasts is equal to six quarters and the lead length of the output gap is two quarters. This is a little above the current objective of the BOE, 2%[4]. A possible explanation is that the BOE alters its inflation target twice during the sample. Therefore, the implied value 2.32% is an average of inflation targets during the period.

Fifthly, the estimations in Table 7 confirm that the BOE is relying on all available information which requires a forward-looking version of the Taylor rule. Take Reg. (9) for illustration: Covering γ_π and γ_y with Eq. (5.9) and Eq. (5.10) gives $\gamma_\pi = 4.79$ with a standard error of 0.465 and $\gamma_y = 5.75$ with a standard error of 1.649. Because γ_π is significantly greater than 1.0, the prediction that the BOE raises its real interest rate in response to expected inflation pressures is indeed statistically significant. A one-percent rise in the projected inflation rate (π^f_{t+6}) will induce the BOE to raise its nominal rate by 0.256%. Holding constant inflation forecasts, a one-percent rise in the expected output induces the BOE to increase nominal interest rate by 0.308%. It is particularly interesting to note that the estimated γ_π and γ_y are much larger than the estimates in Clarida et al. (1998) and Castro (2011) who have a shorter sample period. This motivates the subsample analysis as shown in Reg. (12).

Using the MPC's inflation forecasts (π^f_{t+6}) and the expected TOI (TOI_{t+2}) but for the period before the interest rate hits the effective zero lower bound, Reg. (12) discovers that although the coefficients φ_π and φ_y are still significantly positive for the period of 1993:I-2008:IV they turn out to be much smaller compared to those under the full sample period (1993:I-2013:II). The sum of smoothing parameters (rho) also declines significantly in Reg. (12) but is closer to the estimation in Clarida et al. (1998) and Castro (2011). Covering γ_π and γ_y with Eq. (5.9) and Eq. (5.10) yields $\gamma_\pi = 1.43$ and $\gamma_y = 1.38$ for Reg. (12) which is also closer to the estimated coefficients in the literature. Therefore, it is reasonable to conclude that the difference between the

[4] The BOE introduced an inflation target in October 1992 which is defined as an RPI inflation range of 1-4% a year. In 1995, the inflation goal was modified and the monetary policy would aim consistently to achieve an inflation rate of RPI of 2.5%. In November 2003, the target was defined as 2% for the CPI inflation.

coefficient estimates in this study and those in the literature (for instance, Clarida et al., 1998; Castro, 2011) results from the different time periods examined. Given the different estimates between Reg. (9) and (12), it is essential to re-model the interest rate with a time-varying parameter estimator in the future.

Table 6: Linear Estimates of the Taylor Rule, One Interest Rate Lag

	φ_0		φ_π		φ_y		φ_{fci}		π^*, %	rho	J-stat.		adj. R^2	DW
Reg.1	** 0.515	(0.221)	**-0.180	(0.061)	0.054	(0.112)	---	---	1.88	0.954	---	---	0.961	0.960
Reg.2	** 0.450	(0.197)	**-0.155	(0.056)	** 0.044	(0.016)	---	---	1.83	0.956	---	---	0.964	1.015
Reg.3	** 0.438	(0.206)	**-0.134	(0.058)	** 0.349	(0.105)	---	---	1.83	0.949	---	---	0.963	1.138
Reg.4	0.174	(0.205)	-0.072	(0.059)	** 0.041	(0.017)	---	---	1.46	0.980	---	---	0.961	1.187
Reg.5	**-0.576	(0.117)	** 0.339	(0.070)	** 0.529	(0.060)	---	---	2.34	0.949	15.140	[0.967]	0.968	1.262
Reg.6	**-0.872	(0.153)	** 0.527	(0.091)	** 0.039	(0.008)	---	---	2.26	0.924	16.402	[0.945]	0.962	0.931

*Note: the significant level at which the null hypothesis is rejected: **5%, *10%. The 'optimal' TOI obtained in Section 4.4 is used as the output measure in Reg. (1), (3) and (5). The rolling-PCA-based output index, RTOI is used in Reg. (2), (4) and (6).*

Table 7: Linear Estimates of the (augmented) Taylor Rule, Two Interest Rate Lags

	φ_0		φ_π		φ_y		φ_{fci}		π^*, %	rho	J-stat.		adj. R^2	DW
Panel 1: linear estimates of a Taylor rule (Two lagged interest rates)														
Reg.7	-0.196	(0.130)	* 0.147	(0.081)	** 0.317	(0.056)	---	---	2.34	0.966	14.386	[0.968]	0.971	1.919
Reg.8	**-0.408	(0.089)	** 0.287	(0.053)	** 0.035	(0.005)	---	---	2.23	0.944	16.108	[0.934]	0.970	1.900
Reg.9	**-0.365	(0.100)	** 0.256	(0.072)	** 0.308	(0.053)	---	---	2.32	0.946	16.824	[0.987]	0.973	1.808
Reg.10	**-0.468	(0.061)	** 0.335	(0.045)	** 0.036	(0.005)	---	---	2.22	0.934	16.733	[0.988]	0.971	1.802
Reg.11	**-0.183	(0.066)	** 0.220	(0.046)	** 0.069	(0.008)	---	---	2.27	0.925	17.915	[0.979]	0.974	1.905
Reg.12	** 0.200	(0.077)	** 0.161	(0.051)	** 0.155	(0.030)	---	---	3.46	0.887	13.934	[0.998]	0.924	1.896
Panel 2: linear estimates of an augmented Taylor rule (Two lagged interest rates)														
Reg.13	**-0.324	(0.065)	** 0.251	(0.051)	** 0.219	(0.046)	** 0.234	(0.076)	2.29	0.941	17.787	[0.997]	0.974	1.818
Reg.14	**-0.649	(0.065)	** 0.442	(0.038)	** 0.015	(0.003)	** 0.036	(0.004)	2.53	0.919	17.491	[0.997]	0.973	1.918

*Note: the significant level at which the null hypothesis is rejected: **5%, *10%. The optimal TOI obtained in Section 4.4 is used as the output measure in Reg. (7), (9) and (12). The rolling-PCA-based output index, the RTOI is used in Reg. (8) and (10). In Reg. (13), this study uses the optimal TOI to measure the output gap and uses the optimal FCI (obtained in Chapter 2) to measure the financial market movement. In Reg. (14), it uses the RTOI and the RFCI.*

4.5.3 The Estimates of an Augmented Taylor Rule

Section 4.5.2 concluded that the optimal TOI contains information about the output gap that is not fully reflected in the π^f_{t+6}. Therefore, it would be beneficial to use Eq. (5.5) to model and explain the BOE's interest rate movement. In this subsection, this study extends the baseline model (Eq. 5.4) and considers other factors that the BOE may take into account when setting the interest rate:

$$i_t = \left(1 - \sum_{j=1}^{n} \rho_j\right)\left[\gamma_0 + \gamma_\pi \pi^f_{t+k} + \gamma_y \bar{y}_{t+s} + \gamma_{fci} FCI_{t+l}\right] + \sum_{j=1}^{n} \rho_j \, i_{t-j} + \varsigma_t \tag{5.15}$$

where FCI_t is the financial conditions index created in Chapter 2 and l is the lead length of forecast FCI. Chapter 2 obtains the FCI as the best predictor for the output. However, it is reasonable to hypothesise that the FCI contains information that is not included in the $\bar{y}_{t+s}$. This is because many constituents (such as equity prices, the exchange rate) of the FCI are updated more frequently than the constituents of the output index. In order to test this hypothesis, this study uses the rolling PCA again to construct another index, the RFCI.

Consider the reduced form of Eq. (5.15):

$$i_t = \varphi_0 + \varphi_\pi \pi^f_{t+k} + \varphi_y \bar{y}_{t+s} + \varphi_{fci} FCI_{t+l} + \sum_{j=1}^{n} \rho_j \, i_{t-j} + \varsigma_t \tag{5.16}$$

Combining Eq. (5.16) together with Eq. (2.19) yields a set of orthogonality conditions to explore:

$$E\left\{ i - \left[\varphi_0 + \varphi_\pi \pi^f_{t+k} + \varphi_y \bar{y}_{t+s} + \varphi_{fci} FCI_{t+l}\right] - \sum_{j=1}^{n} \rho_j \, i_{t-j} | v_t \right\} = 0 \tag{5.17}$$

Recover the implied values of γ_0, γ_π, γ_y and γ_{fci} as:

$$(\varphi_0, \varphi_\pi, \varphi_y, \varphi_{fci})' = (1 - \sum_{j=1}^{n} \rho_j)(\gamma_0, \gamma_\pi, \gamma_y, \gamma_{fci})' \tag{5.18}$$

Prior to discussing estimates of the response coefficients, this study employs the SIC again to determine the value of l in Eq. (5.15). When using the SIC, the value of l is decided by choosing the specification with the lowest SIC. Table 8 displays the results. The SIC value is minimised at $l = 1$ implying that the BOE's behaviour of targeting financial market is forward-looking.

Table 8: Lead Length Selection (for FCI) Based on the SIC, 1993:I-2012:IV

l= / s & k=	1	2	3	4	5	6
s=2, k=6	1.00142	1.01679	1.02247	1.02698	1.03489	1.04585

Note: the lead length selection for an FCI is based on the conclusion in Table 5 that the SIC value is minimised given s=2 and k=6 in Eq. (5.12). The estimated SIC in Table 8 is based on based the GMM estimation. The instrument variables include: a constant, 1-6 lags of π_t, TOI, Unem Gap, Yield10yr, ERI, US3m, and FCIs. The formulas for calculating the values of SIC are in Eq. (5.13-5.14).

Reg. (13) and (14) are the GMM estimation of Eq. (5.16). The instruments confirmed by Hansen's J-test include a constant, 1-6 lags of π_t, TOI, Unem, Yield_10yr, ERI, US_rate and FCI. The results indicate that although the response parameter on the FCI_{t+1} is significantly positive when using the RFCI to measure the changes in the financial market (as in Reg. 14), it is much smaller than the coefficient (φ_{fci}) in Reg. (13) which uses the DMA-TVP-FAVAR-based FCI. Therefore, this study is confident in concluding that the FCI produced in Chapter 2 contains information on the financial market that is not reflected in the output gap measure.

Recovering implied values with Eq. (5.18) for Reg. (13), this study obtains $\gamma_\pi = 4.26$ and $\gamma_y = 3.72$ both of which are consistent with the prior theoretical expectation. In addition, the evidence implies that the BOE does not only target the inflation rate and economic growth but also works on stabilising the financial market in the UK. The response coefficient (γ_{fci}) on the financial conditions index is about 3.98. The null hypothesis of $\varphi_{fci} = 0$ is rejected given a significant level of 5%. This suggests that the evolution of the financial market in the UK is stabilised by adjusting the interest rate. A one-percentage rise in the combination of financial indicators above its long-run trend leads to an increase of about 0.234 percent in the nominal interest rate and *vice versa*.

This is an important finding which represents new analysis providing evidence that the BOE is working on promoting both monetary and financial stabilisation. It shows additional evidence to Montagnoli and Napolitano's (2005) argument that central banks target asset prices in making interest rate decisions. Although some earlier studies including Castro (2011) also test the BOE's reaction to variation in the financial market for the UK they fail to find any empirical results supporting the view that the BOE responds to changes in the status of the domestic financial system. An explanation for the different findings rising from this study relates to three issues. Firstly, instead of using the 'substitution' method for expected inflation, this study uses the MPC's central projection which represents its best collective judgement of future inflation. Secondly, this study uses a composite index to measure the output gap. Thirdly, it uses a new approach (i.e., the DMA-TVP-FAVAR model) to estimate the FCI. As in Chapter 1-2, the estimated FCI is expected to capture more variations in the index. In summary, in contrary to the existing studies in this field, this study employs three different, but better, time series data in the regressions.

4.6 Conclusions

This chapter focuses on modelling the short-term interest rate in the UK. It aims to simplify and to explain the process of making monetary policy in the BOE using an interest rate reaction function developed in the literature. It proposes several important issues for discussion. Firstly, there has not been much empirical work that has looked at the UK monetary policy by explicitly using measures of the inflation forecast. The majority of the research done on this topic focuses on the US economy. Secondly, the BOE does not have an explicit definition of the output measure. The existing studies use various measures including the employment rate, deviation of real GDP from its HP filters, the growth rate of real output, etc. However, the current studies fail to clarify why one particular measure is used against the others. Thirdly, following the theoretical literature such as Corsetti and Pesenti (2005), Kontonikas and Montagnoli (2006) and Teranishi (2012) who insist on the interest rate targeting financial variables (e.g., the exchange rate, asset prices and spreads), Castro (2011) and Martin and Milas (2013) model the BOE's interest rate with an augmented (for FCI) Taylor rule. However, the results are not entirely consistent. It is worth noting that different

FCIs are used in their empirical analysis, which is one possible reason for the difference in their findings.

Given the aforementioned three issues, this study is very careful in selecting input data. It employs the MPC's central projection for inflation to measure the BOE's inflation expectation. It also introduces an optimal measure for the output gap which is based on the criterion that an optimal real activity measure should contain as much information about future inflation as possible. This standard is derived from the BOE's monetary transmission mechanism. The optimal measure for the financial market in the UK is from the estimation in Chapter 2.

When empirically modelling the short-term interest rate, this study uses the GMM estimator. The results of the entire study point to the finding that the BOE does not only react to the stabilisation of expected inflation and the output gap but also aims to improve financial stabilisation by bringing the financial market in the UK to its long-run trend. These results are obtained with a linear Taylor rule. However, considering the BOE may react differently to deviations of inflation from its target and also differently to the deviation of real output and financial system from their long-run trends, this study advances the topic of interest rate modelling by using a time-varying parameter estimator in the next chapter.

Appendices:

Appendix 1: Description of the Output Measures and Respective Sources:

No.	Name	Description	Source	Sample
1	RGDP	Domestic gross production (in millions of chained 2010 price)	ONS statistics	1993:I-2013:II
2	RGDPG	Real domestic gross production change on the same quarter one year ago	Authors’ calculation	1993:I-2013:II
3	Unem	Standardised ILO unemployment rate: Seasonally adjusted, Eurostat	ONS statistics	1993:I-2013:II
4	RIPX	Real Industrial Production Index, CPI deflated (base 2010)	ONS statistics	1993:I-2013:II
5	RGVA	Gross Value Added at basic prices: chained volume measures: Seasonally adjusted (base 2010)	ONS statistics	1993:I-2013:II
6	RULCX	Real unit labour cost index, CPI adjusted (2010=100)	OECD statistics	1993:I-2013:II
7	RHDI	Real household disposable income in sterling	ONS statistics	1993:I-2013:II
8	RLP	Real labour productivity, GDP (average) per head, Chained Volume Measures, market prices	ONS statistics	1993:I-2013:II

Note: While estimating a TOI, this study uses the deviation of the above eight variables from their equilibrium levels. Following the existing literature such as Ireland (2007), the equilibrium level of RGDPG is defined as its average. The MPC sets the threshold for the unemployment rate at 7%. The Hodrick-Prescott (HP, 1997) filter is used to estimate the equilibrium of the remaining six variables.

Appendix 2: Description of the TOI Constituents and Respective Covers:

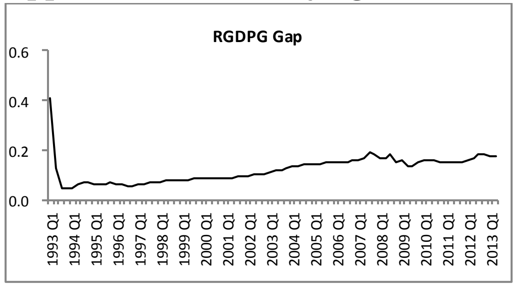
No.	Name	Description	Sample
1	RGDP Gap	The percentage deviation of RGDP from its equilibrium	1993:I-2013:II
2	RGDPG Gap	The difference between RGDP growth and its steady-state level	1993:I-2013:II
3	Unem Gap	The difference between Unem and its threshold rate (7%) set by the MPC	1993:I-2013:II
4	RIPX Gap	The percentage deviation of RIPX from its equilibrium	1993:I-2013:II
5	RGVA Gap	The percentage deviation of RGVA from its equilibrium	1993:I-2013:II
6	RULCX Gap	The percentage deviation of RULCX from its equilibrium	1993:I-2013:II
7	RHDI Gap	The percentage deviation of RHDI from its equilibrium	1993:I-2013:II
8	RLP Gap	The percentage deviation of RLP from its equilibrium	1993:I-2013:II

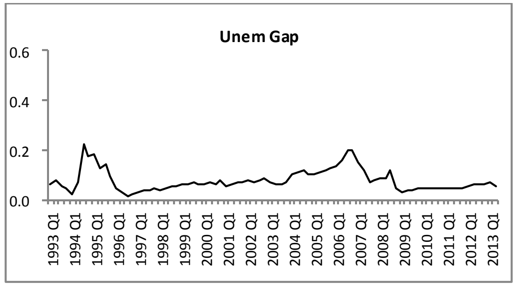
Note: the calculation of all variables in this table are based on the data collection of variables in Appendix 1.

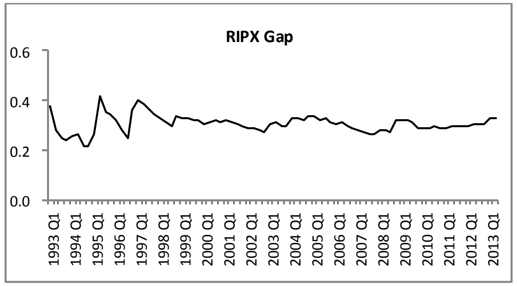
Appendix 3: Description of Other Relevant Variables and Respective Sources:

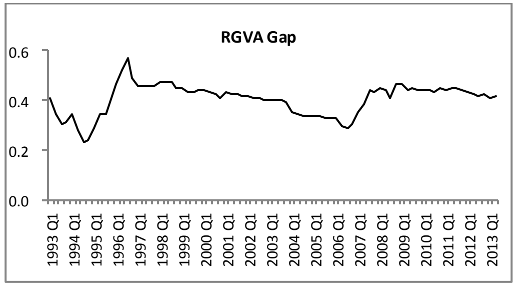
No.	Name	Description	Source	Sample
1	CPI	Consumer price index, seasonally adjusted, quarterly average (2005=100)	ONS statistics	1993:I-2013:II
2	Infl0	Inflation rate, seasonally adjusted, quarterly average	ONS statistics	1993:I-2013:II
3	InflK	Central projection of inflation with K quarters ahead, quarterly average	BOE statistics	1993:I-2013:II
4	Trea3m	3-month treasury bill discount rate, quarterly average	BOE statistics	1993:I-2013:II
5	Libor3m	3-month London inter bank offered rate, quarterly average	BOE statistics	1993:I-2013:II
6	Offic3m	Official central bank interest rate, quarterly average	BOE statistics	1993:I-2013:II
7	ERI	Sterling effective exchange rate index (January 2005=100)	BOE statistics	1993:I-2013:II
8	Yield10yr	10-year quarterly average yield from British Government Securities	BOE statistics	1993:I-2013:II
9	US3m	3-month US treasury bill discount rate, quarterly average	BOE statistics	1993:I-2013:II

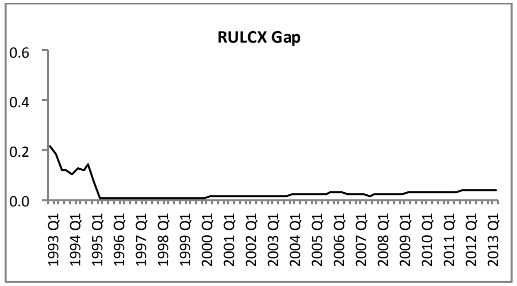
Appendix 4: Time-varying Probabilities of Inclusion to the TOI for Each Measures:

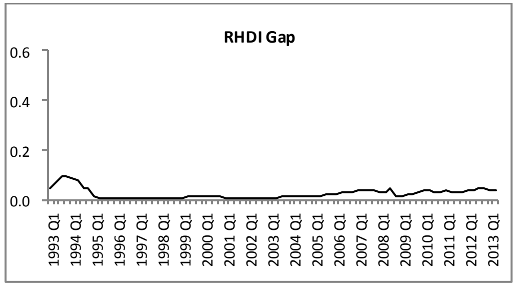


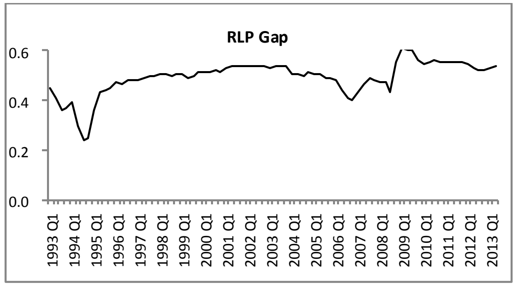












Appendix 5: The Rolling Principal Component Analysis

The rolling PCA is also called the moving window PCA (MVPCA) in literature. It updates at each time point while restricting observations used in the estimation to those which fall within a specified window of time. Ketelaere, Hubert and Schmitt (2013) illustrate how the rolling PCA (MVPCA) works. With each new observation, this window excludes the oldest observation and adds the observation from the most recent time period. Therefore, given the window size H, the information set at time t is $(x_{t-H+1}, \dots, x_t)$ and at time $t+1$ the information set becomes $(x_{t-H+2}, \dots, x_{t+1})$.

This study assumes that the BOE only considers the most recent four quarters when estimating the RTOI and the RFCI. Therefore, it sets $H = 4$. At each point of time, this study extracts a series of a common factor $(f_{t-3}, \dots, f_t)$ from an information set $(x_{t-3}, \dots, x_t)$. The co-movement of financial indicators (output measures) at time t is determined by the value of f_t at time t. The only exception is the RTOI and the RFCI between 1993:I and 1993:IV. The values of the RTOI and the RFCI in the first four quarters of the sample period are estimated using the conventional PCA based on the data in 1993.

CHAPTER 5

SHORT-TERM INTEREST RATE MODELLING USING A TIME-VARYING PARAMETER ESTIMATOR

5.1 Introduction

Recent studies provide strong evidence that a linear reaction function cannot describe movements of the short-term interest rate very well. As in Svensson (1999), Woodford (2003) and Surico (2007), a linear Taylor rule represents an optimal rule under the condition that a central bank minimises a symmetric quadratic loss function. This means that while minimising deviations of inflation, output and the interest rate from their reference values, a central bank has symmetric preferences and assigns the same weights to positive and negative deviations. However, this may not be the case in reality. Surico (2007) indicates that central banks may have asymmetric preferences and therefore follow a non-linear Taylor rule. For example, in the Eurozone, output contraction indicates larger policy responses than output expansion of the same degree for period 1999-2004 (see, Surico, 2007).

For the United Kingdom (UK), Nelson (2000), Trecroci and Vassalli (2010), Castro (2011) and Lafuente, Perez and Ruiz (2014) all agree with the conclusion that the Bank of England (BOE) exhibits shifting preferences for stabilising inflation and output around their target values. Changes in the BOE's monetary policy implementation have been captured by various econometric techniques such as subsample analysis (see, for instance, Neumann and Hagen, 2002), smooth transition regression (see, for instance, Castro 2011), Markov switching (see, for instance, Wesche, 2006), the Kalman filter algorithm (see, for instance, Trecroci and Vassalli, 2010), etc. This chapter aims to use a more advanced model, a time-varying parameter VAR with stochastic volatility (TVP-VAR-SV), to explain the changes in the short-term interest rate in the UK for the period 1993:I-2013:II (the same time period as covered in Chapter 4). This estimator not only allows for the time-variation in response coefficients but also takes possible structural changes into account. This is the first study to use the TVP-VAR-SV model to explain the short-term interest rate in the UK. Furthermore, it is also the first study that examines continuous changes in policy responses to a financial conditions index (FCI) in a TVP-VAR-SV model.

To model and explain the short-term interest rate accurately, this study considers a data modification for (effective) zero lower bound (ZLB). The policy interest rate in the UK was cut to 0.5% in 2009:I. As in Swanson and Williams (2013), since January 2009 the BOE has conducted large-scale asset purchases on a similar scale to the Federal Reserve (Fed) in the United States (US). This implies that over the period under review 50 basis points (bps) is viewed as an effective lower bound on the UK official interest rate[1]. As in Bernanke and Reinhart (2004) and Williams (2014), under the (effective) zero-interest rate circumstance, the monetary transmission mechanism is unlikely to work through the interest rate channel in the same manner as normal circumstances. Therefore, Nakajima (2011b) proposes to modify the nominal interest rate in a TVP-VAR-SV model to account for the effect of the ZLB in Japan. Motivated by Nakajima (2011b) this study applies an effective ZLB while modelling the short-term interest rate in the UK which has never been done before.

The remainder of this chapter is organised as follows: Section 5.2 reviews the literature on monetary policy. Section 5.3 discusses data issues while Section 5.4 introduces the methodologies used in this study. Empirical evidence is given in Section 5.5. Section 5.6 concludes.

5.2 Literature Review

This section of the study considers the existing econometric or statistical models for explaining changes in monetary policy implementation to provide a comprehensive comparison for various models. It is organised as follows: Section 5.2.1 discusses non-linear models used to capture policy changes. Section 5.2.2 introduces Bayesian inference and time-varying parameter (TVP) estimation of the Taylor rule (while considering stochastic volatility). Section 5.2.3 presents empirical literature on monetary policy in the UK.

[1] The website of the BOE also reports that "quantitative easing was first used by the MPC in March 2009. The official interest rate had been reduced to 0.5% and the MPC judged that it could not practically be reduced below that level". It is interesting to note that in August 2016 the BOE cut its bank rate to 0.2841%. However, given the Bank's previous reports regarding the ZLB (as mentioned), it is still reasonable for this study to use the 0.5% as the effective ZLB for the period before August 2016.

5.2.1 Non-linear Models with Constant Volatility

Surico (2007) studies the monetary policy of the European Central Bank (ECB) between 1999 and 2004 using a general framework that allows the ECB to place different weights on positive and negative deviation of the inflation rate, output and the interest rate from their reference values. Empirical analysis using the Generalised Method of Moments (GMM) estimator shows that the objective of price stability is symmetric in the Eurozone but output contractions indicate larger policy responses than output expansions of the same size. Hence, Castro (2011) maintains that if central banks are indeed assigning different weights to inflation and output as described by Surico (2007), a non-linear Taylor rule is necessary to explain the behaviour of the interest rate.

Recently there is a growing interest in the literature in regards to modelling a Taylor rule using non-linear or time-varying parameter models including subsample analysis, Markov switching, smooth transition regression (STR) and (extended) Kalman filter algorithm. Table 1 is a summary of these frequently used methods.

Table 1: Summary: Econometric Models for Exploring Changes in Monetary Policy Implementation

Techniques	Author(s)	Year	Countries
Subsample (use OLS)	Judd and Rudebusch	1998	US
Subsample (use GMM)	Clarida et al.	2000	US
Subsample (use VAR)	Neumann and Hagen	2002	Australia, Canada, Chile, New Zealand, Sweden and UK.
Markov switching	Owyang and Ramey	2003	US
	Wesche	2006	US, UK, Germany
	Kuzin	2006	Germany
Smooth transition regression	Peterson	2007	US
	Gerlach and Lewis	2010	Eurozone
	Castro	2011	US, UK, the Eurozone
Kalman filter	Trecroci and Vassalli	2010	US, UK, Germany, France, Italy
	Kuzin	2006	Germany
	Trehan and Wu	2007	US
Extended Kalman filter	Yuksel et al.	2013	Turkey

5.2.1.1 Subsample Analysis

Subsample analysis requires splitting a sample period at presumed dates and then estimating a reaction function for each period separately.

Clarida, Gali and Gertler (2000) estimate an interest rate reaction function with the GMM estimator for post-war US before and after Volcker's appointment as the Fed chairman in 1979. Their results point to a significant difference between the two subsample periods. Interest rate policy in the Volcker-Greenspan period appears to have much more sensitivity to changes in expected inflation than those in the pre-Volcker period.

Similarly Judd and Rudebusch (1998) consider three subsamples delineated by the identity of the chairmen of the US Fed, Burns (1970-1978), Volcker (1979-1987) and Greenspan (1987-1997). Using the ordinary least squares (OLS) estimator they discover that the coefficients on inflation and output stabilisation for each period vary in ways that seem broadly consistent with the success or failure of controlling the inflation rate during the period.

Parallel to these studies, Neumann and Hagen (2002) discover that for six inflation targeting countries (i.e., Australia, Canada, Chile, New Zealand, Sweden and the UK) the interest rate reaction to inflation and output biases changes with the introduction of inflation targets.

5.2.1.2 Markov Switching Analysis

Wesche (2006) argues that the subsample procedure for the Taylor rule is not attractive in practice because splitting samples will shorten available time series. In the sub-sample analysis, it is generally assumed that long-run inflation and the real interest rate equal their equilibrium values – an assumption which is more likely to be valid if the sample period is long enough. Thus, Owyang and Ramey (2003), Wesche (2006) and Kuzin (2006) employ a Markov switching method which uses all available data and permits independent switching processes for coefficients in a reaction function and residual variance.

Wesche (2006) investigates monetary policy in the US, the UK and Germany via estimating a Markov-switching model. The aim of her work is to study changes in the central banks' attitude towards policy targets. The empirical results show that over time all the central banks (the Fed, the BOE and the Bundesbank) assign changes in weights to inflation and output biases[2]. The coefficients on inflation and output appear to evolve according to two regimes: a 'hawkish' regime (i.e., a low coefficient on output stabilisation and a high coefficient on inflation stabilisation) and a 'dovish' regime (where the opposite holds). Finally, she emphasises the importance of a non-linear rule that an interest rate reaction function could be modelled better with a two-state switching model than with a linear model using the same variables.

Owyang and Ramey (2003) and Kuzin (2006) introduce the same estimator to the Fed and the Bundesbank respectively. For the Fed, Owyang and Ramey (2003) also find that monetary policy switches between an accommodative dovish regime and a less accommodative hawkish regime. Kuzin (2006) reaches the conclusion that the non-linearity in the Bundesbank's reaction function can be interpreted as asymmetric. However, the results do not indicate the same asymmetry as described in Surico (2007).[3] It shows an asymmetry with regard to large increases in inflation where the Bundesbank tries to control strong inflation growth in the beginning phase but does not attempt to impact the direction of inflation in the remaining period.

5.2.1.3 Smooth Transition Analysis

It is worth stressing that the aforementioned papers (e.g., Wesche, 2006; Kuzin, 2006), which use the Markov switching for the interest rate, normally assume a fixed inflation target during a particular regime. Such an assumption is rejected in the STR literature. The STR analysis allows for smooth endogenous regime switches and is able to demonstrate when a central bank changes its policy rule. Empirical studies using STR to estimate TVPs of Taylor rules include Peterson (2007), Gerlach and Lewis (2010) and Castro (2011).

Peterson (2007) discovers that the Fed follows a non-linear Taylor rule during the golden era of monetary policy (1985-2005) and a linear Taylor rule throughout the dark age of monetary policy (1960-1979). Therefore, he suggests that good monetary policy can be associated with a non-linear Taylor rule in the case where the Fed policy responds much more forcefully to inflation.

[2] The bias refers to the deviation of inflation and output from their reference values.
[3] Recall the study of Surico (2007) where the ECB responds to inflation deviation asymmetrically – it strongly reacts to positive deviation from its own inflation target and, on the other hand shows no significant reaction to negative inflation biases.

More recently Gerlach and Lewis (2010) and Castro (2011) concentrate on the ECB's interest rate setting behaviour since 1999. Gerlach and Lewis (2010) find substantial changes in Taylor-rule parameters following the collapse of Lehmann brothers. In addition, according to Castro (2011) the ECB only starts to react to the output gap when inflation is stabilised well below 2.5%. The non-linear Taylor rule in Castro (2011) encompasses several considerations of the ECB. Firstly, promoting price stability is above everything. Secondly, when this is achieved the bank promotes conditions for stabilising output.

5.2.1.4 Kalman Filter Analysis

Some literature also deserves attention in relation to the application of a time-varying parameter model to the analysis of central banks' policy such as Kuzin (2006), Trehan and Wu (2007), Trecroci and Vassalli (2010) and Yuksel, Ozcan and Hatipoglu (2013). They use the Kalman filter algorithm in their econometric estimation which allows parameters to follow either a random walk or autoregressive (AR) process. One of the main reasons for choosing the Kalman filter algorithm is due to its extraordinary flexibility. In this method the dynamics of state variables are assumed to be exogenous, which avoids choices of an explicit transition function and transition variables as in the STR analysis.

Trecroci and Vassalli (2010) perform a time-varying parameter Taylor-rule analysis for countries such as the US, the UK, Germany, France and Italy which are also considered in Wesche (2006). They reach a similar conclusion despite using a different estimation procedure, the Kalman filter algorithm. The estimation method in Trecroci and Vassalli (2010) drops the fixed-parameter hypothesis on which some existing interest rate estimations are based (e.g., Clarida et al., 1998). Coefficients on inflation and output stabilisation are assumed to change and to follow the stationary AR process. However, unlike the TVP-VAR-SV model developed in Primiceri (2005), the fixed-variance hypothesis is still used in Trecroci and Vassalli (2010). With the empirical findings for these five countries, Trecroci and Vassalli (2010) conclude that this TVP algorithm outperforms the GMM based estimation of reaction functions in tracking the policy interest rate.

Kuzin (2006) uses a backward-looking Taylor rule with time-dependent coefficients for Germany only. The Bundesbank's preference on the weighting of inflation and the output gap is allowed to evolve over time. His conclusion is similar to the work of Trecroci and Vassalli (2010) in that the time-varying parameter Taylor rule describes well the evolution of monetary policy in the Bundesbank.

Trehan and Wu (2007) contribute to the estimation of the equilibrium real rate of interest (ERR) in the US. In contrast to studies which are carried out under the assumption that the long-run real rate of interest is time-invariant, Trehan and Wu (2007) estimate a time-varying ERR with a model similar to Leigh (2008). They discover that ignoring a time-varying ERR is likely to exaggerate the amount of interest rate smoothing and lead to a substantial upwards bias in the estimated coefficient on inflation.

Yuksel, Ozcan and Hatipoglu (2013) introduce the extended Kalman filter algorithm as a new method for explaining monetary policy in Turkey. They argue that although the standard Kalman filter algorithm is an influential technique in estimating linear transformations it fails to be a reliable approach for non-linear state-space forms (also see, Harvey, 1990). For example, the non-linear dynamics (state and observations) is written as:

$$x_{t+1} = f_t(x_t) + w_t$$

$$y_t = h_t(x_t) + v_t$$

where $\{w_t\}$ and $\{v_t\}$ are white Gaussian, independent random processes with zero mean and covariance; $x(t)$ is the system state vector. The term $y(t)$ is an observation vector. The function $f_t(\ldots)$ defines the system's non-linear dynamic which distinguishes itself from the standard Kalman filter algorithm that assumes the dynamic is linear. Thus, as in Yuksel at al. (2013), when modelling monetary policy and TVPs simultaneously, this system takes a non-linear form and the extended Kalman filter algorithm would be necessary as the appropriate tool.

Furthermore, Yuksel et al. (2013) include an interest rate pass-through specification with a Taylor rule in the New Keynesian framework. In particular a time-varying interest rate pass-through model is added to the structural model in a simple way in order to account for the monetary transmission mechanism. The empirical findings support non-linearity in a Taylor rule. They show that the extended Kalman filter algorithm outperforms the standard Kalman filter algorithm by reducing 15% of mean squared errors.

5.2.2 Time-varying Parameter Models with Stochastic Volatility

In all the non-linear literature discussed so far, changes in coefficients are well studied while the variance of structural shocks is assumed to be time-invariant over the sample or subsample period. Primiceri (2005) and Nakajima (2011a) highlight the role of stochastic volatility (SV). In many cases, data-generating processes of economic variables seem to have drifting coefficients and stochastic volatility shocks. Hence, the application of a model with TVPs but constant volatility (see, for instance, Trecroci and Vassalli, 2010; Yuksel et al., 2013) may result in biases in estimated coefficients. However, the inclusion of SV makes the estimation process difficult because the likelihood function becomes intractable. For the estimation of the TVP-VAR-SV model, Primiceri (2005) introduces the Markov Chain Monte Carlo (MCMC) in the context of a Bayesian inference which is a natural setup to account for uncertainties in models and parameters.

This subsection begins with a brief introduction to Bayesian inference and then moves on to the MCMC algorithm. It also discusses the use of the TVP-VAR-SV model in the existing studies emphasising some important contributions delivered by this study.

5.2.2.1 Bayesian Inference

Almost all applications of the MCMC, including those used in estimating the TVP-VAR-SV model, are oriented towards Bayesian inference which refers to a procedure of fitting a probability model to a data set and summarising results through a probability distribution on the parameters of that model.

Let the sample space $\mathcal{Y}$ denote the set of all possible datasets from which a single dataset (y) will result. The parameter space Θ represents the set of possible parameter values from which Bayesian analysts hope to identify values that best represent the true population characteristics. Formal Bayesian inference starts with a numerical formulation of joint belief about y and θ stated in terms of probability distributions over Y and Θ. The joint probability distribution $\mathrm{P}(\mathrm{y}, \theta)$ is comprised of two parts, a prior distribution $P(\theta)$ and a likelihood function $P(y|\theta)$:

$$P(y, \theta) = P(y|\theta)P(\theta) \tag{2.1}$$

The posterior distribution $P(\theta|y)$ is obtained from the prior and the likelihood function via Bayes' rule:

$$P(\theta|y) = \frac{P(y|\theta)P(\theta)}{P(y)} = \frac{P(y|\theta)P(\theta)}{\int_{\Theta} P(y|\theta)P(\theta)d\theta} \tag{2.2}$$

where the distribution of θ conditional on y is usually the objective of all Bayesian learning.

Cox (1946, 1961) and Savage (1954, 1972) provide strong theoretical justification for the use of Bayes' rule as a method of quantitative learning. As discussed by Hoff (2009, p.2), the mathematical results in Cox (1946, 1961) and Savage (1954, 1972) show that if $P(\theta)$ and $P(y|\theta)$ represent a rational person's belief then the Bayes' rule is an optimal method of updating this person's beliefs about θ given new information y. According to Box and Draper (1987, p.424), even if $P(\theta)$ fails to accurately represent the belief $P(\theta|y)$ is still useful – if $P(\theta)$ approximates the prior belief, the fact that $P(\theta|y)$ is optimal under $P(\theta)$ means that it serves as a good approximation to what the posterior beliefs should be.

Features of the posterior distribution $P(\theta|y)$, including moments and highest posterior density regions, are all legitimate for the Bayesian inference. All these quantities can be expressed in terms of posterior expectations of functions of θ (Gilks, Richardson and Spiegelhalter, 1996):

$$E[f(\theta)|y] = \frac{\int f(\theta)P(y|\theta)P(\theta)\,d\theta}{\int P(y|\theta)P(\theta)\,d\theta} \tag{2.3}$$

where $f(.)$ denotes the function of interest.

Also according to Gilks et al. (1996), the integrations in Eq. (2.3) are the source of the practical difficulties in the Bayesian inference. In most situations, the analytical evaluation of $E[f(\theta)|y]$ is impossible and alternative methods include numerical evaluation, analytical approximation and Monte Carlo integration. However, the numerical evaluation is not appropriate every time. This motivates the use of the Monte Carlo approach in the case of high-dimensional distributions like the estimation of a TVP-VAR-SV model.

5.2.2.2 Monte Carlo Integration and Markov Chains

The objective of Monte Carlo integration is to obtain a sequence of parameter values $\{\theta^{(1)}, \ldots, \theta^{(S)}\}$ from the posterior distribution and then to approximate:

$$\frac{1}{S}\sum_{s=1}^{S} f\left(\theta^{(S)}\right) \to E[f(\theta)] = \int f(\theta)P(\theta)\, d\theta \tag{2.4}$$

as $S \to \infty$. This means that the average of $\{f(\theta^{(1)}), f(\theta^{(2)}), \ldots, f(\theta^{(S)})\}$ can be used to approximate expected values of $f(\theta)$ under a target probability distribution which always refers to the posterior distribution $P(\theta|y)$. If the sequence $\{\theta^{(1)}, \ldots, \theta^{(S)}\}$ is independent, the approximation can be made as accurate as desired by increasing the value of S in Eq. (2.4).

In the application of the Monte Carlo method, a posterior distribution is sometimes non-standard which raises difficulties in drawing samples $\{\theta^{(1)}, \ldots, \theta^{(S)}\}$ from the target distribution. However, as demonstrated in the existing Bayesian literature, e.g., Gilks et al. (1996, p.4), Hoff (2009, p.96-97) and Gelman, Carlin, Stern, Dunson, Vehtari and Rubin (2004, p.276-277), $\{\theta^{(1)}, \ldots, \theta^{(S)}\}$ need not necessarily be independent. In the case of having a non-standard target distribution, samples can be generated through a Markov chain: to generate a sequence of random variables $\{\theta^{(0)}, \theta^{(1)}, \theta^{(2)}, \ldots\}$ such that at each time $S \geq 0$ the next state $\theta^{(S+1)}$ is sampled from a conditional distribution $P\left(\theta^{(S+1)}|\theta^{(S)}\right)$ which depends only on the current state of the chain $\theta^{(S)}$. Therefore $\theta^{(S+1)}$ is conditionally independent of $\theta^{(0)}, \theta^{(1)}, \ldots, \theta^{(S-1)}$ given $\theta^{(S)}$, which is called the Markov property. A sequence generated in this way is called a Markov chain.

Another issue in the Markov chain is the effect of the starting state $\theta^{(0)}$ on the current state $\theta^{(S)}$. Let $P^{(S)}\left(\theta^{(S)}|\theta^{(0)}\right)$ denote the distribution of $\theta^{(S)}$ given $\theta^{(0)}$. Subject to the regularity condition this chain will gradually forget its initial state. $P^{(S)}\left(\theta^{(S)}|\theta^{(0)}\right)$ will eventually converge to a unique stationary distribution, which does not depend on $\theta^{(0)}$ or S. Define $\varpi(.)$ as the stationary distribution. Therefore, after a long enough burn-in[4] of Q iterations $\left\{\theta^{(Q+1)}, \ldots, \theta^{(S)}\right\}$ could be approximate dependent samples from the distribution $\varpi(.)$. Given the spirit of Eq. (2.4) the output from the Markov chain can be used to estimate the expectation of $f(\theta)$, which is called the MCMC algorithm.

$$\frac{1}{S-Q}\sum_{s=Q+1}^{S} f\left(\theta^{(S)}\right) \rightarrow E[f(\theta)] \tag{2.5}$$

as S $\rightarrow \infty$, where θ has a distribution $\varpi(.)$ and the burn-in samples are discarded from this calculation.

5.2.2.3 Time-varying Parameter VAR with Stochastic Volatility

A vector autoregression model is a system of linear equations, one for each variable. In the primitive form, each equation specifies one of these variables as a linear function of its own lags as well as current and past values of the other variables in the system.

Since its introduction into economics by Sims (1980), the VAR model has been widely used in empirical studies of monetary policy. Jacobson et al. (2001) provide a rationale for using the VAR model instead of a single equation for investigating an interest rate reaction function. As in Jacobson et al. (2001), in the process of setting monetary policy, central banks are faced with a number of empirical questions such as 'does the nominal exchange rate help to predict inflation?', 'how useful are various measures of the output gap and inflation rate?', 'how fast do changes in monetary policy affect economic indicators like output, the inflation rate and the exchange rate?', etc. All these concern complex relationships between variables which are all endogenous and simultaneously determined in the economic system. This implies that a monetary policy reaction model should not only consider central banks' response to

[4] The term 'burn-in' refer to the practice of discarding an initial portion of a Markov chain sample with a purpose of minimising the effect of initial values on the posterior inference.

economic indicators but also take into account the effect of monetary decisions on an economy (also see, Bernanke, Gertler and Watson, 1997).

A common procedure in the literature is to develop a system model like a VAR that intends to handle all questions at one time and to give useful information about aggregate relationships between the variables. As in Jacobson et al. (2001), there is no single equation model that can provide the best possible answers to all relevant questions in the analysis of monetary policy, however a VAR model can serve these purposes. Bernanke et al. (1997) maintain that a VAR model can identify innovations to monetary policy with a shock to some policy indicators like the federal fund rate. As in Bernanke et al. (1997), using an estimated VAR, one can trace out dynamic responses of output, prices and other macroeconomic variables to this innovation and obtain quantitative estimates of how monetary policy innovations affect the economy.

Although a VAR model is a basic and powerful econometric tool for monetary policy analysis, Rudebusch (1998) maintains that a conventional VAR with time-invariant parameters is not sufficient to account for the process of setting the interest rate. Nakajima (2011a) maintains that the TVP-VAR-SV model developed in Primiceri (2005) would enable analysts to capture the potential time-varying nature of the underlying structure in an economy in a flexible and robust manner. All parameters in the TVP-VAR-SV model are set to follow a random walk process thereby allowing both temporary and permanent changes in the parameters.

Regarding estimation procedures, Malik and Banerjee (2013) propose a number of reasons for preferring the Bayesian inference via MCMC on the TVP-VAR-SV model over the classical maximum likelihood (ML) approach. For example, although it is possible to write a likelihood function for such a TVP-VAR-SV model, from a practical perspective it is quite computationally expensive to optimise over such a high-dimensional space. As explained in Section 5.2.2.1, strategies based on the MCMC should do well in the case of estimating a TVP-VAR-SV model.

After developing the TVP-VAR-SV model, Primiceri (2005) studies responses of the short-term interest rate to inflation and the unemployment rate in the US with this tool. Using quarterly data, the sample covers the period 1953:I-2001:III. Primiceri (2005) distinguishes systematic responses which are the responses of the interest rate to inflation and the unemployment rate from non-systematic responses that include policy mistakes and responses to other variables. The results indicate that both systematic and non-systematic monetary policies change throughout the sample period. The interest rate responses to inflation are often gradual, i.e., it takes time for the interest rate to reach a long-run response level after an inflationary shock. By contrast, the Fed reacts to unemployment much faster than inflation. However, the responses of the interest rate to inflation and unemployment both exhibit quite a limited effect on the rest of the US economy.

Following the introduction of the TVP-VAR-SV model by Primiceri (2005), there appears to be a growing interest by academics in studying the Taylor rule with this statistical model (see, for instance, Nakajima, 2011a; Nakajima, Kasuya and Watanabe, 2011; Kengne et al., 2013; Malik and Banerjee, 2013).

Kengne et al. (2013) use the Markov Switching Autoregressive together with a TVP-VAR-SV model to assess the response of monetary policy in the US to house prices and stock prices. They identify a greater response of the interest rate to asset price shocks in bull regimes. This corresponds with a larger effect of interest rate shocks on asset prices during periods of an economic boom.

Malik and Banerjee (2013) use the Livingston Survey data for expected inflation in their TVP-VAR-SV model. Their estimation justifies the use of time variation to address questions concerning the response of the interest rate to shocks in inflation expectations.

Nakajima (2011a) initiates the application of the TVP-VAR-SV model for Japan, where he employs a more technical method, a multi-move sampler, for sampling SVs in a TVP-VAR-SV model. According to Nakajima (2011a), there are mainly two methodologies for sampling SVs in the existing literature. Primiceri (2005) adapts the mixture-sampler which approximates non-linear Gaussian state-space models (including a TVP-VAR-SV model) by normal mixture sampler converting the original model into a linear Gaussian state-space form. In contrast, the multi-move sampler approaches the model by drawing samples from an exact condition posterior density of the original model. The empirical application shows the time-varying nature of the dynamic relationship between output, inflation and the interest rate.

In another paper, Nakajima together with Kasuya and Watanabe (2011) again use the multi-move sampler and estimate a four-variable (including the policy rate, inflation, the index of industrial production and the monetary base) TVP-VAR-SV for the Bank of Japan. For comparative purposes, they calculate the marginal likelihood of their TVP-VAR-SV model and other VARs under different priors, lags and sample periods. The results confirm that the TVP-VAR-SV model best fits the Japanese economy.

5.2.3 Empirical Research on UK Monetary Policy

There is a vast discussion on the BOE's interest rate reaction function which includes, but is not limited to, Nelson (2000), Neumann and Hagen (2002), Wesche (2006), Martin and Milas (2004), Cukierman and Muscatelli (2008), Barnett et al. (2010), Trecroci and Vassalli (2010), Castro (2011), Bruggemann and Riedel (2011) and Lafuente et al. (2014).

Nelson (2000) is one of the first studies to estimate the interest rate reaction function in the UK. He uses the subsample analysis by splitting the estimation from June 1972 – May 1997 into 6 distinct regimes and estimating Taylor rules for each regime (except for the EMS period). Both backward-looking and forward-looking versions are used for each subsample period. He discovers that coefficients on inflation and the output gap change across subsamples. The implied Taylor rules are also different in each regime. For example, in 1972-1976 the BOE appeared to shift its short-term interest rate mainly in response to the past output gap and to a very limited extent inflation. However, in 1979-1987 the BOE adjusted its interest rate in reaction to estimated current inflation rather than to inflation expectations. A forward-looking Taylor rule outperformed the backward-looking rule in 1992-1997. With a similar methodology, Neumann and Hagen (2002) also find possible shifts in response coefficients.

Castro applies the STR as a non-linear estimator to real-time data and discovers that the BOE's monetary policy can be better described by a non-linear Taylor rule (against a linear rule). With the same estimator, Martin and Milas (2004) show that the response of monetary policy to inflation is non-linear as the interest rate reacts more vigorously to upward than to downward deviations of the inflation rate from the target. The other studies such as Cukierman and Muscatelli (2008) and Bruggemann and Riedel (2011) also find non-linearities in similar reaction functions.

Trecroci and Vassalli (2010) maintain that reduced-form policy models which do not allow for shifts and asymmetries in behavioural relations may give misleading results. They discuss a number of popular non-linear approaches including a dynamic stochastic general equilibrium (DSGE) model and subsample analysis. Two conclusions in Trecroci and Vassalli (2010) stand out. Firstly, a DSGE model based on the micro-foundation description of an economy imposes many restrictions on the data. In the specific context of interest rate rules, its ability to get qualitative and robust assessments on monetary policy conduct appears problematic. Secondly, subsample analysis cannot capture gradual changes in policy. It will lead to problematic interpretations when actual regimes shifts do not fit exactly into one of the modelled regimes. Such a method only accounts for variation across averages of policy responses in each regime but ignores changes in the responses within each regime.

To capture gradual changes in the BOE' policy response to economic indicators, Trecroci and Vassalli (2010) use the Kalman filter algorithm. The coefficients in the Taylor rule are allowed to evolve at each point of time. The empirical estimates show that the coefficient on inflation turns out to be significant and substantially growing in magnitude over almost the entire sample period. It points to policy stance becoming gradually more inflation-averse over time.

Lafuente et al. (2014) employ the particle filter algorithm to overcome the non-optimality of the Kalman filter algorithm that arises as a result of the non-linear dynamics for time evolution of monetary shocks. For robustness purposes, they also estimate a Markov switching model. They arrive at the same conclusion from both the particle filter algorithm and Markov switching analysis. In the period (such as 1999 and 2008) for which the Markov switching model implies a regime shift towards expansionary monetary policy, the particle filter algorithm discovers that the current inflation target is systematically above its long-run mean and *vice versa*. (note, this finding also ties in with the results in the earlier chapter; recall, Chapter 3 discovers that the estimated 'implicit' short-term inflation target rises when the short-term interest rate is close to the ZLB).

In the aforementioned non-linear literature on the BOE's policy rules, the econometric estimations are all 'one-direction' i.e., they concentrate on the determinants of the interest rate without examining monetary policy transmissions. Recall the argument of Bernanke et al. (1997) and Jacobson et al. (2001) in Section 5.2.2.3, a monetary reaction model is required to consider both interest rate reactions to economic indicators and the effect of monetary policy on an economy. In the UK, the monetary policy committee (MPC) sets the short-term interest rate at which the BOE deals with the domestic money market. Decisions about the interest rate that affects inflation and economic activity through several channels are known collectively as the monetary transmission mechanism (see, MPC, June 2012). In regards to timing, the MPC estimates that monetary policy has its largest effect on output with a lag of around one year and on inflation with a lag of around two years.

In order to consider the BOE's monetary policy response and the impacts of interest rate changes on the UK's output and the inflation rate, a VAR model seems necessary here. As in Rudebusch (2005), the VAR model is typically a reduced-form representation of an economy.

Barnett et al. (2010) investigate how the interaction between inflation expectations and macroeconomic variables (including the retail inflation rate, the three-month treasury bill interest rate, the annual oil price and the annual GDP growth rate) has evolved in the UK over the post World War II period until 2007. They model time-variation with a Markov-switching structural VAR (MS-VAR). A contractionary monetary policy shock is identified by assuming that it leads to a contemporaneous increase in the short-term interest rate, a fall in inflation and reduced GDP growth. The empirical results show a statistically significant but negative reaction of actual inflation to a contractionary monetary policy in the inflation-targeting period. The inflation response is insignificant in the 1970s, but GDP growth always decreases in response to an exogenous monetary policy contraction as expected.

Table 2 summarises the empirical literature on the BOE's interest rate reaction functions in the UK. It is particularly interesting to note that to the knowledge of the author no one has used the TVP-VAR-SV model for modelling the BOE's short-term interest rate yet. This motivates the use of such an estimator in this study. The advantage of using the TVP-VAR-SV model has been discussed in Section 5.2.2.3.

The TVP-VAR-SV analysis presented in the following sections is expected to contribute to the existing literature by presenting a time-varying parameter reduced-form model of the UK economy and quantitatively describing the process of setting monetary policy within the BOE. It considers both the MPC's reaction to inflation and output and how their decision impacts on economic activity. This chapter also evaluates the robustness of the earlier results (in Chapter 4) that the BOE's monetary policy implementation has changed during the sample period.

Table 2: Summary: Empirical Studies on the BOE's Interest Rate Reaction Function

Methods	Author	Year	Relevant Conclusions
Subsample (use OLS and GMM)	Nelson	2000	The BOE's interest setting behaviour change across the subsamples.
Subsample (use VAR)	Neumann and Hagen	2002	The BOE's interest setting behaviour change after the introduction of inflation targeting.
Smooth transition regression	Martin and Milas	2004	The response of the interest rate to inflation is non-linear.
	Castro	2011	The response of the interest rate to inflation is non-linear and asymmetric.
	Cukierman and Muscatelli Bruggemann and Riedel	2008 2011	The STR outperforms a simple linear specification in terms of model fit and ability to track the interest rate.
Kalman filter	Trecroci and Vassalli	2010	The BOE is gradually more inflation-averse over 1980-2005.
Markov-switching	Lafuente et al.	2014	The BOE's monetary policy is modelled better with a two-state switching model than with a linear model.
Particle filter algorithm	Lafuente et al.	2014	For periods of expansionary monetary policies, the particle filter algorithm suggests that the current inflation target is above its long-run mean.
MS-VAR	Barnett et al.	2010	In the inflation-targeting period, inflation and output negatively react to a contractionary monetary shock as expected.

5.3 Data

This study sources data from the BOE, the Office for National Statistics (ONS) and the OECD. The data used is quarterly. The sample period is from 1993:I to 2013:II. During this time the MPC has been operating an inflation targeting approach and reporting its inflation forecasts on a quarterly basis.

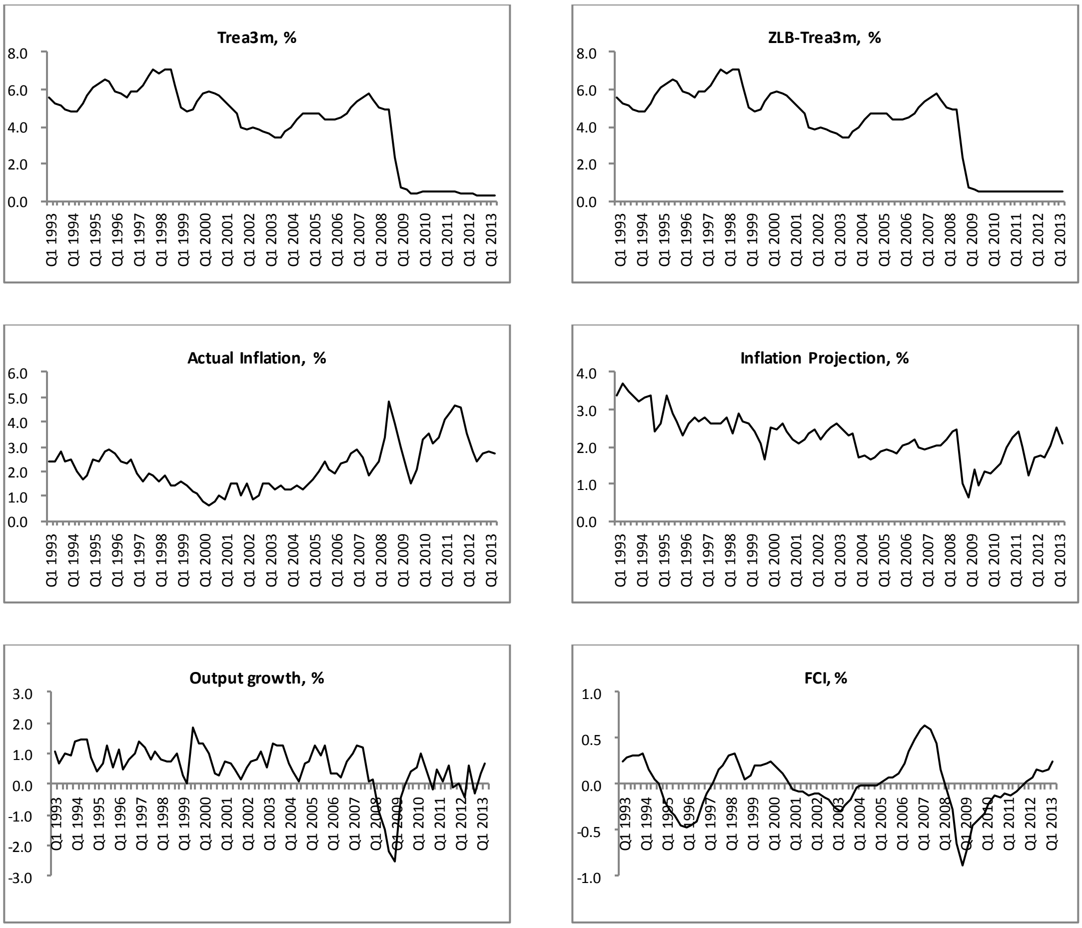


Figure 1: Variables for Modelling the BOE's Monetary Policy

The alternative interest rate measures considered include the official central bank interest rate, the three-month inter-bank sterling lending rate and the discount rate of three-month Treasury bills (Treas3m). Nelson (2000) argues that actual interest rate instruments used by the BOE have changed over time. These rates include the bank rate, the minimum lending rate, the two-week repo rate, etc. To deal with this, Nelson (2000), Martin and Milas (2004) and Castro (2011) argue that the Treas3m has a close relationship with all the interest rate instruments used in the BOE's history.

Consequently this study uses the Treas3m as the nominal interest rate for the sample period analysed.

The evolution of the Trea3m is plotted in Figure 1 (upper left panel). It is particularly important to note that the recent financial crisis has lead the BOE to cut its interest rate to a historical low. As mentioned in Swanson and Williams (2013), since lowering its bank rate to 50 bps in early 2009 the BOE has conducted large-scale asset purchases as an alternative to cutting the interest rate. This implies that 50 bps is viewed as an effective lower bound on the UK monetary policy rate under the period reviewed.

Before August 2016, the MPC also judged that the bank rate could not practically be reduced below 50 bps. Therefore, this study regards 50 bps as the effective zero lower bound over the period under review. It is agreed in studies such as Bernanke and Reinhart (2004), Swanson and Williams (2013) and Williams (2014) that when the short-term policy rate approaches (or is at) the effective ZLB, conventional means of monetary easing are no longer feasible. Under these circumstances, central banks may consider other strategies (such as quantitative easing) to stimulate economies. Hence, Nakajima (2011b) introduces a lower bound (c) to the nominal interest rate in his estimation. When the interest rate drops below c, it is modified back to the lower bound which is also referred to as an effective ZLB[5]. Given the recent low levels of the interest rate in the UK, this study decides to follow this modification:

$$r_t = \begin{cases} r_t & (if\ r_t > 0.5\%) \\ 0.5\% & (if\ r_t \leq 0.5\%) \end{cases} \tag{3.1}$$

where r_t denotes the Trea3m.

Following the BOE, the inflation rate is computed as the annual rate of change in the CPI. However, official CPI statistics in the UK only started in 1996. Historical estimations of inflation back to 1988 are calculated by the ONS based on the retail

[5] Nakajima (2011b) introduces both the short-term interest rate and the medium-term interest rate to his VAR model. When the short-term interest rate is at the zero lower bound, he assumes the Taylor-rule parameter on the inflation rate and output to diminish to zero. Therefore, the Nakajima (2011b) VAR implicitly assumes that central bank adjusts economic activity and inflation using the medium-term interest rate. However, there is no evidence in the UK indicating that the BOE uses the medium-term interest rate as the policy instrument. Hence, instead of following the Nakajima (2011b) settings, this study sets all VAR parameters in a manner which is usually accepted in the existing TVP-VAR-SV literature (see, Eq. 4.10-4.11 for details). However, it would be interesting for future research to introduce other policy instruments (such as money supply) into the TVP-VAR-SV model when studying the zero interest rate period.

price index (RPI). Following the existing literature in this field such as Martin and Milas (2004), this study calculates the inflation rate with the RPI for the period of 1993-1996. To assess the forward-looking behaviour of the BOE, this study employs the MPC's forecasts for inflation. The reported inflation projection is published in the form of charts showing the mean projection (i.e., central projection) together with the estimation of uncertainty based on the historical mean absolute error. This inflation projection represents the MPC's best collective judgement of the outcome for the inflation rate. In Chapter 4, this study estimates that a leading horizon of six quarters will best describe the BOE's forward-looking behaviour. Thus, the same leading horizon of inflation is used in this chapter. This study transforms the CPI (RPI before 1996) as follows:

$$\pi_t = ln(CPI_t) - ln(CPI_{t-4}) = ln\left(\frac{p_t}{100} + 1\right) \tag{3.2}$$

$$\pi_{t+6}^e = ln\left(\frac{p_{t+6}^e}{100} + 1\right) \tag{3.3}$$

where p_{t+6}^e is the MPC's inflation projection expressed in percentage terms and p_t is the actual inflation rate. The t subscripts denote time.

A measure of UK financial market conditions is obtained from Chapter 2. The use of the FCI is to examine whether the BOE reacts to developments in financial markets in a time-varying parameter model. This index measures the deviation between current financial market conditions and their long-run trend. A positive number reflects an improvement in financial conditions and *vice versa*.

For the purpose of comparing with other TVP-VAR-SV literature such as Nakajima et al. (2011), this study uses the output growth rate (measured with real GDP) to assess economic activity in the UK. Let y_t be the real GDP growth rate. It is obtained as follows:

$$y_t = ln\ (real\ GDP_t) - ln\ (real\ GDP_{t-1}) \tag{3.4}$$

5.4 Method

Using a TVP-VAR-SV model, this study proceeds to model an interest rate reaction function and the monetary policy transmission jointly in a VAR structure. It is crucial to test: (i) whether the changes in the BOE's monetary policy could be explained with a time-varying parameter rule and (ii) if so, whether the volatility of the VAR model should be allowed to be time-varying throughout the sample period.

A multivariate TVP-VAR-SV model is an extension of a univariate time-varying parameter regression with stochastic volatility (TVP-R-SV) introduced in Chapter 1. Following Primiceri (2005) and Nakajima (2011a), the TVP-VAR-SV model begins with a basic structural VAR defined as:

$$Ax_t = B_1 x_{t-1} + B_2 x_{t-2} + \cdots + B_s x_{t-s} + \epsilon_t \tag{4.1}$$

where x_t is a vector of five observed variables in this study. In Eq. (4.1), A, B_1, …, B_t are 5×5 matrices of coefficients. The disturbance ϵ_t is a structural shock. Specify simultaneous relations of structural shocks by recursive identification assuming that A is a lower triangular matrix:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & 1 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & 1 \end{pmatrix} \tag{4.2}$$

As in Nakajima (2011a), the specification of a lower-triangular matrix for A is simple and widely used for VAR systems.

Then re-write Eq. (4.1) as the reduced form VAR model:

$$x_t = L_1 x_{t-1} + L_2 x_{t-2} + \cdots + L_s x_{t-s} + A^{-1} \textstyle\sum \tau_t, \tau_t \sim N(0, I_5) \tag{4.3}$$

where $L_i = A^{-1} B_i$, for $i = 1, \ldots, s$ and

$$\Sigma = \begin{pmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5 \end{pmatrix} \tag{4.4}$$

The σ_1, σ_2, …, σ_5 are the standard deviations of structural shocks.

Stack the elements in the rows of the L_i's to form β and define Q_t as:

$$Q_t = I_5 \otimes (x'_{t-1}, x'_{t-2}, \dots, x'_{t-s}) \tag{4.5}$$

where $\otimes$ denotes the Kronecker product. Then the VAR can be written as:

$$x_t = Q_t\beta + A^{-1}\Sigma\tau_t \tag{4.6}$$

At this stage, all parameters (in Eq. 4.6) are time-invariant. Extend the basic VAR to the TVP-VAR-SV model and set both parameters and volatilities to change over time:

$$x_t = Q_t\beta_t + A_t^{-1}\Sigma_t\tau_t \tag{4.7}$$

where $t = s + 1, \dots, n$. The coefficients β_t and the parameters A_t and Σ_t are all time-varying. As proposed in earlier studies such as Kengne et al. (2013), the order of a TVP-VAR-SV model can be determined based on the Schwarz information criterion (SIC) applied to a stable constant-parameter VAR:

$$SIC = -\frac{2LK}{T} + \frac{n\,log\,T}{T} \tag{4.8}$$

where n in Eq. (4.8) is the number of parameters estimated using T observations and LK is the log likelihood that is conducted by looking at the difference between the log likelihood values of restricted and unrestricted versions of an equation:

$$LK = -\frac{T}{2}[1 + log(2\pi) + log(\hat{\epsilon}'\hat{\epsilon}/T)] \tag{4.9}$$

When using the SIC, the length of a lag distribution (n) is determined by choosing the specification with the lowest value of the SIC.

Let a_t be a stacked vector of the lower triangular elements in matrices A_t and $h_t = (h_{1t}, h_{2t}, h_{3t}, h_{4t}, h_{5t})'$ with $h_{jt} = log\sigma_{jt}^2$ for $j = 1, 2, \dots, 5$ and $t = s + 1, \dots, n$.

Following the TVP-VAR-SV literature, this TVP-VAR-SV model assumes a random walk process for all the parameters in Eq. (4.7). This is expected to reduce the number of parameters to estimate:

$$\beta_{t+1} = \beta_t + \epsilon_{\beta t} \tag{4.10}$$

$$a_{t+1} = a_t + \epsilon_{at} \tag{4.11}$$

$$h_{t+1} = h_t + \epsilon_{ht} \tag{4.12}$$

$$\begin{pmatrix} \tau_t \\ \epsilon_{\beta t} \\ \epsilon_{at} \\ \epsilon_{ht} \end{pmatrix} \sim N\left(0, \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & \sum_\beta & 0 & 0 \\ 0 & 0 & \sum_a & 0 \\ 0 & 0 & 0 & \sum_h \end{pmatrix}\right) \tag{4.13}$$

for $t = s+1, \ldots, n$. The initial states for time-varying parameters are $\beta_{s+1} \sim N\left(\epsilon_{\beta 0}, \sum_{\beta_0}\right)$, $a_{s+1} \sim N\left(\epsilon_{a0}, \sum_{a_0}\right)$ and $h_{s+1} \sim N\left(\epsilon_{h0}, \sum_{h_0}\right)$. By treating TVPs as latent variables, the system consisting of Eq. (4.7) and Eq. (4.10-4.13) forms a state-space specification. The shocks to innovations of TVPs are assumed uncorrelated among parameters β_t, a_t and h_t. Furthermore, $\sum_\beta$, $\sum_a$ and $\sum_h$ are assumed to be diagonal, because Nakajima (2011a) suggests that this assumption is insensitive for results compared to a non-diagonal assumption. Drifting coefficients and parameters are modelled to fully capture possible changes in the VAR structure over time.

Following Primiceri (2005), this study estimates the TVP-VAR-SV model using the MCMC procedure in the context of the Bayesian inference. Let $x = \{x_t\}_{t=1}^T$, the latent states $\beta = \{\beta_t\}_{t=s+1}^T$, $a = \{a_t\}_{t=s+1}^T$ and $h = \{h_t\}_{t=s+1}^T$ and the set of hyper-parameters $\omega = \left(\sum_\beta, \sum_a, \sum_h\right)$. The prior probability density is $q(\omega) = \omega$. The MCMC algorithm for the TVP-VAR-SV model is described below:

Step (1): Initialise β, a, h, ω.

Step (2): Sample $\beta | a, h, \sum_\beta, x$.

Step (3): Sample $\sum_\beta | \beta$.

Step (4): Sample $a | \beta, h, \sum_a, x$.

Step (5): Sample $\sum_a|a$.

Step (6): Sample $h|\beta, a, \sum_h, x$.

Step (7): Sample $\sum_h|h$.

Step (8): Go to Step 2, and repeat.

As described in Chapter 1, the simulation smoother, originated by Jong and Shephard (1995), is an effective and efficient tool to sample the TVPs in a TVP-R-SV model. In this study, it is also used in order to sample parameters β and a:

To draw $\beta\sim\pi\left(\beta|a, h, \sum_\beta, x\right)$ in Step (2), write a state-space model with β_t as the state variable:

$$x_t = Q_t\beta_t + A_t^{-1}\textstyle\sum_t\tau_t, \quad t = s+1, \dots, n$$

$$\beta_{t+1} = \beta_t + \epsilon_{\beta t}, \quad t = s, \dots, n-1$$

where $\beta_s = \epsilon_{\beta 0}$ and $\epsilon_{\beta s}\sim N\left(0, \sum_{\beta 0}\right)$. The above system is a combination of Eq. (4.7) and Eq. (4.10). Run the simulation smoother with the variables corresponding to a general state space model (Jong and Shephard, 1995, p. 343):

$$Z_t = Q_t, \quad G_t = \left(A_t^{-1}\textstyle\sum_t, 0_{k_\beta}\right), \tag{4.14}$$

$$T_t = I_{k_\beta}, \quad H_t = \left(0_5, \textstyle\sum_\beta^{0.5}\right), \quad H_0 = \left(0_5, \textstyle\sum_{\beta_0}^{0.5}\right),$$

where k_β is the number of rows in β_t. The standard state space model in Jong and Shephard (1995, p. 343) is expressed as:

$$x_t = Z_t\alpha_t + G_t u_t, \quad (t = 1, \dots, n)$$

$$\alpha_{t+1} = T_t\alpha_t + H_t u_t, \quad (t = 0, \dots, n-1)$$

To draw $a\sim\pi(a|\beta, h, \sum_a, x)$ in Step (4), re-write the state-space model as follows:

$$\hat{x}_t = \hat{Q}_t a_t + \textstyle\sum_t \tau_t, \quad t = s+1, \dots, n \tag{4.15}$$

$$a_{t+1} = a_t + \epsilon_{\beta t}, \quad t = s, \dots, n-1$$

where $a_s = \epsilon_{a0}$ and $\epsilon_{as} \sim N(0, \sum_{a0})$. As in Nakajima (2011a), Eq. (4.15) is important to use the simulation smoother for sampling a, where:

$$\hat{x}_t = x_t - Q_t \beta_t, \quad t = s+1, \dots, n \tag{4.16}$$

$$\hat{Q}_t = \begin{bmatrix} 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ -\hat{x}_{1t} & 0 & 0 & \cdots & 0 & 0 & 0 & \vdots \\ 0 & -\hat{x}_{1t} & -\hat{x}_{2t} & 0 & \cdots & 0 & 0 & \vdots \\ 0 & 0 & 0 & -\hat{x}_{1t} & \cdots & 0 & 0 & \vdots \\ 0 & 0 & 0 & 0 & -\hat{x}_{1t} & \cdots & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & -\hat{x}_{1t} & \cdots & -\hat{x}_{4t} \end{bmatrix} \tag{4.17}$$

Run the simulation smoother with the variables corresponding to the standard state space model in Jong and Shephard (1995, p. 343):

$$Z_t = \hat{Q}_t, \quad G_t = (\textstyle\sum_t, 0_5), \tag{4.18}$$

$$F_t = I_5, \quad H_t = \left(0_5, \textstyle\sum_\beta^{0.5}\right), \quad H_0 = \left(0_5, \textstyle\sum_{\beta_0}^{0.5}\right),$$

For drawing $h \sim \pi(h|\beta, a, \sum_h, x)$ in Step (6), the multi-move sampler method initiated by Shephard and Pitt (1997) is used. Given the diagonality assumption on $\sum_h$ and $\sum_{h_0}$, inference on $h = \{h_{j,t}\}_{t=s+1}^{T}$ is carried out separately for each j $(= 1, \dots, 5)$. Set $x_{i,t}^*$ as the i$^{\text{th}}$ element of $A_t \hat{x}_t$, and write:

$$x_{i,t}^* = exp\left(h_{i,t}/2\right)\tau_{i,t}, \quad t = s+1, \dots, n \tag{4.19}$$

$$h_{i,t+1} = h_{i,t} + \eta_{i,t}, \quad t = s, \dots, n-1 \tag{4.20}$$

$$\begin{pmatrix} \tau_{i,t} \\ \eta_{i,t} \end{pmatrix} \sim N\left(0, \begin{pmatrix} 1 & 0 \\ 0 & \varrho_i^2 \end{pmatrix}\right) \tag{4.21}$$

where $\eta_{i,t}$ is the i$^{\text{th}}$ element of ϵ_{ht}. $\eta_{i,s} \sim N\left(0, \varrho_{i_0}^2\right)$ and ϱ_i^2 is the i$^{\text{th}}$ diagonal elements of $\sum_h$. The multi-move algorithm for drawing the sample $\left(h_{i,s+1}, \dots, h_{i,n}\right)$ is similar to the method for sampling the SVs in a TVP-R-SV in Chapter 1.

Aside from the three sampling processes (for β, a and h) discussed above, implementation of the other steps are quite straight-forward. Following the exposition closely in Malik and Banerjee (2013), this study samples $\sum_{\beta}$, $\sum_{a}$ and $\sum_{h}$ from a Wishart or Gamma distribution under conjugate priors which is similar to the methodology for sampling $\sum$, ϕ, σ_{η} and γ in a TVP-R-SV.

Finally, as discussed by Malik and Banerjee (2013) for the US economy, repeating Step. 1-8 for K iterations the algorithm randomly generates a sequence $\{\beta^k, a^k, h^k, \omega^k\}_{k=1}^{K}$ which is unlikely to be IID (independent and identical distributed) but forms a Markov chain. The distribution of the chain converges to the target distribution $q(\beta, a, h, \omega|x)$. The samples from the joint posterior can be used for the estimation of parameters and state variables using the Markov Carlo method. For example, for a function $p(\beta, a, h, \omega)$ the Markov Carlo estimate of the expected $E[p(\beta, a, h, \omega)|x]$ is $\frac{1}{K}\sum_{k=1}^{K} p(\beta^k, a^k, h^k, \omega^k)$. Therefore, in addition to the convergence of the Markov chain there is the other convergence of $\frac{1}{K}\sum_{k=1}^{K} p(\beta^k, a^k, h^k, \omega^k)$ to expectations $E[p(\beta, a, h, \omega)|x]$. As $K \to \infty$, these two types of the convergence operate simultaneously.

5.5 Empirical Evidence

This section applies the TVP-VAR-SV model, developed so far, to UK macroeconomic data with emphasis on: (i) estimating and explaining the stochastic volatility implied by the model, (ii) examining the response of the interest rate to changes in inflation expectations and economic output, (iii) investigating the monetary policy transmission (i.e., impacts of monetary policy on real economic activity) and (iv) exploring the role of the UK financial system in the BOE's interest rate decisions.

5.5.1 Data and Settings

A five-variable TVP-VAR-SV model is estimated for UK data thereby investigating the time-varying nature of macroeconomic dynamics over the past 20 years in the UK. To this end the following set of variables is examined: (π^e, π, y, fci, r), where π^e is the inflation expectation and π denotes the inflation rate. Both π^e and π are obtained according to Eq. (3.2-3.3). The growth rate of real GDP (y) is obtained according to

Eq. (3.4). The financial conditions index (fci) is considered as the optimal measure of financial conditions in the UK. The nominal Trea3m (r) is modified according to Eq. (3.1). The order of the TVP-VAR-SV model is one which is determined by the SIC applied to a constant parameter VAR. Following Nakajima (2011a), this estimation of the TVP-VAR-SV model assumes that $\sum_{\beta}$ is a diagonal matrix for simplicity.

Since this model is estimated using Bayesian inference, the choice of priors is crucial for solving this model. As in Primiceri (2005), tight priors for the covariance matrix of the disturbances in the random walk process should help avoid unreasonable behaviours of time-varying parameters. He explains that with a prior that favours high degree of time variation (instead of using a tight prior), VAR parameters can change considerably throughout the sample period but just to explain outliers. Their time variation captures much more high frequency variation than the one that would be captured by a pure random walk, which results in implausible parameter estimation. Thus, Primiceri (2005) requires a tighter prior for the time-varying β_t $(t = s + 1, \dots, n)$ than a_t and h_t. Nakajima (2011a) requires a tighter prior for $\sum_{\beta}$ than $\sum_{a}$ and $\sum_{h}$, as the size of structural shocks will fluctuate more severely over time than possible changes in the random walk process. This study follows the default prior specification in the Nakajima (2013) programme[6] and uses the below priors for the ith diagonals of the covariance matrices:

$$(\textstyle\sum_{\beta})_i^{-2} \sim Gamma(20, 10^{-4}) \qquad (\textstyle\sum_{a})_i^{-2} \sim Gamma(4, 10^{-4}) \tag{5.1}$$

$$(\textstyle\sum_{\mathrm{h}})_{\mathrm{i}}^{-2} \sim \mathrm{Gamma}(4, 10^{-4})$$

For initial states of parameters, consistent with Nakajima (2011a, 2013), rather flat priors are assumed $\epsilon_{\beta 0} = \epsilon_{a0} = \epsilon_{h0} = 0$, $\sum_{\beta_0} = \sum_{a_0} = 10 \times I$, and $\sum_{h_0} = 100 \times I$. As illustrated by Nakajima (2011a), there are two ways to specify these priors in the literature. First, set a prior of normal distribution whose mean and variance are determined according to the estimated coefficients of a constant-parameter VAR using a pre-sample period. Second, set a reasonably flat prior for the initial state from the standpoint that we have no information about the initial state a priori. Following Nakajima (2011a, 2013), this study opts to use the second method for simplicity.

[6] Nakajima (2013) provides codes for estimating a TVP-VAR-SV model which is available online [last assessed August 20th 2016]: https://sites.google.com/site/jnakajimaweb/tvpvar

For robustness purposes, this study completes a preliminary sensitivity analysis to check the empirical results with respect to different prior specifications, all of which are normally used in the TVP-VAR-SV literature (e.g., Nakajima, 2011a; Nakajima et al., 2011; Malik and Banerjee, 2011). It does not find any substantial difference in the estimation. Specifically, this means that the estimates are insensitive to prior changes. This finding is consistent with the previous work of Malik and Banerjee (2011). These robustness tests generate voluminous results which this study does not tabulate so as to conserve space. All results are available on request.

Table 3: Estimation Results for the Selected Parameters

Parameter	Mean	Standard Deviation	95% Lower	95% Upper	Convergence Diagnostics	Inefficiency Factor
$(\sum\beta)_1$	0.0023	0.0004	0.0017	0.0033	0.9060	12.29
$(\sum\beta)_2$	0.0023	0.0004	0.0017	0.0032	0.5840	7.620
$(\sum\alpha)_1$	0.0060	0.0031	0.0029	0.0140	0.9340	54.13
$(\sum\alpha)_2$	0.0065	0.0034	0.0030	0.0158	0.2310	63.62
$(\sum h)_1$	0.3202	0.2097	0.0143	0.7692	0.2400	179.4
$(\sum h)_2$	0.1867	0.0900	0.0550	0.3932	0.0640	77.91

Note: in the above table, the following priors are assumed for the ith diagonals of the covariance matrices: $(\sum_\beta)_i^{-2} \sim \mathrm{Gamma}(20, 10^{-4})$, $(\sum_a)_i^{-2} \sim \mathrm{Gamma}(4, 10^{-4})$ and $(\sum_h)_i^{-2} \sim \mathrm{Gamma}(4, 10^{-4})$. For the initial states of time-varying parameters, rather flat priors are set as: $\epsilon_{\beta 0} = \epsilon_{a0} = \epsilon_{h0} = 0$, $\sum_{\beta_0} = \sum_{a_0} = 10 \times I$ and $\sum_{h_0} = 100 \times I$. The estimates of $(\sum\beta)$ and $(\sum\alpha)$ are multiplied by 100.

To generate posterior estimates this study draws $M = 20{,}000$ samples after the initial 2,000 samples are discarded. This burn-in period is determined with the convergence diagnostics (CD):

$$CD = (\bar{x}_0 - \bar{x}_1) \Big/ \sqrt{\frac{\hat{\sigma}_0^2}{n_0} + \frac{\hat{\sigma}_1^2}{n_1}} \tag{5.2}$$

where

$$\bar{x}_j = (1/n_j) \sum_{i=m_j}^{m_j+n_j-1} x^{(i)} \tag{5.3}$$

Table 3 reports the estimation for selected parameters given the above priors in Eq. (5.1). The null hypothesis of convergence to the posterior distribution is not rejected for these parameters at the 5% level of significance based on the CD statistics (in the 6th column). The inefficiency factors are quite low except for $(\sum_h)_1$, which indicates that the burn-in period is enough for the Markov chain to converge in the estimation. Even for $(\sum_h)_1$ the inefficiency factor is roughly 179. This means that there are roughly $20{,}000/179 = 112$ uncorrelated samples obtained which is considered to be sufficient for posterior inference. Therefore, the MCMC algorithm produces posterior draws efficiently.

5.5.2 Time-varying Volatility

Figure 2 presents the series for the estimated stochastic volatility of structural shocks on the five variables based on the posterior mean and 95 percent confidential intervals of the standard deviation of shocks. It shows the time-varying volatility across these five variables.

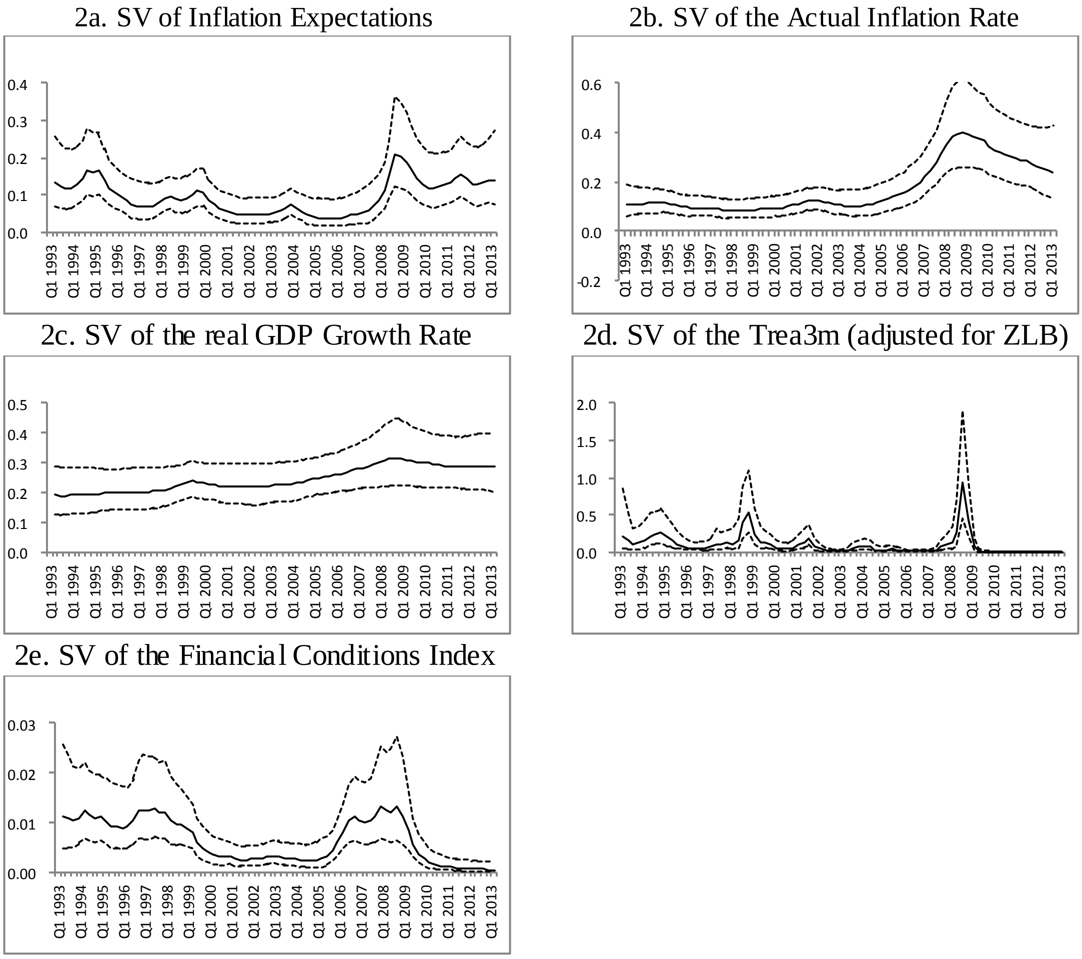


Figure 2: Posterior Estimates for Stochastic Volatility of Structural, Posterior Mean (solid line), +/-1 Standard Deviation (dotted line)

Figure 2a plots the time-varying volatility of inflation expectations. It exhibits several spikes between 1993:I and 2013:II. The first one is in 1994-1995 reflecting the belief that the RPI inflation rate would bottom out in the coming months and would pick up over the next two years (see, Inflation report, November 1994). As illustrated in the bank's Inflation report (February 1995, p.41-43), the sharp rise in expected inflation is attributed to "*the faster-than-expected growth of demand for both labour and output*". The MPC infers that the tightening labour market and quick output growth may contribute together to a faster increase in inflation in the future.

Except for a slight increase around 2000, inflation generally remained subdued between 1996 and 2005. However, the subsequent 2007-2009 period witnessed increased volatility in expected inflation corresponding to the spread of the global financial crisis. In the earlier stage of the crisis, the MPC expected the inflation rate to pick up sharply in the near term, rising to around the threshold at which point the governor of the BOE is required to write to the chancellor. According to the Inflation report published in August 2008 (p.39-41), such expectations reflect higher prices for domestic gas and electricity, petrol and food, and the depreciation of sterling which may push up the level of import prices. The MPC changed its initial expectation later resulting in greater volatility. In its Inflation report in November 2008 (p.36-38), it mentions that even though the inflation rate (3.9%) is still above the official target of 2% the members of the MPC judges that the inflation rate will fall back soon. Therefore, the committee adjusted its inflation forecasts to 1% (from 2.49%) in 2008:IV.

Compared to the volatility of inflation forecasts, the actual inflation rate behaved less smoothly during the recent crisis. Similar to the changes displayed in Figure 1, the volatility of actual inflation π increased substantially in 2007-2009 and hit a peak in 2009:I. Since then it declined but is still at a higher level in comparison with earlier periods. These extremely volatile inflation rates highlight a number of features: (i) high energy, food and import prices pushed up the CPI inflation markedly in 2008:IV (4.8%) doubling that in 2008:I (2.4%); (ii) a sharp decline in inflation was observed around 2009:I which is attributed to two major factors, the fall in energy prices and the cut in VAT in early 2009 (see, Inflation report, February 2009).

Figure 2c plots the volatility of the real GDP growth rate which also picks up substantially between 2007 and 2008. Although the UK's output growth rate rose briskly in 2007:II-2007:III, the 2007:IV period witnessed considerable turmoil in most industrialised economies – the economic prospects deteriorated in the US and many other advanced countries including the UK. Hit by the spread of the crisis, the quarterly growth rate of output in the UK dropped to 0.1% in 2007:IV while the value in 2007:II was still at 1.28% above the post-1993 average. In 2008, disruption to the global economy continued. In the US, GDP growth fell sharply, the labour market weakened and the weakness in the housing market appeared to be spreading to other parts of the US. In the UK, the surveys pointed to output growth slowing (see, Inflation report, August 2008, p.5, 18-22). According to the August 2008 Inflation report, consumer spending appeared to decelerate as household's real incomes were squeezed, residential and business investment prospects deteriorated continually and business intention to invest in plant and machinery was markedly down between 2008:II and 2008:III indicating that these elements of business investment growth may weaken further. All these factors contributed to the substantial fall in the real GDP growth rate and an increase in its volatility during the crisis.

In the existing literature for the UK economy, Barnett et al (2010) also estimate a time-variant VAR model. They combine Markov-switching with a VAR and then show that the volatility of economic growth in the UK is much more stable during 1993-2006 in comparison with the 1980s. Although a similar evolution of volatility is observed in this study, the TVP-VAR-SV model has an obvious advantage over an MS-VAR model in the sense that it does not need to divide the sample into different regimes to confirm the change in the structure of the model.

Apart from that, the volatility of output growth exhibits another spike in 1998-1999 (see, Figure 2c). This reflects the fact that domestic economic growth slowed down sharply which may be caused by the slowdown of the global economy. As in the US DESA's (Department of Economic and Social Affairs, 1999) releases, the recession in Japan, the East Asian crisis and Russian crisis, along with the contagion throughout the financial markets, halved the growth rate of the world economy in 1999. In the UK, two inflation reports in 1998 (August and November 1998) confirm that weaker growth in emerging markets and depreciation of their currencies resulted in lower demand for exports and increased import competition in the domestic consumption markets. This combined with the appreciation of sterling suggests that net trade should lead to a negative contribution to the output growth rate during that time.

In Figure 2d, interest rate volatility has two spikes in 1998-1999 and 2008-2009. The former represents a sudden decline in the short-term interest rate in reaction to the slowdown of the UK economy. The Trea3m fell from 7.13% in 1998:III to 4.87% in 1999:II as a response to the deterioration of the outside economic environment and to bring output activity back to its long-run level. The rise in volatility of the Trea3m around 2009 corresponds to the other adjustment in the short-term interest rate. As already mentioned, the BOE cut its Bank rate to the effective ZLB in early 2009 leading to a sharp structural change. Then volatility remained at a relative low level since the UK experiences a low-interest-rate period. It is worth noting that the results displayed in Figure 2d is consistent with the Lafuente et al. (2014) estimation which shows the BOE's expansionary monetary policy around 1999 and 2008.

The last figure shows the volatility of the FCI as described in Chapter 2. Except for an increase around 1998, the volatility declined between 1993 and 2006. However, it then rose markedly in 2006:III prior to the 2007/09 crisis. This substantial increase is primarily caused by the prosperous market before the financial crisis. In 2007 there are shocks hitting the global financial markets including the UK market, which led to volatility remaining high between 2007 and 2009. As illustrated in two Inflation reports (February 2008, February 2009), the recent decline in asset prices reflects the growing pessimism about growth prospects during that time. The following decrease in the volatility of the FCI corresponds to the moderate recovery of the UK financial system. Expansionary monetary policy (including maintaining the bank rate at 50 bps and continuing asset purchases) resulted in a rise in equity prices and a narrowing of credit spreads.

Overall, as also discussed by Nakajima et al. (2011) the time-varying volatility contributes to the TVP-VAR-SV estimation in this study. It identifies structural shocks with appropriate variances of shock sizes. For the data analysed here, the estimation of a VAR model with constant volatility is likely to result in biases in covariance matrices for disturbances because of the mis-specification of the dynamics.

5.5.3 Time-varying Impulse Responses

Following many existing TVP-VAR-SV studies such as Nakajima (2011a, 2011b), this subsection provides impulse response analysis for the time series in this model.

The impulse response is a basic tool to examine macroeconomic dynamics captured by a VAR model. For a normal VAR with time-invariant parameters, the impulse responses are drawn for each pair of two variables. For the TVP-VAR-SV model estimated here, impulse responses are calculated at each date over the sample using the estimated parameters. It is wroth noting that the shock size for the response estimation is not based on the estimated volatility at point in each time. The shock size for the response is set equal to the time-series average of stochastic volatility for each series. The impulse response summarises the effects of average-sized experimental structural shocks hitting the TVP-VAR-SV model.

5.5.3.1 Policy Reaction to Inflation and Output

Figure 3 and 4 report the responses of the short-term interest rate to positive shocks in inflation expectations and real GDP growth respectively. Several results stand out:

Firstly, Figure 3 presents the impulse response of the Trea3m (modified for ZLB) to a positive shock in either inflation expectations or output growth. It plots the responses at different dates. The dates for comparison include 1995:III, 2000:III and 2005:III which are selected arbitrarily to determine time variant properties. Given the estimation so far, it finds that the responses of the interest rate to a positive shock in either inflation expectations or output conforms to prior expectations but exhibits some time-variant characteristics. Thus, this study infers that a constant-parameter Taylor rule is insufficient to model the short-term interest rate empirically. At the three selected dates, the reaction of monetary policy to a positive shock in real GDP growth persists over 16 quarters.

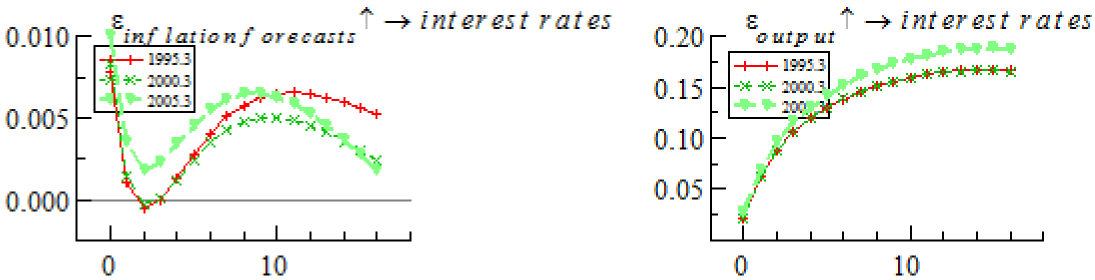


Figure 3: Impulse Responses of the Nominal Interest Rate on Selected Dates

Secondly, Figure 4 includes the evolution of the Trea3m's impulse responses to a positive shock in inflation expectations. Several points are noteworthy in the first sub-figure. (i) The correlation between the one-quarter ahead (short-term) and six-quarter (medium-term) ahead responses is relatively high. (ii) The reaction of the Trea3m to inflation expectation is often gradual. In other words, it takes time for the short-term interest rate to reach the medium term level after having a shock in expected inflation. In particular, it takes over six quarters for the interest rate to react to a positive shock in inflation expectations. (iii) The BOE becomes much less aggressive to inflation expectations around 2004. As reported in the Inflation report (February 2004), output growth in the UK is above trend in the second half of 2003 and business surveys point to further strengthening in 2004. Although the annual CPI inflation rate is much below 2%, it is projected to move up as accumulating pressures on supply capacity add to a modest rise in import prices. Therefore, it is rational for the BOE to focus on bringing aggregate output back to the long-run level, because the inflation rate is quite low.

Thirdly, Figure 4 also incorporates the changes in the impulse response of the Trea3m to a positive shock in real GDP growth. Similarly, the correlation between the short-term and medium-term response is high and the responses of the Trea3m to output growth are also gradual. Apart from that, it is interesting to find that the responses to expected inflation and output growth are highly correlated between 2005 and 2013. This means that the periods when the BOE becomes more aggressive against expected inflation are the same periods (for instance, the pre-crisis phase) in which monetary policy becomes more reactive to output fluctuation. The aggressive response of the interest rate to both output and inflation in 2005-2007 could be explained by the strong economic growth prior to the 2007-9 financial crisis and the growing inflation rate during the same period.

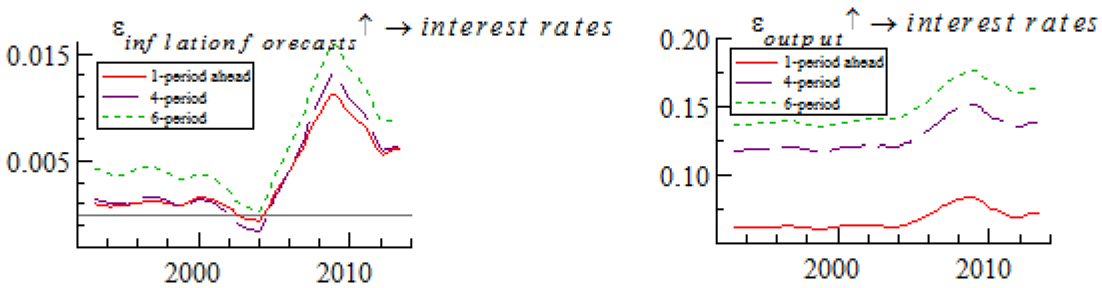


Figure 4: Time-varying Impulse Responses of the Nominal Interest Rate

As mentioned in the literature section, Wesche (2006) studies the BOE's monetary policy for the period 1973:I-2004:II. With a Markov-switching model, she arrives at the conclusion that the coefficients on inflation and output appear to evolve according to two regimes in the UK, a hawkish (i.e., monetary policy becomes more aggressive to inflation and less aggressive to economic stabilisation) and a dovish regime (where the opposite holds). Depending on the other period reviewed (1993:I-2013:II), the TVP-VAR-SV estimation does not find the existence of two such regimes.

5.5.3.2 Policy Impacts on Inflation and Output

To evaluate the effectiveness of BOE monetary policy and to examine timing lags around policy impacts, it is particularly interesting to analyse the impacts of changes in the nominal short-term interest rate on the UK economy. This is considered to be the supplemental component in studying a Taylor rule. The methodology to do so is quite straightforward: observing the time-varying impulse responses of the actual inflation rate and real GDP growth to a positive shock in the Trea3m.

Figure 5 reports the responses of the actual inflation rate and the real GDP growth rate to a positive shock in the short-run interest rate on different dates. As in Figure 3, the dates are selected arbitrarily. Figure 6 is about changes in the impulse responses of inflation and output growth. Several remarks are required in this case:

Firstly, as estimated by the MPC with its own macroeconomic model that a change in monetary policy should shift other wholesale money-market rates very quickly. However, it is expected to take time to have its full impact on the domestic economy. Particularly, the MPC estimates that it takes up to two years for the response to a monetary policy change to have its largest effect on the inflation rate in the UK. The TVP-VAR-SV model here also allows for the examination of the time lags in the BOE's monetary policy transmission. As plotted in the first chart of Figure 6, the peak (i.e., largest) effect is felt after around eight quarters – this is consistent with the estimation of the MPC[7].

[7] The preliminary tests discover that the effect of the interest rate on actual inflation declines after eight quarters. Therefore, it is confident to claim that the maximum effect is felt about eight quarters. This is why this study displays 1-, 4-, 6- and 8-period ahead responses in the first chart of Figure 6.

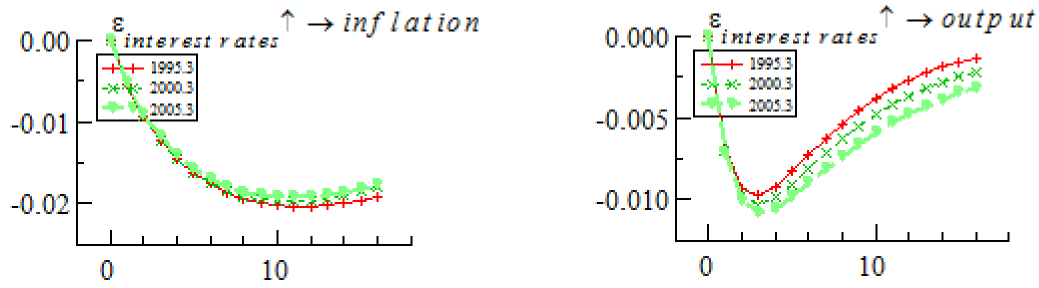


Figure 5: Implpuse Reponses of Inflation and Output Growth on Selected Dates

The second chart in Figure 6 plots the changes in the impulse response of real output growth to a positive shock in the Trea3m. Overall such a positive shock has a negative effect on real GDP growth, which is consistent with the prior expectation and the estimates in Chapter 4 using a linear model. However, the implied time lags differ from the MPC's estimation. The committee suggests that it takes roughly one year for a change in monetary policy to have its largest effect on output growth. The TVP-VAR-SV model estimates that the time lag for the peak effect is at least four quarters but for the full impact it is over six quarters.

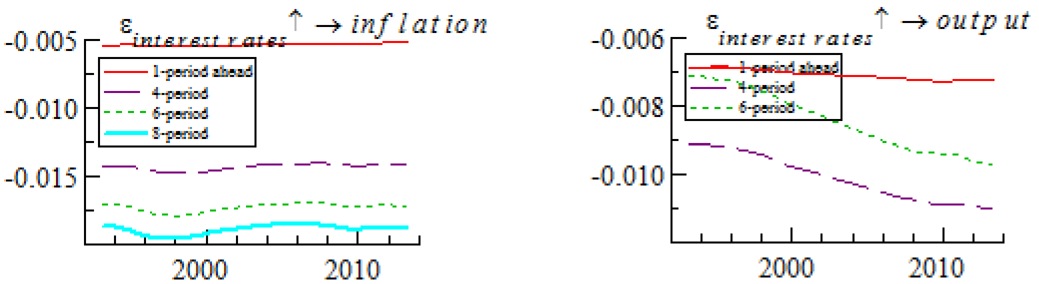


Figure 6: Time-varying Impulse Responses of Inflation and Output Growth

5.5.3.3 Role of an FCI in Monetary Policy

To explore whether the short-term interest rate in the UK can be modelled with an augmented Taylor rule, Chapter 4 includes an FCI into a baseline model. The result provides some evidence that the BOE is promoting financial stabilisation during the sample period. However, the specification in Chapter 4 is time-invariant which motivates this chapter to study whether the impulse responses of the interest rate to a positive shock in financial markets change within the sample period. Figure 7 displays the results. A few remarks are required in this case.

Firstly, the dynamic responses to an FCI behave quite indifferently. In Figure 7, this study expects a positive (negative) financial shock to lead to the higher (lower) asset prices, higher (lower) credit to the private sector and an improvement (decline) in financial market in the UK. This is common in the literature (see, for instance, Fornari and Stracca, 2013). While the condition of the financial system is above its long-term trend, this study expects the BOE to raise the interest rate and *vice versa*. As plotted in the first chart of Figure 7, while receiving a positive shock to the FCI, the response of the interest rate is of a positive nature in 1993-2013 and remains relatively stable during this period. This finding supports to the decision to set the coefficient on the FCI constant.

Secondly, as in Figure 4 the reaction of the interest rate to shocks in an FCI is also gradual. The implied time lag for monetary policy to fully respond to changes in the FCI is over six quarters. It is particularly interesting to find that the one-period-ahead impulse response of the interest rate to expected inflation is much closer to the long-run reaction in comparison to the response to output and the FCI. This implies that the BOE reacts to the inflation expectation stabilisation much faster than to output and the FCI.

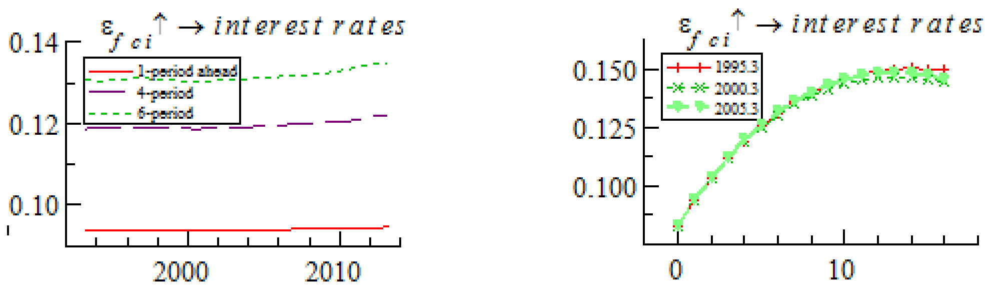


Figure 7: Changes in the Roles of an FCI in the Monetary Policy of the UK

5.6 Conclusions

This study focuses on modelling the short-term interest rate in the UK. It aims to explain the process of setting monetary policy in the BOE using the (augmented) Taylor rule and the TVP-VAR-SV model developed in past literature.

Sims (1980), Bernanke et al. (1997) and Jacobson et al. (2001) have justified the use of a multiple equation model for understanding the reaction of monetary policy to shocks and the impacts of the policy on the rest of the domestic economy. However, almost all the VAR literature on monetary policy in the UK fails to include the possible time variation in volatility. This motivates the use of a TVP-VAR-SV model with the UK data. Furthermore, both the MPC and many existing studies (namely, Swanson and Williams, 2013) identify 50 bps as the effective ZLB for the UK for the period under review. As in Bernanke and Reinhart (2004), Swanson and Williams (2013) and Williams (2014), when the short-term interest rate gets close to (or at) the rate of 50 bps in the UK, conventional means of effecting monetary ease is no longer feasible. However, most of the TVP-VAR-SV literature (such as Primiceri, 2005; Nakajima et al., 2011), except for Nakajima (2011b), fails to consider the required modification of the nominal short-term interest rate when it is below the ZLB. To have an accurate modelling and description of the interest rate in the UK, this study decides to follow Nakajima (2011b) to use the TVP-VAR-SV model and modify data for ZLB.

The empirical results from the TVP-VAR-SV model justifies the use of time variation to address questions concerning the response of the short-term interest rate to the projected inflationary shocks and shocks in the real output growth rate. However, it discovers that the BOE’s reaction to changes in financial markets could be modelled with a linear specification. This suggests that the response of monetary policy to financial market developments is relatively stable throughout the sample period.

The estimates also confirm the existence of changes in volatility (i.e., structural changes within the sample period). Thus, it concludes that a conventional VAR model assuming no time-variation in volatility will lead to bias in the estimation.

Regarding the impact of monetary policy on the rest of the economy, this model discovers that the estimated time lag for the peak effect of the interest rate on the inflation rate is consistent with the estimation of the MPC. Also consistent with its estimate, the TVP-VAR-SV model indicates that it takes over four quarters (but less than six quarters) for monetary policy to have its largest effect on real economic output.

CONCLUSIONS

This thesis makes three principal contribution to the existing literature. Firstly, it produces an optimal estimate of a financial conditions index (FCI) for the United Kingdom (UK). Secondly, it discovers the existence of the unpublished short-term inflation target in the Bank of England (BOE). Thirdly, it investigates the most appropriate method for modelling the interest rate using new data and a new estimator for the UK.

Although previous FCI studies have proposed different models to choose and weight constituent financial variables, there is no consensus about the most appropriate methodology to estimate FCIs. This thesis addresses this problem in Chapter 1 and 2. An optimal FCI is first constructed by evaluating (i) different methods of weighting indicators and (ii) the best combination of constituents at each point in time. Then the estimated FCI is used in other chapters to test whether the BOE works in response to changes in the domestic financial market. The most important assumption in investigating the FCI is that the best FCI predicts economic activity as well as possible. This is derived from the monetary transmission mechanism

Chapter 1 investigates the optimal variable-weighting model. Following Hatzius et al. (2010), it also classifies all FCI weighting methods into two categories: a weighted-sum approach and a principal-component (PC) approach. Then it develops a new weighted-sum model, known as the 'two-step' method, which is based on an aggregate demand equation. It shows that the proposed two-step method is superior to other existing weighted-sum methods.

One of the most important findings obtained from Chapter 1 is that a time-varying parameter factor-augmented VAR (TVP-FAVAR) with stochastic volatility (SV) is the optimal PC method for estimating the weights attached to each constituent in an FCI. Different from conventional Principal Component Analysis (PCA), the TVP-FAVAR with SV model summarises information from a large group of indicators in a rational manner. It does not only seek to extract the co-movement of multiple variables but also takes the purpose of extracting common factors into account. Hence, this is also a useful technique in other research areas. In the following chapter

(Chapter 4), this thesis constructs an optimal total real output index which is defined as the best predictor of the inflation rate. Given the conclusions in Chapter 1, that index is also estimated with the TVP-FAVAR with SV model. As compared to other output measures, it leads to the smaller forecasting errors.

It is worth noting that the TVP-FAVAR with SV model has never been compared to weighted-sum methods to create FCIs. Therefore, the existing literature is unsure about whether this model performs better than weighted-sum methods. Chapter 1 fills this gap in the literature. By comparing the forecasting performance of two FCIs respectively produced by the 'two-step' process and the TVP-FAVAR with SV model, Chapter 1 discovers that the weighted-sum approach (including the proposed two-step process) underperforms relative to PC methods. The TVP-FAVAR with SV model is the best method to weight constituent financial variables in an FCI.

With the results obtained from Chapter 1, Chapter 2 estimates the optimal FCI for the financial markets of the UK by including a variable selection technique, the dynamic model averaging (DMA). Although the DMA model has been used by Koop and Korobilis (2014) to estimate the US FCI, no one has applied it to UK financial data yet. Therefore, previous researchers studying the UK financial market may have no idea about which constituent variables should be included in an optimal FCI. This gap is now filled in Chapter 2.

The DMA model answers two principal questions: (i) which indicators should be included in the optimal FCI at the starting point of constructing this index and (ii) how the constituents of the FCI change at each point in time. Hence, a joint model of the DMA technique and the TVP-FAVAR with SV model (henceforth, DMA-TVP-FAVAR) is expected to address these difficulties in estimating the optimal FCI.

In order to ensure that the DMA model has sufficient candidate constituents to evaluate, Chapter 2 includes a wider range of financial indicators than the coverage of most existing FCIs for the UK. It discovers that the DMA-TVP-FAVAR model provides an FCI with the highest predictive likelihood and the lowest forecasting errors as compared to FCIs based on other methods. This means the FCI obtained in Chapter 2 is the best predictor of economic activity as compared to other FCIs. In other words, Chapter 2 produces an optimal FCI for the UK.

However, the moving trend of all estimated FCIs in this study is different from the Castro (2011) FCI in two sub-periods. The Castro (2011) FCI is positive around 2002-2003 but turns to be negative in 2005-2007. However, this thesis (in Chapter 1-2) obtains negative FCIs for the period 2002-2003 and positive values for 2005-2007. Referring to the BOE's inflation reports, it discovers that the estimation in Chapter 1-2 is closest to the reality. Hence, it is reasonable to infer that the use of inappropriate estimation methodology may lead to inaccurate estimates of FCIs. Given the findings in Chapter 1-2, this thesis believes that the movement of the optimal FCI in Chapter 2 reflects changes in financial markets very well. This index provides significant information for market participants:

Firstly, summarising all financial information in a single index gives investors an overall understanding regarding the trend and the current status of financial markets. A positive value of the estimated FCI indicates that the current financial market is above its long-run trend, while a negative number means it is below the trend. An increase in the index implies an improvement of the financial system and *vice versa*. Secondly, financial markets are a reliable leading indicator of economic activity. Therefore, the estimated FCI in this study offers policy makers and market participants an accurate forecast about future changes in the overall economy of the UK. Since it is obtained with the optimal weighting method and variable-selection technique, this thesis expects that this index has less uncertainty in predicting economic activity in the UK. Thirdly, the estimation of the FCI also supports further investigation on whether the BOE works to promote financial stabilisation in the UK. That would be an important finding in modelling the short-term interest rate.

This thesis then estimates the inflation target from the inflation data in Chapter 3. It raises a hypothesis that in addition to setting the announced target of inflation based on the Bank's medium to long-term considerations, the BOE has another inflation objective for its short-term purpose. This thesis is the first to test the existence of an implicit inflation target in an inflation-targeting central bank.

The empirical results in Chapter 3 discovers that the hypothesised short-term inflation target enters significantly into the BOE's policy rule and it changes significantly throughout the sample period 1993-2013. This new finding changes the traditional thinking of the constant inflation objective in the UK. It shows that the BOE has two inflation targets, a medium/long-term objective which is constant and a short-term target which is time-varying. Technology shocks such as technological developments dominate the changes in this unpublished short-term target.

Overall, the results in Chapter 3 indicate that economic researchers and investors who are concerned about uncertainties in inflation should consider this short-term inflation target. The published inflation objective may not be a reliable indicator in the circumstance that the short-term target deviates substantially from the long-run target. For inflation prediction, technology shock deserves attention. A positive technology shock is defined as an unanticipated change in technology that benefits economic activity. According to the estimation in Chapter 3, a positive technology shock is expected to lower the implicit inflation target which would reduce inflation in the UK.

It is quite important to stress that future researchers must be very careful when applying the methodology in Chapter 3 to other inflation-targeting countries. This is because another inflation-targeting central bank may be different from the BOE whose inflation objective is on the basis of medium to long-run considerations. For future studies, it would be particularly interesting to modify the structural model in Chapter 3 by considering the impacts of the international environment on the short-term inflation target and to draw in some of the empirical conclusions to formulate the theoretical model.

This thesis also studies the Taylor rule using different estimators in Chapter 4 and 5. Chapter 4 includes linear estimates of the Taylor rule, but the subsample analysis in Chapter 4 indicates that the BOE has changed its monetary policy implementation during the sample period which motivates this thesis to use a time-varying parameter estimator in Chapter 5.

An important contribution in Chapter 4 to the existing literature is the construction of an optimal total output index using the DMA-TVP-FAVAR model. According to the monetary transmission mechanism as explained by the MPC (June 2012), this thesis defines the optimal output measure as the best predictor of the future inflation rate among different output indicators. The results show that as compared to the existing output measures in the literature, the estimated optimal output index has lower forecasting errors. Hence, this thesis on the one hand provides firms and individuals with a new way to predict the inflation rate and on the other hand shows the wider applicability of the DMA-TVP-FAVAR model.

The linear estimation in Chapter 4 also supports the previous conclusion in the literature that the BOE works to promote both inflation stabilisation (around the inflation target) and output stabilisation (around its long-term trend). However, it also obtains contrary findings to some of the existing literature studies such as Castro (2011). Using the optimal FCI, Chapter 4 finds strong evidence that the BOE promotes the stabilisation of financial markets in the UK by bringing the market to its long-run trend. Hence, any continuous growth or deterioration in the financial system of the UK cannot happen without significant changes in the long-run trend of the financial system, because the Bank will bring the UK financial market to its long-term trend through the interest rate adjustment. Financial markets should expect the interest rate to rise when they believe market conditions are above the long-run trend and *vice versa*. In other words, if the long-run trend of the financial system does not change, it would be quite risky and dangerous for UK investors to bet on the movement of financial markets in one direction.

Chapter 5 employs a time-varying parameter VAR with stochastic volatility (TVP-VAR-SV) model to describe and explain the short-term interest rate in the UK. The VAR model has several advantages including (i) taking into account the impact of the interest rate, (ii) allowing for the VAR parameters to evolve over time and (iii) considering time-varying volatility of each variable. This thesis contributes to the monetary studies in the UK by exploring time-variation in the BOE's implementation of monetary policy and examining whether the Bank's estimated time lags are reliable.

The empirical findings from the TVP-VAR-SV model justifies the use of time variation to address some questions concerning the response of the interest rate to projected inflationary shocks and shocks in output. Chapter 5 also provides the existing literature with a new important finding that in order to model the interest rate in the UK, both parameters and volatility should be allowed to change throughout the sample period.

Furthermore, Chapter 5 discovers that the BOE's preference for stabilising financial markets is significant and relatively constant in the period 1993-2013. Therefore, another important implication in Chapter 5 is that the reaction of the BOE's short-term interest rate to financial markets can be well modelled using a linear estimator.

Regarding the impact of the interest rate on the rest of the UK economy, this thesis shows that the estimated time lags for the peak effects of the interest rate on the inflation rate and real output are consistent with the estimation of the monetary policy committee of the BOE. This may be informative for many market participants, as a peak effect of a change in the interest rate on real output is usually felt after four quarters. It takes around eight quarters for the interest rate adjustment to have its largest effect on inflation. Therefore, it is quite reasonable for market participants to base their investment decisions on changes in the policy interest rate in the past one or two years (or more).

For future research, adding changes in the money supply into the TVP-VAR-SV model would be interesting. The interest rate was cut to 0.5% in 2009 by the Bank. As in Swanson and Williams (2013), since January 2009 the BOE has conducted large-scale asset purchases on a similar scale to the Fed. This implies that the interest rate hit the (effective) zero lower bound of 0.5% over the period 2009-2013. As in Bernanke and Reinhart (2004), under the zero-interest rate circumstance, the monetary transmission mechanism is unlikely to work through the interest rate channel in the same manner as normal circumstances. Therefore, it seems reasonable to let the money supply (such as, the M4) enter into the VAR during the (effective) zero-interest rate period.

Overall, the results in this thesis suggest that while modelling the interest rate in the UK, it would be beneficial to: (i) allow for the time-variation in the implicit short-term inflation target, (ii) augment the initial Taylor rule for changes in financial markets, (iii) employ better measures as input data (like using a more accurate measure for the central bank's inflation expectation, a composite output index and an optimal FCI) and (iv) use a TVP-VAR-SV model for modelling an extended Taylor rule. However, the response coefficient on financial markets can be set constant for computation convenience.

REFERENCE

Adema, Y. (2004), "A Taylor Rule for the Euro Area Based on Quasi-real Time Data", De Nederland Bank Staff Reports, No. 114

Angelopoulou, E., Balfoussia, H., H. Gibson. (2013), "Building A Financial Conditions Index for the Euro Area and Selected Euro Area Countries: What Does It Tells Us About the Crisis?", European Central Bank Working Paper Series No. 1541

Asteriou, D., Hall, S.G. (2011), *Applied Econometrics,* London: Palgrave Macmillan

Bank for International Settlement. (1995), *Financial Structural and the Monetary Policy Transmission Mechanism*, Basel: Bank for International Settlement

Bank of England. (May 1993), "Inflation Report", Available in PDF at: http://www.bankofengland.co.uk/archive/Documents/historicpubs/ir/ir93may.pdf. Last accessed 31st Jan 2015

Bank of England. (August 1993), "Inflation Report", Available in PDF at: http://www.bankofengland.co.uk/archive/Documents/historicpubs/ir/ir93aug.pdf. Last accessed 31st Jan 2015

Bank of England. (November 1994), "Inflation Report", Available in PDF at: http://www.bankofengland.co.uk/archive/Documents/historicpubs/ir/ir94nov.pdf. Last accessed 31st Jan 2015

Bank of England. (February 1995), "Inflation Report", Available in PDF at: http://www.bankofengland.co.uk/archive/Documents/historicpubs/ir/ir95feb.pdf. Last accessed 31st Jan 2015

Bank of England. (August 1998), "Inflation Report", Available in PDF at: http://www.bankofengland.co.uk/archive/Documents/historicpubs/ir/ir98aug.pdf. Last accessed 31st Jan 2015

Bank of England. (November 1998), “Inflation Report”, Available in PDF at: http://www.bankofengland.co.uk/archive/Documents/historicpubs/ir/ir98nov.pdf. Last accessed 31st Jan 2015

Bank of England. (August 2002), “Inflation Report”, Available in PDF at: http://www.bankofengland.co.uk/archive/Documents/historicpubs/ir/ir02aug.pdf. Last accessed 31st Jan 2015

Bank of England. (February 2003), “Inflation Report”, Available in PDF at: http://www.bankofengland.co.uk/archive/Documents/historicpubs/ir/ir03feb.pdf. Last accessed 31st Jan 2015

Bank of England. (February 2004), “Inflation Report”, Available in PDF at: http://www.bankofengland.co.uk/publications/Documents/inflationreport/ir04feb.pdf. Last accessed 31st Jan 2015

Bank of England. (February 2008), “Inflation Report”, Available in PDF at: http://www.bankofengland.co.uk/publications/Documents/inflationreport/ir08feb.pdf. Last accessed 31st Jan 2015

Bank of England. (June 2008), “Governor/Chancellor Letters on Monetary Policy”, Available in PDF at: http://www.bankofengland.co.uk/monetarypolicy/Documents/pdf/cpiletter080616.pdf. Last accessed 15th June 2016

Bank of England. (August 2008), “Inflation Report”, Available in PDF at: http://www.bankofengland.co.uk/publications/Documents/inflationreport/ir08aug.pdf. Last accessed 31st Jan 2015

Bank of England. (November 2008), “Inflation Report”, Available in PDF at: http://www.bankofengland.co.uk/publications/Documents/inflationreport/ir08nov.pdf. Last accessed 31st Jan 2015

Bank of England. (February 2009), “Inflation Report”, Available in PDF at: http://www.bankofengland.co.uk/publications/Documents/inflationreport/ir09feb.pdf. Last accessed 31st Jan 2015

Bank of England. (February 2010), "Inflation Report", Available in PDF at: http://www.bankofengland.co.uk/publications/Documents/inflationreport/ir10feb.pdf. Last accessed 31st Jan 2015

Bank of England. (August 2010), "Governor/Chancellor Letters on Monetary Policy", Available in PDF at: http://www.bankofengland.co.uk/monetarypolicy/Documents/pdf/cpiletter100817.pdf. Last accessed 15th June 2016

Bank of England. (February 2011), "Inflation Report", Available in PDF at: http://www.bankofengland.co.uk/publications/Documents/inflationreport/ir11feb.pdf. Last accessed 31st Jan 2015

Bank of England. (November 2011), "Governor/Chancellor Letters on Monetary Policy", Available in PDF at: http://www.bankofengland.co.uk/monetarypolicy/Documents/pdf/cpiletter111115.pdf. Last accessed 15th June 2016

Barnett, A., Groen, J.J.J., H. Mumtaz. (2010), "Time-varying Inflation Expectations and Economic Fluctuations in the United Kingdom: A Structural VAR Analysis", Bank of England Working Paper Series No. 392

Barro, R.J., Gordon, D.B. (1983), "Rules, Discretion and Reputation in a Model of Monetary Policy", *Journal of Monetary Economics*, vol. 12, no. 1, pp, 101-121

Bates, B.J., Plagborg-Moller, M., Stock, J.H., M.W. Watson. (2013), "Consistent Factor Estimation in Dynamic Factor Models with Structural Instability", *Journal of Econometrics*, vol. 177, no. 2, pp. 289-304

Batini, N., Turnbull, K. (2002), "A Dynamic Monetary Conditions Index for the UK", *Journal of Policy Modelling*, vol. 24, no. 3, pp. 257-281

Beechey, M., Osterholm, P. (2012), "The Rise and Fall of U.S. Inflation Persistence", *International Journal of Central Banking*, vol. 8, no. 3, pp. 55-86

Benati, L. (2003), "The Inflation-targeting Framework from A Historical Perspective", Available in PDF at:

http://www.bankofengland.co.uk/publications/Documents/quarterlybulletin/qb050203.pdf. Last accessed 31st Mar 2014

Bernanke, B.S., Boivin J., P. Eliasz. (2005), "Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach", *The Quarterly Journal of Economics*, vol. 120, no. 1, pp. 387-422

Bernanke, B.S., Gertler, M. (1995), "Inside the Black Box: The Credit Channel of Monetary Policy Transmission", *Journal of Economic Perspectives*, vol. 9, no. 4, pp. 27-48

Bernanke, B.S., Gertler, M., M. Watson. (1997), "Systematic Monetary Policy and the Effects of Oil Price Shocks", *Brookings Papers on Economic Activity*, vol. 28, no. 1, pp. 91-157

Bernanke, B.S., Reinhart, V.R. (2004), "Conducting Monetary Policy at Very Low Short-Term Interest Rates", *American Economic Review*, vol. 94, no. 2, pp. 85-90

Blanchard, O., Fischer, S. (1989), *Lectures in Macroeconomics*, Cambridge: MIT Press

Blinder, A.S. (1998), *Central Banking in Theory and Practice*, Cambridge: MIT Press

Boivin, J., Ng, S. (2006), "Are More Data Always Better for Factor Analysis?", *Journal of Econometrics*, vol. 132, no. 1, pp. 169-194

Borio, C., Lowe, O. (2002), "Asset Prices, Financial and Monetary Stability: Exploring the Nexus", Bank for International Settlements Working Paper Series No. 114

Box, G.E.P., Draper, N.R. (1987), *Empirical Model-building and Response Surfaces*, New York: John Wiley & Sons Inc

Brave, S., Butters R.A. (2011), "Monitoring Financial Stability: A Financial Conditions Index Approach", *Economic Perspectives Federal Reserve Bank of Chicago*, vol. 35, no. 1, pp. 22-43

Breitung, J., Eickmeier, S. (2011), "Testing for Structural Breaks in Dynamic Factor Models", *Journal of Econometrics*, vol. 163, no. 1, pp. 71-84

Bruggemann, R., Riedel, J. (2011), "Nonlinear Interest Rate Reaction Functions for the UK", *Economic Modelling*, vol. 28, no. 3, pp. 1174-1185

Bryant, R.C., Hooper, P., Mann, C. (1993), *Evaluating Policy Regimes: New Research in Empirical Macroeconomics.* Washington DC: Brookings

Bullard, J., Schaling, E. (2002), "Why the Fed Should Ignore the Stock Market", *Federal Reserve Bank of St. Louis Review*, vo. 84, no. 2, pp. 35-41

Calvo, G.A. (1983), "Staggered Prices in a Utility Maximising Framework", *Journal of Monetary Economics*, vol. 12, no. 3, pp. 383-398

Carlin, W., Soskice, D. (2006), *Macroeconomics: Imperfections, Institutions, and Policies*. Oxford; New York: Oxford University Press

Castro, V. (2011), "Can Central Banks' Monetary Policy Be Described by A Linear (Augmented) Taylor Rule or A Nonlinear Rule?", *Journal of Financial Stability*, vol. 7, no. 4, pp. 228-246

Chadha, J.S., Sarno, L., G. Valente. (2004), "Monetary Policy Rules, Asset Prices, and Exchange Rates", *International Monetary Fund*, vol. 51, no. 3, pp. 529-552

Chib, S., Greenberg, E. (1995), "Understanding the Metropolis-Hasting Algorithm", *The American Statistician*, vol. 49, no. 4, pp. 327-335

Clarida, R., Gali, J., M. Gertler. (1998), "Monetary Policy Rules in Practice: Some International Evidence", *European Economic Review*, vol. 42, no. 6, pp. 1003-1067

Clarida, R., Gali, J., M. Gertler. (1999), "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature*, vol. 37, no. 4, pp. 1661-1707

Clarida, R., Gali, J., M. Gertler. (2000), "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory", *Quarterly Journal of Economics*, vol. 115, no. 1, pp. 147-180

Cliff, M.T. (2003), *GMM and MINZ Program Libraries for Matlab,* Available from Computer Programs and Data, Michael Cliff's Homepage, Web site: https://sites.google.com/site/mcliffweb/programs [Accessed: September 10, 2014]

Clyde, M., George, E.I. (2004), "Model Uncertainty", *Statistical Science,* vol. 9, no. 1, pp. 81-94

Cogley, T., Paoli, B., Matthes, C., Nikolov, K., T. Yates. (2011), "A Bayesian Approach to Optimal Monetary Policy with Parameter and Model Uncertainty", Bank of England Working Paper Series No. 411

Cogley, T., Sargent, T.J., (2005), "Drift and Volatilities: Monetary Policies and outcomes in the post WWII U.S", *Review of Economic Dynamics,* vol. 8, no. 2, pp. 262-302

Coibion, O., Gorodnichenko, Y., J. Wieland (2012), "The Optimal Inflation Rate in New Keynesian Models: Should Central Banks Raise Their Inflation Targets in Light of the Zero Lower Bound", *Review of Economic Studies*, vol. 79, no. 4, pp. 1371-1406

Contessi, S., Pace, P.D., M. Guidolin. (2013), "How Did the Financial Crisis Alter the Correlations of U.S Yield Spreads", Federal Reserve Bank of St. Louis Working Paper Series No. 2013-005D

Cooper, J.P., Fischer, S. (1972), "Simulations of Monetary Rules in the FRB-MIT-Penn Model", *Journal of Money, Credit and Banking,* vol. 4, no. 2, pp. 384-396

Corsetti, G., Pesenti, P. (2005), "International Dimensions of Optimal Monetary Policies", *Journal of Monetary Policies,* vol. 52, no. 2, pp. 281-305

Cox, R.T. (1946), "Probability, Frequency and Reasonable Expectation", *American Journal of Physics*, vol. 14, no.1, pp. 1-13

Cox, R.T. (1961), *The Algebra of Probable Inference,* London: The Johns Hopkins Press

Cristadoro, R., Veronese, G. (2011), "Monetary Policy in India: Is Something Amiss", *India Growth and Development Review,* vol. 4, no. 2, pp. 166-190

Cukierman, A., Muscatelli, A. (2008), "Nonlinear Taylor Rules and Asymmetric Preferences in Central Banking: Evidence from the United Kingdom and the United States", *The B.E. Journal of Macroeconomics*, vol. 8, no. 1, pp. 1-29

Curdia, V., Woodford, M. (2009), "Credit Frictions and Optimal Monetary Policy", Bank for International Settlements Working Papers No. 278

Dickey, D., Fuller, W. (1979), "Distribution of the Estimators for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association*, vol. 74, no. 366, pp. 427-431

Diebold, F.X., Mariano, R.S. (1995), "Comparing Predictive Accuracy", *Journal of Business and Economic Statistics*, vol. 13, no. 3, pp. 134-144

Doz, C., Giannone, D., L. Reichlin. (2011), "A Two-step Estimator for Large Approximate Dynamic Factor Models Based on Kalman Filtering", *Journal of Econometrics*, vol. 164, no. 1, pp. 188-205

Draper D. (1995), "Assessment and Propagation of Model Uncertainty", *Journal of the Royal Statistical Society Series B (Methodological)*, vol. 57, no. 1, pp. 45-97

Driffill, J., Rotondi, Z., Savona, P., C. Zazzara. (2006), "Monetary Policy and Financial Stability: What Role for the Futures Market?", *Journal of Financial Stability*, vol. 2, no. 1, pp. 95-112

Driscoll, J.C. (2000), "On the Microfoundations of Aggregate Demand and Aggregate Supply", Manuscript, Brown University

Dudley, W., Hatzius, J. (2000), "The Goldman Sachs Financial Conditions Index: The Right Tool for a New Monetary Policy Regime", Global Economics Paper No. 44.

Durbin, J., Watson, G.S. (1951), "Testing for Serial Correlation in Least Squares Regression, II", *Biometrika*, vol. 38, no. 1/2, pp. 159-179

Eickmeier, S., Lemke, W., M. Marcellino. (2009), "Classical Time-varying FAVAR Models – Estimation, Forecasting and Structural Analysis", Discussion Paper Series 1: Economic Studies No. 04/2011 Deutsche Bundesbank

Eickmeier, S., Lemke, W., M. Marcellino. (2011), "The Changing International Transmission of Financial Shocks: Evidence from a Classical Time-varying FAVAR", Deutsche Bundesbank Discussion Paper Series 1: Economic Studies No. 05/2011

Erceg, C.J., Levin, A.T. (2003), "Imperfect Credibility and Inflation Persistence", *Journal of Monetary Economics*, vol. 50, no. 4, pp. 915-944

Federal Reserve Bank of St. Louis. (2015), "St. Louis Fed Financial Stress Index", Available at: https://research.stlouisfed.org/fred2/series/STLFSI. Last accessed 3rd Sep 2015

Fornari, F., Stracca, L. (2013), "What Does A Financial Shock Do? First International Evidence", European Central Bank Working Paper Series No. 1522.

Fourcans, A., Vranceanu, R. (2004), "The ECB Interest Rate Rule under the Duisenberg Presidency", *European Journal of Political Economy*, vol. 20, no. 3, pp. 579-595

Freedman, C. (1994), "The Use of Indicators and the Monetary Condition index in Canada", in Balino, T.J.T., Cottarelli, C. (eds), *Frameworks for Monetary Stability: Policy Issues and Country Experiences*, pp. 458-476, IMF, Washington, D.C.

Freedman, C. (1995), "The Role of Monetary Conditions and the Monetary Condition Index in the Conduct of Policy", *Bank of Canada Review (Autumn)*, pp. 53-60

Friedman, B., Kuttner, K. (1996), "A Price Target for U.S. Monetary Policy. Lessons from the Experience with Money Growth Targets", *Brookings Papers on Economic Activity*, vol. 27, pp. 77-125

Friedman, M., (1968), *Inflation: Causes and Consequences* in Dollars and Deficits: Living with America's Economic Problems, pp. 21-71, Englewood Cliffs, New Jersey: Prentice-Hall

Friedman, M. (1983), *Program for Monetary Stability*. New York: Fordham University Press

Fuhrer, J.C. (1997), "The (Un)Importance of Forward-looking Behaviour in Price Specification", *Journal of Money, Credit and Banking*, vol. 29, no. 3, pp. 338-350

Fuhrer, J.C., Moore, G. (1995), "Inflation Persistence", *The Quarterly Journal of Economics*, vol. 110, no. 1, pp. 127-159

Fuhrer, J.C. (2000), "Habit Formation in Consumption and Its Implications for Monetary Policy Models", *American Economic Review*, vol. 90, no. 3, pp. 367-390

Gali, J. (2015), *Monetary Policy, Inflation and Business Cycles*. Princeton: Princeton University Press

Gali, J., Gertler, M. (1999), "Inflation Dynamics: A Structural Economic Analysis", *Journal of Monetary Economics*, vol. 44, no. 2, pp. 195-222

Gali, J., Monacelli, T. (2005), "Monetary Policy and Exchange Rate Volatility in A Small Open Economy", *Review of Economic Studies*, vol. 72, pp. 707-734

Garratt, A., Koop, G., Mise, E., Vahey. S.P. (2009), "Real-time Prediction with UK Monetary Aggregates in the Presence of Model Uncertainty", *Journal of Business and Economic Statistics*, vol. 27, no. 4, pp. 480-491

Gauthier, C., Graham, C., Y. Liu. (2004), "Financial Conditions Indexed for Canada", Bank of Canada Working Paper No. 2004-22

Gelman, A., Carlin, J.B., Stern, H., Dunson, D.B., Vehtari, A., D.B. Rubin. (2014), *Bayesian Data Analysis,* 3^{rd} ed, London: Chapman & Hall

Gerald, P., Dwyer, P. (1993), "Rules and Discretion in Monetary Policy", Federal *Reserve Bank of St. Louis Review*, vol. 75, no. 3, pp. 3-14

Gerlach, S., Lewis, J. (2010), "The Zero Lower Bound, ECB Interest Rate Policy and the Financial Crisis", De Nederlandsche Bank Working Paper Series No. 254

Geweke, L. (1992), "Evaluating the Accuracy of Sampling-based Approaches to the Calculation of Posterior Moments", in J.M. Bernardo, J. Berger, A.P. Dawid, A.F.M. Smiths (eds.), Bayesian Statistics, vol. 4, Oxford University Press, Oxford, pp. 169-188

Gilks, W.R., Richardson, S., D.J. Spiegelhalter. (1996), *Markov Chain Monte Carlo in Practice*, London: Chapman & Hall

Goodfriend, M. (1991), “Interest Rates and the Conduct of Monetary Policy”, *Carnegie Rochester Conference Series on Public Policy*, vol. XXXIV, pp. 7-30

Goodhart, C., Hofmann, B. (2000), “Financial Variables and the conduct of Monetary Policy”, Sveriges Riksbank Working Paper Series No. 112

Goodhart, C., Hofmann, B. (2001), “Asset prices, Financial Conditions, and the Transmission of Monetary Policy”, Paper presented at the conference on Asset Prices, Exchange Rates and Monetary policy, Stanford University, March 2-3, 2001

Goodhart, C., Hofmann, B. (2002), “Asset Prices and the Conduct of Monetary Policy”, Paper presented at the conference on Royal Economic Society, August 29, 2002

Gordon, R.J. (1997), “The Time-Varying NAIRU and its Implications for Economic Policy”, *The Journal of Economic Perspectives*, vol. 11, no. 1, pp. 11-32

Guerrero., V.M. (2008), “Estimating Trends with Percentage of Smoothness Chosen by the User”, *International Statistical Review*, vol. 76, no. 2, pp. 187-202

Guichard, S., Haugh, D., D. Turner. (2009), “Quantifying the Effect of Financial Conditions in the Euro Area, Japan, United Kingdom and United States”, OECD Economic Department Working Paper Series No. 677

Haberler, G. (1946), *Prosperity and Depression: A Theoretical Analysis of Cyclical Movements*, New York: United Nations

Hansen, L. (1982), “Large Sample Properties of Generalised Method of Moments Estimators”, *Econometrica*, vol. 50, no. 4, pp. 1029-1054

Hall, S., Walsh, M., A. Yates. (2000), “Are UK Companies’ Prices Sticky?”, *Oxford Economic Paper*, vol. 52, no. 3, pp. 425-446

Hannan, E.J., McDougall, A.J., D.S. Poskitt. (1989), “Recursive Estimation of Auto regressions”, *Journal of the Royal Statistical Society Series B*, vol. 51, no. 2, pp. 217-233

Hatzius, J., Hooper, P., Mishkin, F., Schoenholtz, L.K., M.W. Watson. (2010), "Financial Condition Indexes: A Fresh Look after the Financial Crisis", NBER Working Paper Series No. 16150

Harvey, A.C. (1990), *Forecasting, Structural Time Series Models and Kalman Filter*, Cambridge: Cambridge University Press

Hock, T., Zimmerman, P. (2005), "Forecasting Monetary Policy in Switzerland: Some Empirical Assistance", *Swiss Society for Financial Market Research*, vol. 19, no. 2, pp. 201-212

Hodrick, R.J., Prescott, E.C. (1997), "Postwar U.S. Business Cycles: An Empirical Investigation", *Journal of Money, Credit and Banking*, vol. 29, no. 1, pp. 1-16

Hoff, P.D. (2009), *A First Course in Bayesian Statistical Methods*, London: Springer

Hollo, D., Kremer, M., M.L. Duca. (2012), "CISS – A Composite Indicator of Systemic Stress in the Financial System", European Central Bank Working Paper Series No. 1426

Hsing, Y. (2004), "Estimating the Bank of Japan's Monetary Policy Reaction Function", *Banca Nazionale del Lavoro Quarterly Review*, vol. 57, no. 229, pp. 169-183

Jacobson, T., Jansson, P., Vredin, A., A. Warne. (2001), "Monetary Policy Analysis and Inflation Targeting in a Small Open Economy: A VAR Approach", *Journal of Applied Econometrics*, vol. 16, no. 4, pp. 487-520

Jorgensen, B., Song, P.X.K. (2007), "Stationary State Space Models for Longitudinal Data", *The Canadian Journal of Statistics*, vol. 35, no. 4, pp. 461-483

Jong, P., Shephard, N. (1995), "The Simulation Smoother for Time Series Models", *Biometrika*, vol. 82, no.2, pp. 339-350

Judd, J.P., Rudebusch, G.D. (1998), "Taylor's Rule and the Fed: 1970-1997", *Federal Reserve Bank of San Francisco Economic Review*, no. 3, pp. 3-16

Ireland, P.N. (2004), “Technology Shocks in the New Keynesian Model”, *The Review of Economics and Statistics*, vol. 86, no. 4, pp. 923-936

Ireland, P.N. (1997), “A Small, Structural, Quarterly Model for Monetary Policy Evaluation”, *Carnegie-Rochester Conference Series on Public Policy*, vol. 47, pp. 83-108

Ireland, P.N. (2004a), “Technology Shocks in the New Keynesian Model”, *The Review of Economics and Statistics,* vol. 86, no. 4, pp. 923-936

Ireland, P.N. (2004b), “Money’s role in the Monetary Business Cycle”, *Journal of Money, Credit and Banking,* vol. 36, no. 6, pp. 969-983

Ireland, P.N. (2007), “Changes in the Federal Reserve’s Inflation Targets: Causes and Consequences”, *Journal of Money, Credit and Banking*, vol. 39, no. 8, pp. 1851-1882

Joyce, M., Tong, M., R. Woods. (2011), “The United Kingdom’s Quantitative Easing Policy: Design, Operation and Impact”, Bank of England Quarterly Bulletin 2011 Q3

Kapetanios, G., Price, S., G. Young. (2015), “A Financial Conditions Index Using Targeted Data Reduction”, Paper presented at the 9th International Conference on Computational and Financial Econometrics, December 12-14, 2015

Kengne, B.D.S., Miller, S., R, Gupta. (2013), “Evolution of Monetary Policy in the U.S.: The Role of Asset Prices”, University of Pretoria, Department of Economics Working Paper Series No. 2013-43

Ketelaere, B.D., Hubert, M. Schmitt. E. (2013), “A Review of PCA-based Statistical Process Monitoring Methods: for Time-dependent, High-dimensional Data”, Department of Mathematics, University of Leuven Working Paper Series, Available online at: https://wis.kuleuven.be/stat/robust/papers/2013/deketelaere-review.pdf Last accessed 5th Sep 2016

Keynes, J.M. (1937), *The Theory of Interest* in Lessons of Monetary Experience, New York: Farrar & Rinehart. Inc

Klein, P. (2000) "Using the Generalized Schur Form to Solve a Multivariate Linear Rational Model", *Journal of Economic Dynamics and Control*, vol. 24, no. 10, pp. 1405-1423.

Kim, C.J., Nelson, C.R. (2006), "Estimation of A Forward-looking Monetary Policy Rule: A Time-varying Parameter Model Using Ex-post Data", *Journal of Monetary Economics*, vol. 53, no. 8, pp. 1949-1966

Kim, D.H., Osborn, D.R., Sensier. M. (2005), "Nonlinearity in the Fed's Monetary Policy Rule", *Journal of Applied Econometrics*, vol. 20, no. 5, pp. 621-639

King, M. (1997), "Changes in UK Monetary Policy: Rules and Discretion in Practice", *Journal of Monetary Economics*, vol. 39, no. 1, pp. 81-97

King, R.G., Watson, M.W. (1994), "The Post-war U.S. Phillips Curve: a Revisionist Econometric History", *Carnegie-Rochester Conference Series on Public Policy*, vol. 41, pp. 157-219

Kiyotaki, N., Moore, J. (1997), "Credit Cycles", *Journal of Political Economy*, vol. 105, no. 2, pp. 211-248

Kontonikas, A., Montagnoli, A. (2006), "Optimal Monetary Policy and Asset Price Misalignments", *Scottish Journal of Political Economy*, vol. 53, no. 5, pp. 636-654

Koop, G., Korobilis, D. (2012), "Forecasting Inflation Using Dynamic Model Averaging", *International Economic Review*, vol. 53, no. 3, pp. 867-886

Koop, G., Korobilis, D. (2013), "Large time-varying parameters VARs", *Journal of Econometrics*, vol. 177, no. 2, pp. 185-198

Koop, G., Korobilis, D. (2014), "A New Index of Financial Conditions", *European Economic Review*, vol. 71, pp. 101-116

Kuzin, V. (2006), "The Inflation Aversion of the Bundesbank: A State Space Approach", *Journal of Economic Dynamics & Control*, vol. 30, no. 9-10, pp. 1671-1686

Kydland, F., Prescott, E.C. (1977), "Rules Rather than Discretion: The Inconsistency of Optimal Plans", *Journal of Political Economy,* vol. 85, no. 3, pp. 473-492

Kwiatkowski, D., Phillips, P., Schmidt, P., Y. Shin. (1992), "Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root: How Sure Are We that economic Time Series Have a Unit Root", *Journal of Econometrics,* vol. 54, no. 1-3, pp. 159-178

Lafuente, J.A., Perez, R., J, Ruiz. (2014), "Time-varying Inflation Targeting after the Nineties", *International Review of Economics and Finance,* vol. 29, pp. 400-408

Laidler, D. (1991), *Price Stabilisation in the 1990s: A Summary, Kumiharu Shigehara, (ed.) Price Stabilisation in the 1990s,* The MacMillan Press Ltd

Leamer, E.E. (1978), "Specification Searches", New York: John Wiley & Sons, Inc

Lean, W.V. (2000), "A Review of the Rules versus Discretion Debate in Monetary Policy", *Eastern Economic Journal,* vol. 26, no. 1, pp. 29-39

Leduc, S., Sill, K., T. Stark. (2007), "Self-fulfilling Expectations and the Inflation of the 1970s: Evidence from the Livingston Survey", *Journal of Monetary Economics,* vol. 54, no. 2, pp. 433-459

Leigh, D. (2008), "Estimating the Federal Reserve's implicit inflation target: A state space approach" *Journal of Economic Dynamics & Control,* vol. 32, no. 2013-2030

Leith, C., Moldovan, L., R. Rossi. (2009), "Optimal Monetary Policy in a New Keynesian Model with Habits in Consumption", European Central Bank Working Paper Series No. 1076

Lown, C.S., Rich. R.W. (1997), "Is There an Inflation Puzzle?", *Federal Reserve Bank of New York Economic Policy Review*, pp. 51-69

Lubik, T., Schorfheide, F. (2007), "Do Central Banks Respond to Exchange Rate Movements? A Structural Investigation", *Journal of Monetary Economies,* vol. 54, no. 4, pp. 1069-1087

Madigan, D., Raftery, A.E., (1994), "Model Selection and Accounting for Model Uncertainty in Graphical Models Using Occam's Window", *Journal of American Statistical Association*, vol. 89, no. 428, pp. 1535-1546

Malik, S., Banerjee, A. (2013), "The Changing Role of Expectations in US Monetary Policy: A New Look Using the Livingston Survey", Banque de France Working Paper Series No. 376

Martin, C., Milas, C. (2004), "Modelling Monetary Policy: Inflation Targeting in Practice", *Economica*, vol. 71, no. 282, pp. 209-221

Martin, C., Milas, C. (2013), "Financial Crises and Monetary Policy: Evidence from the UK", *Journal of Financial Stability*, vol. 9, no. 4, pp. 654-661

Mayes, D., Viren, M. (2001), "Financial Condition Indexes", Bank of Finland Working Paper Series No. 17

McCallum, B.T. (1988), "Robustness Properties of a Rule for Monetary Policy", *Carnegie-Rochester Conference Series on Public Policy*, vol. 29, pp. 173-204

McCallum, B.T. (1989), *Monetary Economics: Theory and Policy*, New York: Macmillan Publishing Company

McCallum, B.T. (1993), "Specification and Analysis of a Monetary Policy Rule for Japan", *Bank of Japan Monetary and Economic Studies*, vol. 11, no. 2, pp. 1-45

Meenagh, D., Minford, P., Nowell, E., Sofat, P., N. Srinivasan. (2009), "Can the Facts of UK Inflation be Explained by Nominal Rigidity", *Economic Modelling*, vol. 26, no. 5, pp. 978-992

Mehra, Y.P. (1999), "A Forward-Looking Monetary Policy Reaction Function", *Economic Quarterly – Federal Reserve Bank of Richmond*, vol. 85, no. 2, pp. 33-53

Melosi, L. (2012), "Signaling Effects of Monetary Policy", Federal Reserve Bank of Chicago Working Paper Series No. 2012-05

Meyer, L.H. (1997), "The Economic Outlook and Challenges for Monetary Policy", Remarks at the Charlotte Economics Club, N.C., 16th January

Monetary Policy Committee. (August 2013), "Monetary Policy Trade-offs and Forward Guidance", Available in PDF at: http://www.bankofengland.co.uk/publications/Documents/inflationreport/2013/ir13augforwardguidance.pdf. Last accessed 31st Mar 2014

Monetary Policy Committee. (June 2012), "The Transmission Mechanism of Monetary Policy", Available in PDF at: http://www.bankofengland.co.uk/publications/Documents/other/monetary/montrans.pdf. Last accessed 31st Mar 2014

Monetary Policy Committee. (March 2013), "Remit for the Monetary Policy Committee", Available in PDF at: https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/221566/chx_letter_to_boe_monetary_policy_framework_200313.pdf. Last accessed 31st Mar 2014

Montagnoli, A., Napolitano, O. (2005), "Financial Condition Index and Interest Rate Settings: A Comparative Analysis", Istituto di Studi Economici Working Paper Series No. 8

Nakajima, J. (2011a), "Time-Varying Parameter VAR Model with Stochastic Volatility: An Overview of Methodology and Empirical Applications", *Monetary and Economic Studies*, vol. 29, pp. 107-142

Nakajima, J. (2011b), "Monetary Policy Transmission under Zero Interest Rates: An Extended Time-Varying Parameter Vector Autoregression Approach", Department of Statistical Science, Duke University Working Paper Series No. 11-08

Nakajima, J., Kasuya, M., T, Watanable. (2011), "Bayesian Analysis of Time-varying Parameter Vector Autoregressive Model for the Japanese Economy and Monetary Policy", *Journal of the Japanese and International Economies*, vol. 25, no. 3, pp. 225-245

Nakajima, J. (2013), "Time-varying Parameter VAR Model Using TVP-VAR Package (for Matlab)", Available in PDF at https://sites.google.com/site/jnakajimaweb/tvpvar [Assessed on August 20th 2016]

Negro, M.N., Otrok, C. (2008), "Dynamic Factor Models with Time-varying Parameters: Measuring Changes in International Business Cycles", Federal Reserve Bank of New York Staff Reports No. 326

Nelson, E. (2000), "UK Monetary Policy 1972-1997: A Guide Using Taylor Rules", Bank of England Working Paper Series No. 120

Neumann, M.J.M., Hagen, J.V. (2002), "Does Inflation Targeting Matter", *The Federal Reserve Bank of St. Louis*, vol. 84, no. 4, pp. 127-148

Newey, W., West, K. (1987), "A Simple Positive Semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix", *Econometrica*, vol. 55, no. 3, pp. 703-708

Onorante, L., Raftery, A.E., (2014), "Dynamic Model Averaging in Large Model Spaces Using Dynamic Occam's Window", Available in PDF at Cornell University Library: http://arxiv.org/abs/1410.7799 [Assessed on 9th January 2016]

Orphanides, A. (2001), "Monetary Policy Rules Based on Real-time data", *American Economic Review*, vol. 91, no. 4, pp. 964-985

Orphanides, A. (2003), "Historical Monetary Policy Analysis and the Taylor Rule", *Journal o Monetary Econometrics*, vol. 50, no. 5, pp. 983-1022

Orphanides, A. (2007), "Taylor Rules", Finance and Economics Discussion Series No. 2007-18

Orphanides, A., Williams, J.C. (2002), "Robust Monetary Policy Rules with Unknown Natural Rates", *Brookings Papers on Economic Activity*, vol. 2002, no. 2, pp. 63-118

Osterholm, P. (2005), "The Taylor and Real-time Data – A Critical Appraisal", *Applied Economics Letters*, vol. 12, no. 11, pp. 679-685

Owyang, M.T., Ramey, G. (2003), "Regime Switching and Monetary Policy Measurement", The Federal Reserve Bank of St. Louis Working Paper Series No. 2001-002B

Paries, M.D., Maurin, L., D. Moccero. (2014), "Financial Conditions Index and Credit Supply Shocks for the Euro Area", European Central Bank Working Paper Series No. 1644

Parzen, E. (1962), "On the Estimation of A Probability Density Function and the Mode", *Annals of Mathematical Statistics*, vol. 33, no. 3, pp. 1065-1076

Petersen, L. (2007), "Does the Federal Reserve Follow A Non-linear Taylor Rule?", University of Connecticut Working Paper Series No.2007-37

Phillips, A.W. (1958), "The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957", *Economica*, vol. 25, no. 100, pp. 283-299

Phillips, P.C.B., Perron, P. (1988). "Testing for a Unit Root in Time Series Regression", *Biometrika*, vol. 75, no. 2, pp. 335-346

Primiceri, G.E. (2005), "Time Varying Structural Vector Autoregressions and Monetary Policy", *Review of Economic Studies*, vol. 72, no. 3, pp. 821-852

Price, S., Kapetanios, G., G. Yong. (2015), "A Financial Condition Index Using Targeted Data Reduction", Paper presented at the Department of Economics Spring Term Seminar Series 2015, City University London, June 3rd, 2015

Raftery, A.E., Madigan, D., J.A. Hoeting. (1997), "Bayesian Model Averaging for Linear Regression Models", *Journal of the American Statistical Association*, vol. 92, no. 437, pp. 179-191

Raftery, A.E., Karny, M., Andrysek, J., P. Ettler. (2007), "Online Prediction under Model Uncertainty via Dynamic Model Averaging: Application to A Cold Rolling Mill", Department of Statistics University of Washington Technical Report No. 525

Raftery, A.E., Karny, M. P. Ettler. (2010), "Online Prediction Under Model Uncertainty via Dynamic Model Averaging: Application to a Cold Rolling Mill", *Technometrics*, vol. 52, no. 1, pp. 52-66

Reifschneider, D., Tetlow, R., J. Williams. (1999), "Aggressive Disturbance, Monetary Policy and the Macroeconomic: the FRB/US Perspective", Federal Reserve Bulletin, January 1999

Risk-Metrics. (1996), "Technical Document", fourth ed. Available in PDF at: http://www.riskmetrics.com/system/files/private/td4e.pdf [Assessed on 4th August 2015]

Roos, C.F., Szeliski, V.V. (1942), "The Determination of Interest Rates", *Journal of Political Economy*, vol. 50, no. 4, pp. 501-535

Rotemberg, J.J. (1982), "Sticky Prices in the United States", *Journal of Political Economy*, vol. 90, no. 6, pp. 1187-1211

Rudebusch, G.D. (1998), "Do Measures of Monetary Policy in a Var Make Sense?", *International Economic Review*, vol. 39, no. 4, pp. 907-931

Rudebusch, G.D. (2005), "Assessing the Lucas Critique in Monetary Policy Models", *Journal of Money, Credit, and Banking*, vol. 37, no. 2, pp. 245-272

Rudebusch, G.D., Svensson, L. (1999), "Policy Rules for Inflation Targeting", NBER Working Paper Series No. 6512

Sack, B. (1998), "Uncertainty, Learning and Gradual Monetary Policy", FEDS Working Paper Series No. 98-34

Salemi, M.K. (2006), "Econometric Policy Evaluation and Inverse Control", *Journal of Money, Credit and Banking*, vol. 38, no. 7, pp. 1737-1764

Samuelson, P.A., Solow, R.M. (1960), "Analytical Aspects of Anti-Inflation Policy", *The American Economic Review*, vol. 50, no. 2, pp. 177-194

Sauer, S., Sturm, J.E. (2007), "Using Taylor rules to Understand the ECB Monetary Policy", *German Economic Review*, vol. 8, no. 3, pp. 375–398

Savage, L.J. (1954), *The Foundations of Statistics*, New York: John Wiley & Sons Inc

Savage, L.J. (1972), *The Foundations of Statistics*, New York: Dover Publications Inc

Shephard, N., Pitt, M. (1997), “Likelihood Analysis of Non-Gaussian Measurement Time Series”, *Biometrika*, vol. 84, no. 3, pp. 653-667

Simons, H.C. (1936), “Rules verse Authorities in Monetary Policy”, *Journal of Political Economy*, vol. 44, no. 1, pp. 1-30

Sims, C.A., (1980), “Macroeconomics and Reality”, *Econometrica*, vol. 48, no. 1, pp. 1-48

Sims, E. (2010), “The Simple New Keynesian Model”, The University of Notre Dame, Available in PDF at: http://www3.nd.edu/~esims1/new_keynesian_model.pdf

Steinsson, J. (2003), “Optimal Monetary Policy in an Economy with Inflation Persistence”, *Journal of Monetary Economics*, vol. 50, no. 7, pp. 1425-1456

Stiglitz, J.E. (1979), “Equilibrium in Product Markets with Imperfect Information”, *The American Economic Review*, vol. 69, no. 2, pp. 339-345

Stock, J.H., Watson, M.W. (1999), “Forecasting Inflation”, *Journal of Monetary Economics*, vol. 44, no. 2, pp. 293-335

Stock, J.H., Watson, M.W. (2002), “Macroeconomic Forecasting Using Diffusion Index”, *Journal of Business & Economic Statistics*, vol. 20, no. 2, pp. 147-162.

Surico, P. (2007), “The Monetary Policy of the European Central Bank”, *Scandinavian Journal of Economics*, vol. 109, no. 1, pp. 115-135

Svensson, L.E.O. (1999), “Inflation Targeting as a Monetary Policy Rule”, *Journal of Monetary Economics*, vol. 43, no. 3, pp. 607-654

Swanson, E.T., Williams, J.C. (2013), “Measuring the Effect of the Zero Lower Bound on Yields and Exchange Rates in the UK and Germany”, Federal Reserve Bank of San Francisco Working Paper Series No. 2013-21

Taylor, J.B. (1980), “Aggregate Dynamics and Staggered Contracts”, *Journal of Political Economy*, vol. 88, no. 1, pp. 1-23

Taylor, J.B. (1993), “Discretion versus Policy Rules in Practice”, *Carnegie-Rochester Conference Series on Public Policy*, vol. 39, pp. 195-214

Taylor, J.B. (1998), "A Historical Analysis of Monetary Policy Rules", NBER Working Paper Series No. 6768

Taylor, J.B. (1999), "The Role of the Exchange Rate in Monetary-Policy Rules", *Policy Implications of Macroeconomic Research*, vol. 91, no. 2, pp. 263-267

Taylor, J.B. (2008), "Monetary Policy and the State of the Economy", Testimony before the Committee on Financial Services, U.S House of Representatives February 26th, 2008

Teranishi, Y. (2012), "Credit Spread and Monetary Policy", *Economics Letters*, vol. 114, no. 1, pp. 26-28

The U.S, Department of Economic and Social Affairs. (1999), "World Economic Situation and Prospects for 1999", Available online in PDF at: http://www.un.org/en/development/desa/policy/wesp/wesp_archive/1999wesp.pdf. Last accessed 31st Sep 2014

Trecroci, C., Vassalli, M. (2010), "Monetary Policy Regime Shifts: New Evidence from Time-varying Interest Rate Rules", *Economic Inquiry*, vol. 48, no. 4, pp. 933-950

Trehan, B., Wu, T. (2007), "Time-varying Equilibrium Real Rates and Monetary Policy Analysis", *Journal of Economic Dynamics & Control*, vol. 31, no. 5, pp. 1584-1609

Wacker, K.M., Lodge, D., G. Nicoletti. (2014), "Measuring Financial Conditions in Major Non-Euro Area Economics", European Central Bank Working Paper Series No. 1743

Watanabe, T., Omori, Y. (2004), "A Multi-mover Sampler for Estimating Non-Gaussian Time Series Models: Comments on Shephard and Pitt (1997)", *Biometrika*, vol. 91, no. 1, pp. 246-248

Wesche, K.A. (2006), "Estimating Central Banks' Preference from a Time-varying Empirical Reaction Function", *European Economic Review*, vol. 50, no. 8, pp. 1951-1974

Wicksell, K. (1898), *Interest and Prices,* London: Macmillan

William, J.C. (2014), "Monetary Policy at the Zero Lower Bound Putting Theory into Practice", Federal Reserve Bank of San Francisco, Available online in PDF at: http://www.brookings.edu/~/media/research/files/papers/2014/01/16-monetary-policy-zero-lower-bound/16-monetary-policy-zero-lower-bound-williams.pdf. Last accessed 7th Jun 2016

Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy,* Princeton, NJ: Princeton University Press

Yuksel, E., Ozcan, K.M., O. Hatipoglu. (2013), "A Survey on Time-varying Parameter Taylor Rule: A Model Modified with Interest Rate Pass-through", *Economic Systems*, vol. 37, no. 1, pp. 122-134

Zheng, G., Wang, Y. (2014), "Financial Conditions Index's Construction and Its Application on Financial Monitoring and Economic Forecasting", *Procedia Computer Science*, vol. 31, pp. 32-39